P. R. Z. Royal.

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CONTENTS.

BAKER, J. G. :— PAGE

On the Fern Flora of the Seychelles. (*With Plates XXVIII. to XXXI.*), ... 509

[Published August 26, 1875.]

BALL, R. S. :—

Account of Experiments upon the Retardation experienced by Vortex Rings of Air, when moving through Air. (*With Plates IV. to X.*), 135

[Published August, 1872.]

The Theory of Screws: a Geometrical Study of Kinematics, Equilibrium and small Oscillations of a Rigid Body, (*With Plates XI. and XII.*), ... 157

[Published December, 1872.]

Screw Co-ordinates, and their Applications to Problems in the Dynamics of a Rigid Body, ... 295

[Published May, 1874.]

COLLINS, E. W. :—

On Accessory Lobes of the Human Lungs. (*With Plate XVI.*), ... 329

[Published May, 1874.]

DOBERCK, W. :—

On the First Comet of 1845, ... 459

[Published April 12, 1875.]

On μ^2 *Bootis* (*considered as a Revolving Double Star*), ... 481

[Published April 12, 1875.]

On the Binary Stars, σ *Coronæ,* τ *Ophiuchi,* γ *Leonis,* ζ *Aquarii,* 36 *Andromedæ, and* ι *Leonis,* 581

[Published August, 1875.]

HART, A. S.:— PAGE

On the Nine-point Contact of Cubic Curves, ... 559
[Published May 24, 1875.]

JELLETT, J. H.:—

Researches in Chemical Optics, ... 371
[Published February 8, 1875.]

KING, W.:—

Report on the Superinduced Divisional Structure of Rocks, called Jointing, and its relation to Slaty Cleavage. (*With Plates XXXIV. to XXXVIII.*), ... 605
[Published November 8, 1875.]

MACALISTER, A.:—

Observations on Muscular Anomalies in Human Anatomy. (*Third Series.*) *With a Catalogue of the Principal Muscular Variations hitherto published.* (*With Plates I. to III.*), ... 1
[Published August, 1872.]

A Monograph of the Anatomy of Chlamydophorus truncatus (Harl.); *with Notes on the Structure of other Species of Edentata.* (*With Plates XIII. to XV.*), ... 219
[Published April, 1873.]

Report on the Anatomy of the Insectivorous Edentates. (*With Plates XXVI. and XXVII.*), ... 491
[Published April 12, 1875.]

MACKINTOSH, H. W.:—

Researches on the Structure of the Spines of the Diadematidæ (Peters). (*With Plates XXXI.* to XXXIII.*), ... 519
[Published August, 1875.]

MALET, J. C.:—

Some Theorems in the Reduction of Hyper-elliptic Integrals, ... 279
[Published April, 1874.]

On certain Symmetric Functions of the Roots of an Algebraic Equation, 337
[Published June 22, 1874.]

M'NAB, W. R.:—

Experiments on the Movements of Water in Plants. Part I. (*With Plates XVII. to XXIV.*), ... 343
[Published July 6, 1874.]

M'NAB, W. R. :— PAGE

Experiments on the Movements of Water in Plants. Part II., ... 567

[Published June 21, 1875.]

STONEY, B. B. :—

Report on the Strength of Single-riveted Lap Joints. (With Tables, and Plate XXV.), ... 451

[Published February 8, 1875.]

*** The date of reading will be found attached to each Paper. The date of publication, as here inserted, is that when the Paper was laid in a printed form on the Table of the Academy; or, when the Academy was not sitting, the date when it was forwarded to the Academy's Booksellers.

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TRANSACTIONS

OF THE

ROYAL IRISH ACADEMY.

I.—*Additional Observations on Muscular Anomalies in Human Anatomy (Third Series), with a Catalogue of the Principal Muscular Variations hitherto published.* By ALEXANDER MACALISTER, L. R. C. S. I., L. M., T. C. D., Professor of Zoology, University of Dublin.

Read January 23, 1871.

IN the Proceedings of the Royal Irish Academy for 1866 and 1867, I have given a detailed account of two series of Muscular Anomalies which I have met with in the Dissecting-Room of the Royal College of Surgeons in Dublin. Since the publication of the last of these Papers I have met with several new and important deviations from the ordinary muscular arrangements, which are embodied in the present Paper.

As the literature of anomalous Myology is very extensive and peculiarly diffuse, some of the most curious and significant forms of muscle-variations being described in obscure or local journals, I have felt in working up this subject a great want of a catalogue of the chief anomalies hitherto described, and for my own convenience I have been at considerable pains and trouble to prepare a list of all the anomalies hitherto described with which I have become acquainted, and as this has not been done elsewhere, I trust that it may be found by anatomists to be as useful in anomalous Myology as Krause and Telgmann's work is in anomalous Neurology, or any of the numerous catalogues of anomalous Angeiology. This catalogue I have embodied in this Paper, as I

have in my experience of the dissection of over six hundred and ninety subjects seen and verified a very large number of those observed by others. The list does not pretend to be a complete one of all the existing anomalies in Myology, but I trust that it may be accepted by anatomists as an attempt to systematize a host of desultory observations, and to indicate briefly the relations of the different anomalies of individual muscles. For purposes of convenience I have drawn up this list in a strictly anatomical order, and thus have departed from the method laid down in my other Paper. I have not added any comments or comparative anatomy notes, except in a few instances, as such additions would have swelled this paper to an unseemly bulk, nor have I appended any remarks on muscular homologies, as I wish to make this Paper as practical and as purely a record of facts as possible.

MUSCLES OF THE HEAD AND NECK.

1. The frontalis muscle is constant in its origin from the galea aponeurotica, but I have found it to vary in extent, volume, and insertion. I have never met with any case in which this muscle was directly continuous with the occipitalis, but such an occurrence is described by Mayer (tom. iii. p. 159). 2. I have seen the fleshy portions of muscles of each side united together in the median line for nearly their whole length, as described by Sömmerring. 3. I have found its position of origin to vary, sometimes being opposite the line of the coronal suture, but far more frequently falling in front of this line, and most commonly the fleshy fibres begin midway between the coronal suture and the supra-orbital ridge. 4. The muscle may be thick and in separate fascicles, instead of as usual forming a continuous sheet. Its insertions are variable, and a singular discrepancy exists in the works of anatomists as to the normal anterior attachment of the muscle. Thus, our latest and best authority on the subject, Professor Halbertsma ("Musculus Frontalis," in "Verslagen en Mededeelingen der k. Akademie van Wetenschappen natuurk. Deel," vii., and Archiv. Bd. ii. Hft. i., p. 48), gives as the normal insertion the internal angular process of the os frontis, the skin of the eyebrow, a slip to the levator labii superioris alæque nasi and the integument of the forehead; to these I would add, with Wilson, the deep surface of the orbicularis palpebrarum. These are the commonest anterior insertion points; but superadded to or in place of some of these we often find

other arrangements: thus it may be attached (5) to the supraorbital margin, as seen by Arnold, Bock, and myself; (6) to the prominent superciliary arch, described by Bock, Fles, Hyrtl, Langenbeck, Meyer, and Weber; (7) to the glabella, by Arnold, Fles, and Hyrtl; (8) to the nasal bone itself on one or both sides, Arnold, Bock, Langenbeck, Böhmer, Rosenmüller, Theile, Henle, and Weber; (9) to the corrugator supercilii (10); into the fronto-maxillary suture. 11. I have found it inserted into the orbicularis palpebrarum alone, and into the integuments of the eyebrow. 12. It has often a slip into the nasal process of the superior maxillary bone—this I have seen, and its occurrence has been noted by Meckel, Bock, and Böhm. 13. The slip to the levator labii superioris alæque nasi is very constant, and should rather be reckoned as an ordinary insertion than as an anomaly; it is described by Lauth, Langenbeck, Rosenmüller, Theile, Albinus and Courcelles. 14. A slip to the upper surface of the tendo oculi deserves also to be ranked as an interesting anomaly, and one of rare occurrence. 15. A complete suppression of this muscle I have once seen, which is a condition not before described. 16. Theile has recorded the existence of a slip from the temporal crest of the frontal bone as an external supplement to this muscle, a condition I have never seen. 17. An external fascicle has been found passing to the external angular process of the frontal bone. 18. A decussation of the lowest fibres, as described by Henle (p. 135), I have also seen.

Occipitalis I have found less variable than the frontalis. 1. Suppression of it I have once seen, and the occurrence of this anomaly was noticed by Cassebohm. 2. A condition of fasciculation is very common in it, and is a step towards the rarer anomaly, No. 1. 3. A variety of this form of fasciculation is the splitting into superior and inferior portions as described by Portal ("Cours d'Anat. Med.": Paris, 1803, tom. II., p. 50), but this inferior part seems to be the upper portion of the retrahens aurem. 4. I have never seen a union of these muscles in the middle line, but they sometimes approach very closely, so as to be partly contiguous, or they may decussate ("Sömmerring," p. 79). 5. I have seen it united to the retrahens aurem, as described by Lieutaud, p. 121, or more commonly crossed by it, or by the next muscle. 6. Fibres pass occasionally from the insertion of the sterno-cleido-mastoid to this muscle.

Transversus nuchæ is a muscle which, though not described in the ordinary

anatomical text-books, yet it is by no means a rare constituent of the human body. It has been first systematically described by Professor F. Eilhard Schultze, of Rostock ("Schmidt's Jahrbuch," Bd. cxxvii., p. 288). It usually runs from the occipital protuberance and the portion of the superior curved occipital line contiguous to this process, and passing outwards, varies in its mode of termination. Thus it may be inserted (1) into the outer part of the superior occipital line; (2) into the posterior border of the sterno-cleido-mastoid muscle or of its insertion tendon; (3) into the retrahens aurem muscle, and when this variety is present, as the junction-line of the two muscles is tendinous, it gives the retrahens a digastric appearance. Cases of this kind have long been known, and are mentioned by Henle ("Muskellehre," p. 138). Cases without the tendinous inscription of the origin of the whole of the retrahens from the occipital protuberance are mentioned by Gibson ("Anatomy," 1716, p. 489); Hallett ("Ed. Med. Journal," 1849, p. 2). 4. The transversus nuchæ may, in the whole or in part, underlie the trapezius, and in that case it may run (*a*) from the occipital crest to the insertion of the splenius, or (*b*) likewise into the retrahens aurem, or into the sterno-mastoid; cases of this kind have been seen by Walther and Sömmerring (vol. iii., p. 81), or (*c*) interlacing with and inserted into the fibres of the trapezius at its occipital origin, which I believe to be the commonest form of arrangement, when the muscle is deeply seated and not extending as far outward as the sterno-mastoid. This muscle is always symmetrical, and the two opposite sides usually are connected in the middle line. Professor Theile regards the lower portion of the muscle, or that which passes to the sterno-cleido-mastoid insertion, as being the most important, and hence he objects to the name given by Schultze ("Schmidt," p. 288). Professor Schultze has met with this muscle in eighteen out of twenty-five subjects in some form or other. My own experience does not lead me to believe that this is a common degree of frequency, and I would rather regard seven out of twenty as its rate of occurrence in Irish subjects. Often, when it is absent, a tendinous, band takes its place across the level of the superior curved line of the occipital bone.

Retrahens aurem I have found in many conditions: (1) rarely as a continuous undivided sheet; (2) most commonly in two unequal fasciculi; of these the lower is that most commonly joined to the preceding, and the frequency of that fusion is, I believe, correctly stated by Hallett (loc. cit., p. 2), as five times

in two hundred subjects; but this occurs in females as well as males, though certainly it is more frequent in the latter sex. Hallett never found this slip but in robust males. 3. Doubling of the muscle is described by M'Whinnie ("Lond. Med. Gaz.," 1846, p. 185), and by Walther ("Disput. Anat. Select.," Göttingen, 1751, vol. vii., p. 615); and the latter saw (4) this muscle receiving fibres from the sterno-cleido-mastoid muscle—a condition likewise found by Sömmerring. 5. Origin of the whole muscle from the transversus nuchæ has been seen by Hallett, but this is rare. 6. I have seen it arising not only from the mastoid process, but by its lower fascicle from the cervical fascia. 7. It is described as having its sole origin from the latter in Rowley's "Schola Medicinæ nova Universalis," 1794, vol. i., p. 48. 8. I have seen it continuous with the occipito-frontalis, as described by Valsalva ("De Aure," cap. i., p. 11). 9. It may have three fascicles, a not uncommon arrangement figured and described by Casserius (Tab. 2, Fig. 1), Sandifort ("Historia Muscul.," p. 65). 10. Or it may be in four fascicles, still rarer, described by Valsalva (loc. cit.), Morgagni (Epist. iv., No. 4), Sömmerring, (p. 81), and Haller ("Elem. Physiol.," lib. xv., p. 191.) 11. Its origin when thus multiple may extend to the external part of the occipital bone, as described by Sandifort. 12. Duverney describes it as having been seen in five or six bands—a degree of fasciculation I have never met with. 13. Its absence is very rare, but I have occasionally met with this anomaly. 15. It may also be replaced by a tendinous band.

Attollens aurem is subject to few varieties: 1. I have noticed its absence; 2. Its being split into several fascicles; 3. Its arising as far as the galea aponeurotica; 4. Or only being attached to the temporal aponeurosis; 5. Or its being continued by a few scattered fibres to the next muscle; 6. Or to the transversus auriculæ; 7. Many fibres often stop short of the auricle, and are inserted into the temporal aponeurosis. Attrahens aurem is more fertile in anomalies; but from my dissections, I think it to be more deserving of the name given to it by Henle—epicranio-temporalis—than that by which it is usually described, as I have rarely found a sufficiently pronounced ear-insertion to justify the common name. Its commonest origin is from the anterior part of the galea aponeurotica, higher than usually represented in most of the text-books, and when as strongly marked as represented in the plate in "Gray's Anatomy" (Fig. 148), it is nearly an inch higher than it is there pourtrayed as

being. I have never seen it as described by Harrison ("Dublin Dissector"), Wharton Jones ("Todd's Cyclopædia," vol. II., p. 352), and others arising from the zygomatic arch, and inserted by a short tendon into the anterior surface of the helix; this form is, however, described by Cruveilhier, as a "Musculus Auricularis anterior profundus" (Henle, p. 138). This occurrence I would look upon as a very rare form of anomaly. I have seldom found its ear-attachment stronger than a lamina of condensed areolar tissue. In no case have I ever seen any fibres arising from the zygomatic arch, and very rarely a weak band from the temporal crest on the frontal bone. It has been seen in two thin strata (Walther "Anat. Musc. Tenuior," in Haller's "Disput. Anat. Select.," vol. VI., p. 613); but in the only case of this that has occurred to me, the superficial stratum was little better than a layer of areolar tissue with a few muscular fibres in it. More commonly it is divided into several unconnected fasciculi on the same plane; occasionally it gets some fibres from the temporal aponeurosis, and it has been seen joined to the last described muscle. Perfect suppression is commoner in the case of this muscle than in either of the two other auricular muscles. In some cases of doubling of this muscle one portion has been found inserted into the helix and the other into the concha. Henle found a fascicle connecting the lower border of this muscle with the platysma (loc. cit., p. 137), which is interesting as showing the relation between the platysma proper and the occipito-frontal group of muscles.

The proper muscles of the auricle are seldom sufficiently developed to present much in the way of variety. Absence of one or more of them is the most constant variety which I have met with; the order of constancy seems to be—1. Helicis major; 2. Tragicus; 3. Antitragicus; 4. Helicis minor; 5. Transversus auriculæ. Not uncommonly the muscular fibres of the latter are replaced by tendinous threads. The tragicus not unfrequently sends a slip to the helix; or may be united to the helicis major—a condition described by Sömmerring ("De Corp. Hum. Fabric.," III., p. 32), and Albinus ("Hist. Musculorum," p. 183). The helicis major and minor may also be united.

A very singular ear muscle was described by Garengeot ("Myotomie," p. 68), by Lauth ("Mem. de la Société d' Histoire Naturelle de Strasbourg," tom. I., 1830, p. 65, and Souchon and Rambaud ("Gaz. Med.," 1861, No. 37, p. 585), arising from the styloid process, and passing below the parotid to the

external meatus, where its cartilaginous and osseous portions unite, as well as to the cartilaginous part itself. The same muscle is described in 1749, by Duverney ("L'Art de Dissequer Methodiquement les Muscles du Corps.Humain," Paris, pp. 11, 12). This muscle was again found by Hyrtl ("Œsterreich. Med. Jahrbuch," vol. xxx., 1840, p. 345; and Lehrbuch, I., Aufl., p. 291, and 5 Aufl., p. 329), and described by him as the stylo-auricularis. Professor Wenzel Gruber, of St. Petersburg, has considered this as an auricular slip to the stylo-glossus, and has described it ("Bulletin math. Class. Akad. der Wissensch," in St. Petersburg, 1854, vol. XIII., p. 257, and vol. xv., p. 206, "Henle u. Pfeufer's Zeitschrift," 3, Reihe x., p. 368). I have never been fortunate enough to find a well-marked instance of this remarkable muscle, but its place is often occupied by a fibrous cord. It has been found by these authors monogastric, with a tendinous or with a fleshy origin, or digastric, or as a continuous slip passing from the ear to the tongue, auriculo-glossus, or bicipital, or represented by a double tendon, or by a single belly with two tendons. Lauth calls it depressor auriculæ.

Of the internal ear muscles, the only anomalies with which I have met are—1. Doubling of the stapedius muscle; 2. Absence of the laxator tympani major. Doubling of the tensor tympani has been met with by Böhmer (p. 7.), Cassebohm ("De Aure Humani," tract. IV., p. 64). Sömmerring describes a slip from the laxator minor as being continued to the apex of the malleus (p. 84). This muscle may be absent also.

Of the internal orbital muscles few anomalies have occurred within my experience; indeed in this simple and definite group of muscles very few abnormalities have ever been described. The levator palpebræ superioris I have seen joined to the superior rectus at its origin as described by Albinus; and Dr. Kelly, Demonstrator of Anatomy at the College of Surgeons, has communicated to me a case in which this muscle arose from the posterior margin of the frontal part of the roof of the orbit, in front of, and above the optic foramen, separate from the other muscles. Doubling of this muscle was described by Albinus, the second muscle (comes obliqui superioris) being slender. This accessory oblique has been described by Bochdalek, Jun., of Prag., as the gracillimus orbitis ("Prager Vierteljahrsschrift," IV., 1868); it is also mentioned by Sandifort ("Exercitationes Anat. Lugd." Batav., 1783, p. 80).

Still earlier even than this we find Vesalius noticing an occasional duplicity

of this muscle. A slip of this nature was shown to Molinetti, by Boldrini ("Dissert. Anat. et Pathol.," Patav., 1669, p. 25), and was described by him as a fifth rectus; this muscle is inserted into the trochlea according to this author. Another case of the same kind is mentioned by Albinus (p. 176), and Kulmus ("Descriptio Anatomica fœtus Monstrosi," 1724, p. 14).

A muscle has been more recently described as the tensor trochleæ, by Professor Julius Budge ("Henle u. Pfeufer's Zeitschrift," Reihe 3, Bd. VII., p. 273), which is a lateral offshoot from the levator palpebræ superioris inserted by several slips into the trochlea for the superior oblique tendon. This I have met with, and it is to be regarded as the same muscle as the muscle of Albinus, Molinetti, and Kulmus. I have found it in two stages, as an external slip from the outer border of the levator palpebræ, only separating from the rest of the muscle at its insertion, and as a slip separate for the anterior two-thirds of the orbit. The instances described by Budge, of still further differentiation, connect these to the perfectly distinct tensor trochleæ of Molinetti. A band from the insertion of the obliquus superior to that of the inferior is described by some of the older anatomists, but their descriptions are not always to be relied on; for we find Realde Columbus unacquainted with the trochlea of the superior oblique, his tertius oculi ("De Re Anatomica," lib. v., cap. 8).

In one of my former papers I described a case of absence of the levator palpebræ superioris existing in a subject without ptosis. I have seen it attached, α. To the upper margin of the tarsal cartilage; β. Into the tarsal cartilage, and into the conjunctiva of the superior palpebral sinus; γ. Into the latter alone; δ. Into the ciliary fibres of the orbicularis palpebrarum; ϵ. Or a triple insertion into all three.

A fusion of the recti internus and inferior at their origin always exists, but its extent is variable. I have seen them united for the entire posterior third of the orbit. The rectus externus I have seen with its two heads closely joined; more rarely I have found them farther sundered than usual, but never to the extent described by Zagorsky and Albinus, who found a double external rectus. Absence of the outer head of this muscle I have at least twice noticed. I have never seen any of the recti suppressed, but in some rare cases of divergent strabismus, the internal rectus is absent; and in equally rare cases of con-

vergent squint, the external is absent. Instances of this nature are described by Wrisberg "Götting. Gelehrten Anzeigen," 1781, p. 683; absence of all the eye muscles is described by Klincosch of Prag. (quoted by Otto "Patholog. Anat.," South's translation, p. 243). A depressor palpebræ inferioris was met with by Caldani, "Institut. Anatomic.," tom. XI., p. 41, consisting of a superficial band of fibres inserted into the tarsal cartilage of the lower lid and lying over the orbicularis palpebrarum. This I think is nothing but an upward prolongation of the platysma, and as such I have referred to instances like it under the head of platysma. I have never met with this anomaly.

An additional orbital muscle, the musculus transversus orbitis, is a singular structure described by Professor Bochdalek, of Prag.—passing from the anterior and superior portion of the os planum, across the upper surface of the eyeball to the outer wall of the orbit, closely connected to the levator palpebræ superioris. This singular band seems to be a displaced deep slip of the upper arcuated fibres of the palpebral part of the orbicularis palpebrarum ("Prager—Vierteljahrsschrift," IV., 1868). I have found deep fascicles of the orbital portion of the orbicularis nearly in the same situation, but they, and the fibres of the palpebral segment of the muscles, have always been external to the levator.

Among the facial muscles many anomalies have occurred in the past two years; indeed they are singularly prone to variety, as the following list will show.

The orbicularis palpebrarum is very variable, and in nothing is this variability better shown than in the very different methods of division adopted by anatomists in describing it. Thus, among the more modern authors, we find Malgaigne dividing it into orbital ("external" of Riolan) palpebral, and ciliary; Arlt ("Ueber den Ringmuskel der Augenlieder," in "Gräfe, Archiv. für Ophthalmologie," Bd. IX., 1ste Abth., p. 64), divides it into, 1st. The peripheral or accessory; 2nd. The part from the orbital margin; 3rd, from the ligamentum palpebrale; 4th. From the lachrymal bone. On careful examination in general it is possible to separate this muscle into, 1st. A slip not extending as far as the external canthus, and lying close to the tarsal cartilage; this is the pars subtarsalis of Möll ("Bydragen tot der Anatomie en Physiologie den Oogleden"); 2nd. The ciliary, (Riolan "Anthropographia," Lib. v., cap. 2); 3rd. Palpebral; 4th. Proper orbital; and, 5th. Extra-orbital (Richet). Besides these normal parts, there are two extra muscles which are variable in their frequency of

existence, and in the amount of their union with the proper orbicularis; these are, 1st. The tensor tarsi, described by Duverney ("L'Art de Dissequer Methodiquement les Muscles du Corps Humain," Paris, 1749, p. 37); by Rosenmüller, as musculus sacci lacrymalis ("Compendium Anatomicum," Lipsiæ, 1816, p. 241); and afterwards by Horner. This muscle is also known by the names lacrymalis posterior (Olschewsky and Henke); dilatateur inferieur du sac (Bourjut St. Hilaire); lacrymalis (Krause, Arnold). It has been seen by me doubled, completely separate from the orbicularis, and very rarely suppressed; it is usually more or less united to the deep surface of the orbicularis; 2nd. The depressor of the eyebrows, of Arlt. (loc. cit., p. 76), a triangular muscle with its apex below and its base above, situated under the skin of the inner half of the brow; its origin is below at the apex from above the ligamentum palpebrale internum, and its insertion into the skin along its basal line. This muscle is described also by Professor Lesshaft, now of Kasan, in his admirable and comprehensive paper, "Ueber den Musculus Orbicularis Orbitæ" ("Reichert u. Du Bois Reymond, Archiv.," 1868, p. 265), as the depressor supercilii. This is the same as the "dilatateur superieur" of the lacrymal sac described by Bourjut St. Hilaire ("Considerations Generales sur les Voies lacrymales," "Journal des Connaissances Medico-chirurg.," Feb., 1853), but he considers its base as its origin, and its apex as its insertion; and it is also the same as the lacrymalis anterior of Henke ("Gräfe's Archiv.," Bd. iv., 2te Abth., p. 73). This muscle is by no means constant, indeed its presence as a distinct muscle cannot be demonstrated in the majority of subjects. Of anomalies in the orbicularis muscle itself, the following have been met with:—1. A fascicle surrounding the lacus lacrymalis is described by Theile, and this slip may be attached to the border of the lid alone, or to the lacrymal canal (Foltz "Journal de la Physiologie," v., p. 226). This may be joined to Horner's muscle, or may co-exist with it separately; 2. A slip into zygomaticus minor—this is very common, and is figured by Eustachius (fig. 1, A. B.); 3. The entire zygomaticus minor may arise from the orbicularis, as I have seen; 4. Or a slip from this muscle may pass parallel and internal to the zygomaticus, and perfectly distinct from it (Santorini, "Observ. Anat. Venetiis," 1724, p. 9; Walther, "Teniorum Muscul. Anat. repetita" in Haller's "Disput. Anat. Select.; "p. 607, vol. vi.); this I have also seen. This slip has been seen going to the musculus incisorius

by Walther (loc. cit.); 6. And the same author describes having seen the peripheral fibres of the orbicularis continuous from those of the corrugator supercilii—this observation supports the opinion of Fabricius, Langer, and others, that the corrugator and orbicularis are parts of the same muscle, an opinion with which I cannot agree; 7. A slip to the levator labii superioris is mentioned by Henle; 8. An absence of the peripheral fibres I have once seen; 9. A slip of the platysma continued into it, I have several times noted; and I would regard the depressor palpebræ inferioris of Caldani as an anomaly of this kind, in which the platysmal slip overlay instead of fusing with the orbicularis; 10. Varieties of cleaving in the muscle are often seen thus:—I have seen the Extra-orbital fibres very sharply separated all round from the orbital, and Sandifort ("Descriptio Musculorum, Lugd. Batav.," 1781), describes the ciliaris as distinct from the rest of the muscle; 11. I have seen a great variety in the strength of the development of the ligament of the external commissure; 12. Mr. T. W. Jones, in "Todd's Cyclopædia," vol. iii., p. 82, describes that many of the fibres of this muscle are inserted into the skin of the eyelids. This, I think, is by no means a normal arrangement, as I have failed to verify, by dissection, this mode of insertion, except in a very few rare cases.

The corrugator supercilii, I have seen (though rarely) perfectly separate from the orbicularis, and inserted into the skin of the brow above the orbital fibres of the latter: I cannot therefore adopt the opinion of Fabricius that this is but a part of the last muscle. I have seen it—1. Divided into several fasciculi with fleshy origins; 2. M'Whinnie describes it as having been found by him arising by two or three tendinous slips; 3. Complete fusion of the whole muscle with the orbicularis (Hyrtl and others) is not common, but I have seen it as well as (4.) continuity of its outer fibres with those of the frontalis; 5. Suppression is rare, but I have several times failed to find the muscle; 6. In one subject it was bilaminar; 7. Walther describes its upper fibres as being sometimes continuous with the extra-orbital fibres of the orbicularis.

Pyramidalis nasi (procerus nasi of Santorini, "Obs. Anat." p. 12), may be a single slip continued on the nose (Casserius, "De Nasi Fabrica," L. 3, c. 7); this I have seen. More commonly it is double the two slips placed side by side, or divergent as they descend (Santorini, "Observationes

Anatomicæ," Venetiis, 1724, p. 12, pl. 66). The muscle is sometimes absent, according to Harrison, but this occurrence is rare in my experience. Above, it is almost always directly continuous with the frontalis; below, its insertion may vary, but most commonly it is attached to the expansion of compressor nasi, more rarely into the perichondrium of the lateral cartilages, the lower border of the nasal bone, or the integument of the nose; still more rarely it is continuous into the inner margin of the levator labii superioris alæque nasi. I have found it separate at origin from the occipito-frontalis; the same has likewise been described.

The compressor nasi I have found—1. Divided into separate fasciculi; 2. As a single sheet; 3. As two bands, an upper and lower; 4. So undeveloped or atrophied as to be barely discernible; 5. Rectangular in outline; 6. Fused with the musculus caninus at its origin; 7. With fibres continued into the alar insertion of the levator labii superioris alæque nasi.

The dilator naris posterior (Santorini) I have found—1. Absent; 2. It may be purely sesamoidal in origin and cutaneous in insertion; 3. Or fused with a slip of the orbicularis oris. The dilator anterior may be—1. Absent, or 2. in two slips. The compressor narium minor is very frequently absent. The depressor alæ nasi is also liable to fasciculation, fusion with part of the orbicularis oris, or absence. The depressor septi mobilis narium (Krause, Berres, "Anthropotomie," Band. i., Wien, 1835), or naso labialis is generally so connected with the external fibres of the orbicularis oris as to be indistinguishable from it.

The orbicularis oris is only liable to vary, as far as my observations extend, in proportional development, and in the number and extent of attachment of its accessorial bands from the alveolar arches; I have never been able to separate the two parts described by Bichat, marginal and internal; I have always found them too tightly bound into one: various parts of it have been differentiated and described as special muscles, by Merkel ("Stimm u. Sprachorgans," 1857), as the Protractor labii superioris and inferioris, dexter and sinister, co-angustator labii inferioris, constrictor labii inferioris and superioris, and levator prolabii superioris, or nasalis labii superioris. For another paper, by Langer, on this muscle, see "Oester. Med. Jahrb," 1851, No. 2, p. 8.

The levator labii superioris alæque nasi—1. May have only one head, its

internal or nasal origin, all the orbital fibres forming the levator proprius ; 2. This nasal head (caput angulare, Henle), may be double as described by Bell, or separate ("pyramidalis socius, Santorini," tab. i.) ; 3. It may receive an accessory fascicle from the frontalis, or ; 4. From the pyramidalis nasi as above described ; 5. Very frequently this muscle is inseparable from the next.

Levator labii superioris proprius, when separate from the last, which it is not in one out of every three subjects, may be—1. Double—this I have seen, and it also has been seen by Sandifort ; 2. Triple (*ibid.*) ; 3. Joined to the zygomaticus minor, rather rare ; 4. Bicipital (Eustachius), its outer head being from the malar bone (the jochbeinzacke of Henle), very common ; 5. Its outer head from the orbicularis palpebrarum (Cant and Henle) ; 6. A slip inserted into the compressor nasi; 7. A part of the malar origin continuous into the levator anguli oris. Absence of this muscle is not common, but has been described by Otto ; I have never met with an instance of this.

Beneath this muscle there lies very often a singular slip, the Musculus Anomalus of Albinus ("Historia Musculorum," p. 167), or the Rhomboideus of Santorini ("Observationes Anatom., tab. 1, f.), described also by Sömmerring (p. 102), and Sandifort ("Exercitationes Acad. Lugd. Batav.," 1783). This I have seen in various conditions ; but usually, when present, it passes from the nasal process of the superior maxilla to the neighbourhood of the canine fossa. It is usually perfectly distinct from all the other facial muscles ; but rarely its origin is connected with the caput angulare of the levator labii superioris alæque nasi, while sometimes its insertion is fused with the origin of the musculus caninus. I have, however, never found these two muscular connexions co-existing. I have seen it with a round tendinous origin, and a round fleshy belly at its insertion, or it has been a flat band, musculo-tendinous, or muscular for its whole extent. It has sometimes exceeded an inch in length. Allied in its nature to the musculus anomalus is a singular tendinous slip, which I met with in a subject in 1867—namely, a fibrous band, commencing below the tendo palpebrarum internum from the nasal process of the superior maxillary bone, passing downwards and outwards, under cover of the inferior fibres of the orbicularis palpebrarum, to be inserted into the superior maxillary bone, midway between the foramen infra-orbitale, and the middle inferior rim of the orbit. When the upper maxillary bone was cleaned, this

ligament projected from it as a tendinous arch, whose use or relations appear obscure. It was only present on one side, and I have never seen it but in the one subject. I am not aware of its having ever been noticed before.

The risorius of Santorini ("Observations Anatom.," Venetiis, 1724, p. 33), or the slip from the lateral fascia of the face or neck, to the angle of the mouth I have seen in a variety of forms, arising—1. From the integument over the upper third of the sterno-cleido-mastoid (Hallett); 2. From the parotidean fascia; 3. From the fascia over the masseter muscle; 4. Doubled, as described by Santorini; 5. Triple, as mentioned by the same author; 6. With a supplemental zygomatic slip; or 7. More rarely, it arises entirely from the zygoma, according to M'Whinnie—a form which I have never met with; 8. Arising from the external ear (Albinus)—I have never seen this form; 9. Or from the platysma directly; 10. From fascia over the mastoid process, and in this form, but very rarely, a slip may be continued into it from the transversus nuchæ (Schultze); 11. It may be inserted into the depressor anguli oris.

The zygomaticus minor muscle is very variable—1. Its absence is a condition often observed. 2. Or its not extending as far as the mouth, but, midway between the zygoma and the upper lip, being lost in the soft parts and muscles of the cheek. 3. I have seen it fused with the major; 4. Strengthened by an accessory band from the risorius Santorini; 5. Or from the orbicularis palpebrarum; 6. More rarely have I seen it joined to the levator labii superioris; 7. And Eustachius describes a remarkable and rare union of this muscle with the outer fibres of the frontalis. 8. I have seen it arising entirely from the orbicularis palpebrarum. 9. Doubling of the muscle is described by Malgaigne, M'Whinnie, and other observers, and I have several times met with this condition; 10. Triplicity at its insertion is much rarer; it is described only by Morgagni ("Adversaria Anatomica," xi., p. 23).

The zygomaticus major I have seen—1. Fused with the last; 2. Or with the risorius; 3. Or double; 4. Double at insertion, and single at origin; (Bell, "Engravings of Bones, Muscles, and Joints," London, 1819); 5. This muscle may be suppressed; 6. Or very broad, and joined at insertion to the depressor anguli oris; 7. Sometimes its origin is below the zygoma from the masseteric fascia; 8. Or it may be continuous with the platysma; 9. Or connected to the extra-orbital fibres of the orbicularis palpebrarum.

Depressor anguli oris—1. With its origin inseparably united to the platysma; 2. Or exclusively from the bone; 3. A band of its inner fibres may curve downwards, forwards, and inwards, from their origin into the mesial line of the neck, behind and below the chin; this not uncommon band is described by Sömmering, p. 98: it is the transversalis menti of Santorini, sous-symphysien of Cruveilhier, or the doppelkinnmuskel of the Germans. It was found by Hallett, Zagorsky, and others; 3. The depressor may be split into fascicles, some of which are over, and others under the facial artery; generally this takes the form of a threefold division into two lateral, and one central slip; 4. It may be found with the depressor labii inferioris; 5. Or its fibres may be continuous with those of the zygomaticus major, or levator labii superioris, or levator anguli oris. Depressor labii inferioris I have seen—1. Continuous with the platysma; 2. United to the last; 3. Split into two by the mental nerve.

Levator menti I have only found varying in shape and size; rarely it is in two parallel slips.

Levator anguli oris has been found—1. Split; 2. Joined to the compressor nasi; 3. Or to the depressor anguli oris.

Depressor labii inferioris I have seen in two or more parallel fascicles.

The first of the neck muscles, whose anomalies I have collected, is the platysma myoides; and as this is the diminished rudiment of the panniculus of lower animals, we may expect to find it presenting many varieties. I have not found this muscle, as a rule, to be stronger in females than in males, as described by Mayer (Tom. III., p. 203); its development seems to me to be, as a rule, pretty equal in the two sexes. 1. I have found this muscle to be suppressed entirely on one side (at least twice), and on both once. 2. Or in the opposite condition of a strong red sheet, with bony connexions at each end. 3. Not uncommonly it is divided into detached fasciculi. 4. It has been found arising by a direct origin from the clavicle; this I have seen but seldom. 5. Mr. Wood describes a slip as arising from the pre-sternal fascia, as far down as the level of the third costal cartilage; this portion formed a band, which passed over the origin of the sterno-mastoid to the inner border of the platysma. 6. I have found a broad clavicular origin, and a round sternal fascicle, joining with a separate fascial supra-acromial slips, and forming the body of the platysma. 7. A very rare anomaly consists in the union of the lateral muscles in the mesial line; this I

have but once seen. 8. The two muscles are not unfrequently united by aberrant fibres below the pomum Adami, an anomaly which has not hitherto been noticed. 9. A tranverse slip below the chin uniting the muscle is described by Zagorsky (Mem. de l'Acad. Impériale de St. Petersburg, Tom I., 357); and by Gantzer ("Dissert Anat. Musculorum Variet. Sistens, p. 6); and Fleischmann ("Erlangen Abhandlung," vol. I., p. 28). This is probably a form of the doppelkinnmuskel as supposed by Hallett; 10. It is often continuous into the depressor anguli oris; 11. Still more frequently into the depressor labii inferioris (Cloquet); 12. I have seen it connected to the zygomaticus major by a band representing the risorius of Santorini; 13. More rarely it is continued into the zygomaticus minor; 14. Even rising still higher, the fibres may be attached to the peripheral fasciculi of the orbicularis palpebrarum; 15. The inner fibres have been seen inserted into the thyroid cartilage; 16. The outer fibres passing into the mastoid process (Zagorsky, loc. cit.); 17. Into the lower part of the concha of the ear (Fallopius, p. 63); 18. Into the anterior part of the lobe and lowest part of the concha ("Albinus Hist. Musculorum," p. 194; "Barkow Monstra Duplicia, Lipsiæ," 1828, p. 10); 19. By a thick round slip to the occiput (Zagorsky, l. c.); 20. By a separate slip distinct from the depressor anguli oris into the angle of the mouth (Cloquet); 21. A slip rising as high as the skin of the forehead (Rolfincius, "Dissertatio, Anatom., Noribergæ," 1656, p. 521); 22. To the attrahens aurem (Henle); 23. A transverse slip is described by Fleischmann from the ramus of the jaw on one side to the corresponding part on the other side, crossing transversely across the anterior bellies of the digastric (Fleischmann, loc. cit.). This is probably a detached "doppelkinnmuskel"; 24. Several forms of union between this muscle and the transversus nuchæ of Schultze I have met with, and others have been described. Some of these constitute the corrugator posticus, or occipitalis teres or minor of Santorini ("Observationes Anatomicæ," cap. I., §. 4., p. 6), or the peauciers sous occipitaux of Cruveilhier. One of these is described by Theile (p. 183) as a flat thin slip from the middle line of the back of the neck, passing forwards to the face; and another is described by Kelch ("Beiträge," p. 30), which, from the mastoid process and parotid fascia, passes forward beneath the chin to its fellow in the median line. A case of this kind is described by Wood. Its nature may be a junction of the transversus nuchæ (displaced forwards), with

the submental slip above described by Gantzer, &c.; 25. Of other aberrant slips of platysma, I have seen a slip from the acromial end of the clavicle passing down into the fascia over the deltoid; 26. Slips from the clavicle to the skin over the upper lobes of the mammary gland; 27. A slip from the sternal fascia between the second and third costal cartilage, crossing downwards and outwards, joining with the tendon of the latissimus dorsi, is described by Mr. Wood (1866); 28. One from the clavicle passing outwards to be lost in the skin over the deltoid (Gantzer, loc. cit., p. 6), like No. 25, but more horizontal; 29. Henle mentions slips of the platysma crossing over the sternum to be inserted into the second and third rib-cartilage of the opposite side, decussating with corresponding opposite fascicles (p. 108.) This is interesting, as throwing light upon some varieties of the musculus sternalis, to be hereafter referred to; 30. As other parts of the panniculus occasionally developed, we may enumerate here the existence of subcutaneous fibres in the axilla stretching parallel to the border of the great pectoral, but placed immediately below the skin. These "fibres of Lucas" are by no means constant, but I have frequently found them strong and red; 31. Wandering fibres of the platysma have been seen lying on the anterior part of the serratus magnus, opposite its fourth and fifth costal origins; 32. On the infraspinatus, close to the posterior border of the scapula, and joining to the lower margin of the trapezius near the spine of the scapula; 33. From the fascia over the acromion to the trapezius, opposite the spine of the first dorsal vertebra; 34. Fibres from the aponeurosis over the gluteus medius, blending with the external oblique; (31–34 are described by Turner, "Journal of Anatomy and Physiology," vol. i., p. 252); 35. A few superficial fibres over the pectoralis major near the axilla; 36. Oblique fibres crossing those of the trapezius at an angle of 60°, but unconnected with them; 37. Fibres over the latissimus dorsi in the middle dorsal region, few and scattered, without any bony attachment. These three last mentioned have occurred during the last two sessions in the Dublin College of Surgeons' Dissecting Room (1869-70); 38. The platysma was seen to extend to the fourth rib by Sömmerring (p. 110); 39. A platysmal slip from the sternal end of the clavicle to the fascia over the deltoid is described as a tensor fasciæ brachii by Ibsen ("Ugeskrift for Laeger," Copenhagen, 1842, Bd. 7, p. 456). This was unconnected with the rest of the platysma. 40. A slip over the trapezius at its

origin from the level of the second to that of the sixth dorsal spine is described by Turner ("Journal of Anat.," vol. v., p. 117). A similar rudiment from the spinous process of the eight or ninth dorsal to the second dorsal spine was found by Perrin ("Journal of Anat.," vol. v., p. 233).

Beneath the superficial group of muscles just described in the facial region, the masticatory muscles are subject to few varieties, except individual variations of size. The masseters vary only in the relative development of the two layers, which are sometimes very separate (Sömmerring, p. 105), in the presence of a bursa between its laminæ described by Monro (Icones bursarum edit. Rosenmüller, 1799, p. 32, Tab. ii.); and Hyrtl describes one between the deep layers of this muscle and the joint-capsule ("Topographisch. Anat.," Band i., p. 299). I have found the inner fibres of the deep layer coalescing with some of the lowest of the fibres of the temporal; and Haller describes a coalescence of the masseter and the buccinator ("Op. Anat. Argument. Minor," vol. iii., Lausanne, 1768, p. 18). Dumeril found these muscles absent in a subject with phocomelia ("Bull. de la Soc. Philom.," vol. iii., p. 122). A slip of the deep layer also may take origin from the external lateral ligament of the lower jaw.

The temporal is also singularly free from variety. The fibres from the crista sphenoidale I have seen separate, and also those from the zygoma. Horner describes the lower fibres of this muscle as continuous with the upper border of the external pterygoid muscle, a condition which I have seen in a case in which the internal maxillary artery ran under the latter muscle ("Special Anatomy," Philadelphia, vol. i., p. 372). Massa describes the temporal as having been found double ("Liber. Introductorius," c. 35, p. 77). The tendon of insertion sometimes extends as far forwards as the last molar tooth.

The buccinator; 1. I have found possessing common fasciculi with the buccopharyngeus muscle; 2. And also to vary in the relative amount of its superior and inferior maxillary origins; 3. A case of its coalescence with the masseter occurring in a monster is noticed under the head of the latter muscle; 4. With a special slip arising from Steno's duct; 5. Thin and almost deficient in the centre.

Pterygoideus externus. I have very often (about twelve times) found a nearly vertical slip passing from the crest on the great wing of the sphenoid to

the tuber palati, or to the posterior margin of the external pterygoid plate. This has been described by Henle (p. 164), Gruber, and Theile (p. 59), as the pterygoideus proprius, and recently cases have been reported by Mr. Wagstaffe ("Journal of Anat.," vol. v. p. 281.) In these latter cases the maxillary head of the muscle was absent, and some of the ordinary fibres arose from these vertical bands. I have also found a muscular slip passing along the ligamentum pterygo-spinosum of Civinini from the spina angularis to the tooth in the external pterygoid plate ("Schmidt's Jahrbuch," Bd. 23, p. 277), or even replacing the ligament. A form like this is mentioned by Theile, (p. 60). An origin for the internal pterygoid muscle from the pyramidal process of the palate bone is very common, and I have seen a slip to the tensor palati from the origin of this muscle. I have often found the separate fascicle from the tuber palati beside the external pterygoid plate, which finally merges into the rest of the muscle; cases of its occurrence are mentioned by Wagstaffe (l. c. p. 282). A fascicle from the pterygoid fossa to a band stretching from the spina angularis to the angle of the jaw is described by Prof. Gruber ("Neue Anomalien," p. 13). Moser found it giving origin to the styloglossus ("Meckel's Archiv," vol. vii., p. 226).

The external pterygoid has been found joined to the temporal, as described under the head of the latter muscle, and one case is on record of its union with the digastric ("G. Meckel, de Duplicitate Monstrosa," p. 42). A separate part of the lower or pterygoid head to the capsule of the lower jaw is described by Fäsebeck ("Müller's Archiv," 1842, p. 475). The second head may be perfectly separated as far as its insertion from the first. The relation of the internal maxillary artery to the muscle is also very variable, as described in all the works on arterial anomalies. A curious slip—a form of pterygoideus proprius—I have found six times passing from the crest on the great wing of the sphenoid to the tuber maxillare, lying superficial to the external pterygoid muscle. The sphenoidal head may be partly tendinous.

Sterno-cleido mastoideus has afforded to anatomists many varieties, and I have met with some interesting forms of its variation: 1. I have found it perfectly cleft into a separate sterno-mastoid and a cleido-mastoid muscle, with the spinal accessory nerve in the plane between the two, or beneath both, and separating the cleido-mastoid from the cleido-occipital; 2. Or its two heads I have found

inseparable, but this I regard as a very rare anomaly—it is also described by M'Whinnie ; 3. A double sternal head I have found more than once, and (4). an absence of the sternal origin I have also noticed ; 5. A double clavicular and a single sternal head was described by Mr. Wood, and I have, last year, met with a case of it ; 5. I have, however, never met with a complete quadriplicity such as that described by Hallett, in which four heads, two sternal and two clavicular, coexisted ; 6. On the other hand, I have met with a case in which the clavicular origin was as narrow and nearly as tendinous as the sternal ; 7. I have found the clavicular head fasciculated, split into slightly joined bundles for its whole extent ; 8. Still more rarely a single muscle belly was connected, not only with a doubled origin, but also with a doubled insertion. This form of anomaly was described by Hallett ; 8. The cleido-mastoid I have found doubled in two strata, the deeper layer being inserted by a long tendon into the mastoid process immediately over the digastric groove. A case of this kind is described by G. Meckel ("De Duplicitat. Monstros.," p. 40) ; 9. The origin of the cleido-mastoid may extend out to the middle third of the clavicle (from its fusion with the next muscle), or even to the outer third. This anomaly Hallett has found once in every eight cases—a very unusual proportion : I have not found it nearly so frequent. 10. Connexions between the sternal head of the sterno-mastoid and the rectus thoracicus have been described, and I have at least twice seen it in six hundred subjects, so it seems to be a rare anomaly. Cases of it, however, are described by Fabricius ("Idea Anatomiæ Practicæ," Wetzlar, 1741, p. 17), Bourrienne ("Roux' Journal de Médecine," Tom. xxxix., 1773), and Otto ("Patholog. Anat." South's Tr. p. 247) ; 11. A slip from the anterior margin of the muscle into the front of the mastoi dprocess ; 12. One into the tympanic ring ; 13. One into the stylo-maxillary ligament ; and, 14. one into the angle of the jaw, have been found by me at different times. The last of these is the only one with a description of which I am acquainted, it having been met with by Brugnone (loc. cit.), and Theile (p. 164). 15. A triple clavicular origin is described by M'Whinnie. One of these was probably the cleido-occipital of Wood ; 16. In a monster Barkow found a muscle from the left mastoid process joined by a slip from the hyoid bone, and passing to the manubrium ; the hyoid portion divided into two, one, the largest, ran as described ; the smaller joined the normal sterno-

mastoid ("Monstra Duplicia," Leipzig, p. 20.) A third head for the sterno-mastoid has been also described by Ehlers ("Henle u. Pfeufer's Zeitschrift," vol. xxi., p. 297). Schwegl, of Prag, found this muscle joined to the omo-hyoid and sterno-hyoid by a cross slip ("Sitzungsberichte der k. Akad. in Wien," Bd. xxxiv., p. 51). Cleido-occipital of Wood is a muscle ascending from the clavicle, external to the cleido-mastoid, to the occipital bone, usually into the outer third of the superior transverse ridge. This muscle is described by several anatomists incidentally (see "Horner," p. 319), but was never systematically named and pourtrayed until Mr. Wood described it in his Papers in the "Proceedings of the Royal Society" for 1866, p. 230. It may be perfectly separate from the cleido-mastoid (Wood, Murie, in a Bushwoman, "Journal of Anatomy," vol. i., p. 189, and myself), or joined to it below, and separate above; this I have seen—or separate below, and joined above, as described by Mr. Wood, an arrangement which I have also met with. Its place is usually superficial to that of the cleido-mastoid. A much rarer variety is that in which the cleido-occipital, arising with the cleido-mastoid, is inserted into the trapezius. A second head was found by Mr. Wood in one case arising from the sternal end of the clavicle, passing across the clavicular origin of the cleido-mastoid, joining the rest of the normal cleido-occipital. A double cleido-occipital was found also by Wood. This I have never seen, but sometimes I have seen it fasciculated. I have also found it attached to the trapezius, and not to the cleido-mastoid, as has been described by Professor Gruber. The frequency of this muscle is computed by Mr. Wood as being about once in 2·7 subjects. It was present twenty-six times out of seventy subjects, and eleven times out of thirty-two. Wood, Phil. Trans. vol. 160, p. 90.

As an anomaly of the sterno-mastoid some of the older anatomists described a muscle which was first recognised as a separate human muscle by Mr. Wood, the levator claviculæ. Thus, Theile (z. c. p. 153) describes a specimen in which this muscle arose from the oblique processes of the fourth and fifth cervical vertebræ, and was inserted into the middle third of the clavicle external to the sterno-cleido-mastoid; and Kelch ("Beiträge," p. 32), describes an instance in which this muscle passed from the transverse process of the sixth cervical vertebra to the humeral end of the clavicle. The muscle has been seen by Mr. Wood and myself, arising from the transverse process of the lower cervical

vertebræ, and inserted into the acromion, into the outer third of the clavicle under the trapezius, or into the trapezius itself. The true atlantic form so common among the lower animals, of trachelo-acromial (Duverney), or omo-atlantic (Haughton), I have not met with, but a case of it is given by Wagner (Heusinger's "Zeitschrift für Organische Physik, Band iii., p. 337. Murie, in the "Journal of Anatomy and Physiology" (vol. i., p. 199), describes a case in which it arose from the transverse processof the third cervical vertebra, passing over the second levator anguli scapulæ, and was inserted into the fascia behind the clavicle covering the axillary surface of the first tooth of the serratus magnus. And Gruber found it arising from the transverse process of the second vertebra (Vier Abh. p. 33), see Phil. Trans. vol. clx., p. 88.

The omo-hyoid is very fertile in anomalies, which form the subject of an interesting monograph by Professor Turner, of Edinburgh ("Edin. Medical Journal," 1861, p. 982). 1. Suppression of both muscles was described by Büchner, "Miscellen," 1727, p. 252, Von Behr., p. 118); by Cheselden the absence of one of them was described, "Anatomy," 1740, p. 79; and the same was seen by Hallett (loc. cit)., Schwegl (loc. cit.) 2. Absence of the anterior belly is described by Krause ("Handbuch der Mensch. Anat.," Hanover, 1833, 1st. Bd., 1. Abthg., p. 289), in which case the posterior belly ended in a tendon which was lost in the cervical fascia; this form is denominated by Krause coraco-cervicalis. The same was found by Hallett, and this latter author regards its frequency as once in every hundred subjects. I have not succeeded in finding a single specimen of it out of six hundred subjects. A similar muscle has been found existing with a normal omo-hyoid. 3. The posterior belly has never been found wanting, but Hallett has found it in one instance very small, and composed of a few weak fasciculi, but connected by the ordinary intermediate tendon, with a strong red anterior belly. I have not seen this variety. 4. The posterior belly may arise entirely from the transverse ligament of the notch in the scapula; or from, 5. The outer half of the superior costa of the scapula; still more rarely the origin may be flattened out, and extended nearly along the whole superior costa; these varieties have fallen under my own observation. In the first of these forms (No. 4), the supra-scapular artery has been seen piercing the origin of the muscle. 6. A slip of the origin not unfrequently extends as far forward in point of origin as the

coracoid process, or it may take sole origin from this process (the coraco-hyoid of Gruber). 7. Its origin may miss the scapula, and be attached to the first rib (Wagner, "Heusinger's Zeitschrift," p. 335, Schwegl, loc. cit., and Gruber, Abhandl. aus dem Gebiete der medico-chirurg. Anat., Berlin, 1847): this form is considered as a variety of the serratus magnus by Theile; 8. A still farther extension of this displacement of origin is seen in the cases in which the posterior belly either entirely or in part arises from the acromion process. 9. The next step consists in the attachment of the posterior belly by an accessory head to the clavicle; this occurred five times out of seventy subjects (Wood), or seventeen times in three hundred and seventy-three cases (Turner)—that is, about once in every twenty subjects. Hallett describes its frequency as about once in fifteen; but this is, I think, an unusual degree of frequency. Schwegl found it twice in a hundred cases. This accessory head almost always runs into the posterior belly before that portion of the muscle becomes tendinous (Albinus), but sometimes this has a few tendinous fibres in it at its termination (Kelch, p. 31). 10. A purely clavicular origin is described by Petsche (l. c., p. 769); Rosenmüller; Luschka (Müller's "Archiv," 1856, p. 284); Cruveilhier (p. 173); Hallett, who has found it in three per cent. of the subjects which he has examined; Wood, M'Whinnie, and myself. This origin may be from the middle third of the clavicle (Wood, M.,) or from the outer third (M.); it may have a tendinous inscription, but more commonly it is a monogastric muscle, more or less united to the sterno-hyoid. When the intersection is present it may be only partial, or merely as a superficial white line (Haller). Probably the case of absence of the muscle described by Cheselden, and referred to above, may have been a complete case of the fusion of a monogastric clavicular omo-hyoid with the sterno-hyoid, as in the description he states that the sterno-hyoid was unusually broad; another case of this anomaly, under the name cleido-hyoid, is described by Professor Schmidtmüller, of Landshut ("Reil u. Autenrieth's Archiv," vol. viii., p. 269). 11. Doubling of the whole muscle has been described by Kelch and Gubler. In a case described by Koster ("Verslagen en Mededeelingen der konink. Akad. van Wetenschappen, Afdeeling Naturk., 2nd Reeks, IV. Deel.), the supernumerary omo-hyoid was also split: the anterior muscle passed to the sterno-hyoid, the posterior was attached to the os-hyoides. Another case of doubling is described by Gruber, in

which two anterior and two posterior bellies are attached to the one central tendon ("Vier Abhandl." p. 13); of these the anterior superior joined the sterno-hyoid. Hallett has also put on record a case of doubling of this muscle, in which the posterior belly shortly after its origin divided into two, the upper of which was normal, and the inferior ran into the sterno-hyoid, midway between the sternum and the hyoid bone. A similar case is described by Sels, in which the lower part united with the sterno-thyroid ("De Musculorum Varietatibus," Berlin, p. 6). 12. Doubling of the posterior belly is described by Winslow and Duille ("Dissertatio Anatomica nonnullas Musculorum Varietates exhibens," Landshuti, 1813). 13. Duplicity of the anterior belly was found by Mr. Wood four times in seventy subjects, that is, nearly once in seventeen. In one of these cases the superior slip was normal, the inferior joined the sterno-hyoid by fascia. In another case I have found the superior passing with the hyoglossus, and the middle constrictor receiving a slip from the sterno-thyroid. Another instance of the kind is described by Gruber ("Vier Abhandl." p. 13), 14. A normally attached muscle was found by Mr. Wood to exhibit no tendinous inscription, and this occurred twice out of seventy subjects. In one case Haller describes his finding a very slight white line as the only sign of division ("Elem. Phys.," Lib. IX., p. 415). I have seen the opposite extreme—namely, a tendon of fully an inch in length uniting the two bellies. 15. Mr. Wood found the anterior belly once triple, the middle part being normal, and the uppermost going to the superior cornu of the thyroid cartilage, while the lowest was lost in the cervical fascia. 16. Blending of the anterior belly with the sterno-hyoid was found by Professor Turner four times in three hundred and seventy-three subjects. Winslow found the entire anterior belly inseparably fused with sterno-hyoid. 17. The opposite of this, or a slip from the sterno-hyoid to the omo-hyoid, without a complete blending of the muscles, is described by Wood. 18. A corresponding slip from the omo-hyoid to the sterno-thyroid I have met with, and also a slip (19) to the mylo-hyoid; 20. to the great cornu of the os hyoides; 21, to the sheath of the vessels. 22. A curious arrangement described by Mr. Wood will be found described in connexion with the stylo-hyoid. 23. M'Whinnie describes a slip of this muscle as extending over the hyoid bone to the inferior maxilla, close to the symphysis menti, beneath the mylo-hyoid, and close to the genio-hyoidei. 24. Sels (loc. cit., p. 4), and

Rudolphy (quoted by Otto, "Path. Anat.," p. 247) describe this muscle as sending a slip to the second cervical vertebra, but do not give a precise description of this curious anomaly, which seems to be a fusion of the omo-hyoid and the trachelo-acromial. 25. Hallett describes the posterior belly as being bound down on a level with the clavicle once in every five subjects, thus obliterating the posterior inferior triangle of the neck. The sheath binding it down varies in strength and distinctness. 26. Union of the stylo-hyoid and omo-hyoid was seen by Sömmerring. 27. Schwegl found it in one subject united to the sterno-mastoid by a fasciculus, and at the same time to the sterno-hyoid by another abnormal fasciculus.

The sterno-hyoid may vary in the following ways:—1. It has been found with no clavicular origin; 2. Or with a wide origin from the inner third of the clavicle; 3. Or from the middle third (Kelch, Retzius); 4. With no sternal origin; this form Schwegl calls the cleido-hyoid ("Sitzungsber, d. k. Akad." Wien, Bd. xxxiv., p. 51.) He has found it three times in a hundred subjects. 5. Or a distinct origin from the posterior sterno-clavicular ligament; 6. In one subject I found its sole origin to be from the posterior sterno-clavicular ligament, and from the cartilage of the first rib; 7. It has been found joined by decussating fibres to the companion muscle of the opposite side (Albinus); 8. Or merely continuous above, without a mesial separation between them; 9. Or it has been seen united to the sterno-thyroid (Albinus); 10. Or sending a slip into the mylo-hyoid (M'Whinnie); 11. Or fused with the anterior belly of the omo-hyoid; 12. Or under it; 13. The origins of the two sterno-hyoids are rarely close together in the middle line, but this anomaly I have seen; 14. A linea transversa occurs occasionally opposite the level of the inscription in the omo-hyoid muscle, but rarely a second one is present above this level, on a line with the insertion of the sterno-thyroid. I have never found one below the level of the primary intersection above noticed; 15. A rare condition of this muscle is that of splitting into two parallel muscles; this condition I have once seen, and it is described by Oribasius ("Op.," p. 202). It is described as double by Sömmerring (p. 115). Meckel found it to have a slip from the root of the coracoid process coexisting with the omo-hyoid ("Archiv," vol. viii., p. 585). Schwegl found it bicipital,

and in another instance found this muscle joined to the omo-hyoid and sterno-cleido-mastoid.

The sterno-thyroid is more variable.—1. Otto describes a case of its absence; and, 2. Kelch, Wood, and Gunz ("Mém. de l'Académie des Sciences," vol. i., p. 286), have met with cases of duplicity in this muscle, a variety which I have also met with ; 3. Splitting of the muscle is described by Tarin ("Myographie," Paris, 1753, p. 59, fig. 37), and Hallett. A case of this kind is also described by Cowper, in which the inner portion was small; 4. A second head from the back of the middle third of the clavicle (Wood) ; in one instance of this, the clavicular slip, arising still farther out, was seen by the same author separated from the normal head of the muscle by the omo-hyoid which wound round it like a pulley ; 5. The extent of its attachment to the cartilage of the second rib is liable to great variety. I have seen a separate slip coming from this place, and Haller also describes this variety ; in other cases the muscle has had no costal origin ; 6. Occasionally there is a single linea transversa, and I have found this either above or below the level of the inscription in the omo-hyoid ; 7. An additional and not very complete intersection on the level of the top of the sternum I have found coexisting with the before-mentioned one. 8. This muscle is frequently united to its fellow, either by decussating bands of muscular fibres from the inner borders, as described by Girardi ("De Re Anatomica Oratio," Parma, 1781, p. 36), and Wood; this I have seen, and I have also seen single slips of muscle passing from one side to the other. 9. Union to the sterno-hyoid is described by Albinus and myself; 10. And I have also seen it united to the levator glandulæ thyroidei ; 11. To the inferior constrictor (M'Whinnie and Haller ("Elem. Phys.," vol. iii., 1761, p. 381) ; and, 12. To the thyro-hyoid ; 13. It occasionally sends a slip to the cricoid cartilage (M'Whinnie) ; 14. A slip has been traced from the omo-hyoid into this muscle, which passed up to the greater cornu of the hyoid bone, blending with the fibres of the hyo-glossus and middle constrictor; 15. I have also found a separate slip passing up to the os hyoides; 16. Sels describes a slip from the posterior belly of the omo-hyoid as running into the middle of this muscle. 17. I have seen a slip from the anterior belly similarly uniting them together. 18. The sterno-thyroid has been found with a slip from the sheath of the vessels in the neck.

Levator glandulæ thyroidei of Sömmerring (p. 117) has been found arising —1. From the angle of the thyroid cartilage (M.); 2. From the hyoid bone to the isthmus of the thyroid body; 3. As an unsymmetrical band on one side; 4. Or as a mesial slip either single or with a trace of a central division; 5. More rarely as a symmetrical muscle on each side; 6. I have found it inserted into the apex of the pyramid of Lalouette; 7. Or into the fibrous capsule of the lateral lobe at its junction with the isthmus; 8. Or united to the sterno-thyroid; 9. A triple form is described by Professor Gruber, of St. Petersburgh, one slip coming from the right process of the lower border of the thyroid cartilage, passing beside the isthmus; a second from the most posterior bundle but one of the thyro-hyoid muscle, passing into the right lobe at the apex; and a third from the hindermost bundle of the thyro-hyoid with a slip from the thyro-pharyngeus (Reichert and Du B. Reymond's "Archiv," 1868, p. 644, taf. xv., D). Slips from the thyro-hyoid may form simply a levator.

Thyro-hyoid.—1. Continuous by slips with the sterno-thyroid; 2. By a slip to the omo-hyoid; 3. Having a slip from the cricoid cartilage ("Haller," p. 383); 4. Sending a slip, or several slips, to the levator glandulæ, or to the hyo-trachealis; 5. Occasionally one or more fasciculi are separate from the rest, giving an example of variation by fission; this I have seen.

Thyro-hyoideus superior, or minor, or azygos, is an occasional muscle, not referred to in our ordinary handbooks, but described by Sömmerring (p. 117), by Weber-Hildebrandt ("Handbuch," p. 358); Morgagni ("Epist. Anatomica," 11. Art. 43," Pat., 1764, p. 112), Haller ("Elem. Phys.," Tom. 3, lib. ix., p. 383). This muscle arises from some part of the inner portion of the upper border of the thyroid cartilage, and is inserted into the lower edge or anterior margin of the hyoid bone, in general near the middle of its body, but sometimes to the great cornu. Most commonly the fibres arise from near the incisura medialis of the thyroid cartilage. One on the right side is described and figured by Professor Gruber ("Reichert's u. Du Bois Reymond's Archiv," 1868, Taf. xv., fig. B., β): bands like these are also described by Professor Luschka ("Anat. des Menschlichen Halses." Tübingen, 1862, p. 275. They are frequently present as rudiments of this muscle, continuous with the thyro-hyoid.

The crico-thyroid.—1. May have fibres from the upper ring of the trachea;

2. May send a band into the insertion of the sterno-thyroid; 3. May send fibres into the pharynx; 4. It may be bilaminar; 5. Or may be united to the kerato-cricoid muscle; all these varieties I have seen. The first is the rarest, then the fourth; the third is the most common; 6. The anterior fibres may be separate, forming a crico-thyroideus superior muscle, like that described in Hylobates albifrons, by Eschricht ("Müller's Archiv," 1834, p. 218); or they may, arising in common with the rest of the muscle, have a separate insertion, corresponding to that of Eschricht's muscle.

Kerato cricoid.—This muscle was described first by Tourtual ("Neue Untersuchung über den Bau des Menschlichen Schlund u. Kehlkopfs," Leipzig, 1846, p. 105), as the crico-corniculatus; then it was described by Merkel, ("Anthropophonik," p. 132, fig. 44), as kerato-cricoid, and more recently by Bochdalek, junior, of Prag ("Œsterreich. Zeitschrift," 1864, No. iv.), under the name of crico-thyreoideus posticus. Probably this muscle is the second part of the double crico-thyroid described by Gunz ("Mémoires des Savans Étrangers," p. 287), by Tarin ("Myographie," p. 61, fig. 39), and also by Courcelles, Vesalius, and Rolfincius ("Dissertatio Anatomica," p. 532). It passes from the lowest part of the cricoid cartilage below the crico-thyroid joint, to the lower border of the thyroid cartilage, below and behind the crico-thyroid. In one case in which I found this there was no superior cornu of the thyroid cartilage, a condition described by Professor Gruber, of St. Petersburgh and by Professor Luschka ("Virchow's Archiv, 1868," p. 478). According to Professor Turner ("Edin. Med. Journal," 1870, p. 744), this muscle was present seven times out of thirty-two subjects; four times on the right, twice on the left, and once on both sides. Bochdalek has found it only on one side, and in females; but Patruban has seen it on both sides, and I have seen it twice in males, and four times singly. I have seen it united to the crico-thyroid.

Crico-hyoid is described by Zagorsky ("Mémoires de l'Académie Imp., Petersbourg," T. 1., p. 353) as a separate muscular slip from the cricoid cartilage to the great cornu of the hyoid bone. I have never met with it, but a few fibres of the thyro-hyoid have sometimes been extended downwards to the cricoid, or to the crico-thyroid muscle which made some slight approach to a complete crico-hyoid. It has not, as far as I know, been described by any other author.

Kerato-hyoideus or thyro-hyoideus lateralis is a name given by Professor Wenzel Gruber to a muscular fascicle passing from the superior cornu of the thyroid cartilage to the great cornu of the hyoid bone. I have never seen this muscle; it is described in Professor Gruber's Paper on "New Anomalies" (Berlin, 1849, p. 14). This muscle is closely related to, if not identical with, the thyreo-syndesmicus of Sömmerring, which is inserted into the posterior thyro-hyoid ligament. I have myself found a thyreo-triticeal muscular slip stretching to the corpus triticea, in one subject; these three are all but different forms of the same muscle. Morgagni describes a triangular muscle from the upper border of the thyroid cartilage near its upper cornu passing to the great cornu of the hyoid bone, and formed by some fibres of the sterno-thyoid ("Epist. Anatomica," x., p. 362).

Hyo-trachealis is another muscle which has been found by Professor Gruber ("Reichert's Archiv," 1868, p. 644), from the anterior end of the great cornu of the hyoid bone, sometimes with a slip from the body of the os hyoides; it passed behind the isthmus of the thyroid body, then becomes tendinous or aponeurotic at the level of the first ring, and is inserted into the perichondrium of the three upper rings of the trachea.

Thyreo-trachealis is described by the same indefatigable observer ("Bulletin de l'Acad. Imp. de St. Petersbourg," Tom. 3, p. 153, 1861), as a muscle arising from the right side of the inner border of the middle part of the thyroid cartilage anterior to the origin of the crico-thyroid with two bundles, one median, and one lateral; it passed over the crico-thyroid muscle, and over the isthmus of the thyroid body to be inserted into the fourth ring of the trachea. This was found by Gruber in twenty-one out of eighty larynges—eighteen times in men, and three times in women. This is a degree of frequency much greater than I have met with in Irish subjects, as I have sacrcely seen it five times out of eighty subjects. It appears like a deep detached band of the sterno-thyroid.

Crico-trachealis I have once found passing from the lower border of the anterior part of the cricoid cartilage, below the origin of the crico-thyroid muscle, and inserted into the fifth ring of the trachea; it passed behind the isthmus of the thyroid body. This muscle seems to be of the same nature as the preceding than which it is much rarer.

Thyreo-mediastinal, is a remarkable slip, shown to me by Dr. Hewitt, and published in my last paper in the "Proc. R. I. A.," going from the capsule of the thyriod body down along the trachea, into the anterior mediastinum to the region of the upper sterno-pericardial ligament.

Thyreoideus transversus anomalus, or thyreoideus marginalis inferior, is a small muscle described by Professor Gruber ("Neuer Anomaler Kehlkopfsmuskel," in the "Œsterreich. Med. Jahrbuch," Bd. 52, 1845, Wien, p. 148, and "Reichert's Archiv," 1868, p. 635), which passed from one side of the inferior medial thyroid notch to the other, across the upper part of the crico-thyroid membrane. A similar muscle is described by Eschricht as existing in Hylobates albifrons ("Müller's Archiv," 1834, p. 218).

Arytenoideus proprius is liable to few varieties.—1. In one subject I found all the fibres nearly transverse; 2. A few fibres I have found continuous with those of thyro-arytenoideus ("Sömmerring," p. 137); 3. The proportion of oblique and transverse fibres may vary; 4. One or two of the oblique fibres may reach the corniculum of Santorini; but I have never found this band in any degree separate from the rest of the muscle. Some of the oblique fibres may arise from the cricoid cartilage or outwards from the epiglottis ("Sömmerring," p. 136).

Crico-arytenoideus lateralis. I have never found this muscle to present any noteworthy variations. Girardi found it with a slip from the thyroid cartilage (loc. cit).

Crico-arytenoideus posticus I have seen—1. Split; 2. Bilaminar, joined to the crico-keratic muscle, or sending a slip to the kerato-cricoidean articulation.

Thyreo-arytenoideus I have found split, the smaller part being the thyreo-arytenoideus minor of Albinus (Tab. 12, fig. 6), and Sandifort; this may send a slip to the epiglottis ("Santorini," p. 110); this is often absent ("Sömmerring," p. 138).

Kerato-arytenoideus is a muscle described by Professor Gruber as the "schildknorpelhorn-giessbeckenknorpels-muskel;" it lies between the outer border of the crico-arytenoideus posticus and the hinder margin of the thyroid cartilage; its origin is generally tendinous, and is attached to the middle of the posterior border of the inferior horn of the thyroid cartilage, beside the posterior inferior kerato-cricoid ligament; it becomes fleshy, and is inserted into the muscular process

of the arytenoid cartilage ("Reichert's Archiv," 1868, p. 640). I described a specimen of this muscle passing from the inferior cornu of the thyroid to the arytenoid, in the "Proceedings of the Royal Irish Academy," January, 1868, p. 9.

Hyo-epiglotticus is a small muscle from the posterior surface of the os hyoides to the point of the epiglottis above the epiglottic gland; it is described by Fabricius, p. 1, c. 3, by Avicenna and Vesalius, (pp. 213 and 299); also by Lauth ("Mém. de l'Acad. Royale de Médecine," T. 4, 1835, p. 112); but it is exceedingly rare as a human muscle, though common in other animals. (See Luschka, "Reichert's Archiv," 1868, p. 224.)

Thyreo-epiglotticus major and minor were described by Tarin ("Myographie," p. 62); there is frequently one of these on each side, but very rudimentary. I have never met with them doubled.

Crico-epiglotticus is described by Verheyen (p. 188), but it has not been recognised by any modern anatomist.

Glosso-epiglotticus, consisting of fibres in the frænum epiglotticus, is figured by Eustachius (T. 42, fig. 5), and described by Heucher ("Opera," p. 564); it is very commonly present, but it is usually only a continuation of the genio-epiglotticus so commonly present (Luschka, op. cit., supra).

Aryteno-epiglotticus minor and major, most accurately described by Santorini ("Obs. Anat.," p. 11), and recently by Mr. Hilton, is frequently absent.

Under the name of transversus colli Professor Luschka, of Tübingen, has described ("Sitzungberichte der k. k. Akad. in Wien, mathemat. physik. Klasse." Band xxxiii., p. 18), a muscle similar to one which I had formerly published as the costo-facialis cervicalis, which arose from the first rib, passed obliquely between the sterno-hyoid and sterno-thyroid muscles, behind the sternal end of the clavicle, to be inserted into the endo-thoracic fascia (septum thoracico-cervicale of Sir Astley Cooper), which it tenses, and it occasionally meets its fellow of the opposite side in the median line; in the instance which I described, there was an origin from the back of the clavicle between the sterno-hyoid attachment and a clavicular origin of the omo-hyoid, coexisting with a costal head.

Another cleido-fascial muscle has been described by Rambaud and Carcassone in a tall muscular subject ("Gazette Méd. de Paris," 3rd Series, vol. 19,

p. 191), similar to the muscle which I described; this passed nearly horizontally as in Professor Luschka's case, quoted above.

Aryteno-corniculatus obliquus and rectus, or depressores cartilaginis Santorini, are muscles described by Professor Luschka; and their names denote their positions; the former are differentiated portions of the arytenoideus obliquus proprius, and the latter straight bands from the arytenoid base to the corniculum laryngis. This latter has been seen by Tourtual ("Neue Untersuchung über den Menschlichen, Schlund-u-Kehlkopfs," 1846, p. 105); they are both described by Professor Luschka ("Reichert's Archiv," 1869, p. 595).

The digastric is very variable.—1. Albinus describes having found it double; 2. An accessory slip of origin for the posterior belly from the styloid process, coexisting with the insertion of the stylo-hyoid into the mesial tendon (Wood); 3. An accessory slip for the posterior belly from the angle of the lower jaw, behind and below the internal pterygoid, Henle ("Muskellehre," p. 112); 4. A fasciculi of fibres from the splenius I have seen united to its origin 5. Its pit of origin may be deep and narrow, or shallow and broad; in the former case the fibres extend to a large extent on the mastoid process proper; 6. This muscle is described as arising from the styloid process in the "Bibliotheca Anatomica," vol. i.; 7. M'Whinnie found it to possess a large posterior and a very small anterior belly in a case in which no stylo-hyoid was present (Lond. "Med. Gazette," 1846, vol. ii., p. 186); 8. Cases in which the posterior belly was inserted into the middle of the ramus of the lower jaw are described by Platner ("De Musculo digastrico Maxillæ Inferioris," Leipsic, 1737, p. 14,) and by M'Whinnie (loc. cit.); in examples of this kind the muscle is monogastric, it is a very common mammalian plan of arrangement; 9. A third belly anteriorly is a variety of not uncommon existence being present once in seventeen subjects according to Mr. Wood, and once in fifteen according to Hallett: this may be—*a.* merely a splitting of the anterior belly with normal attachments; *β.* or a secondary anterior belly may arise from the tendon and pass above the lesser cornu of the hyoid bone, dividing it into two slips which decussate with similar slips on the opposite side; cases of this kind have been recorded by Hallett, Fleischmann, Gantzer, Sömmerring, Theile, Henle, Wood, and myself, as well as by Cloquet, Monro ("Essays of a Society at Edinburgh, &c.," vol. ii., p. 266), and Weitbrecht ("Acta Petropol.," vol. ix., p. 265); *γ.* or a

belly like this may be present on one side, while a normal anterior belly exists on the other. A case of this kind is described by J. Zachariah Platner (loc. cit. Tab. m); δ. the second part may unite with its fellow in a median raphe below the chin, overlapping the mylo-hyoid muscle (Wood and M.); ε. or the same may occur on one side, while the opposite muscle is normal: in this case the insertion of the aberrant belly is into the median line of the mylo-hyoidei. I have seen this three times. ζ. A slip of the anterior belly may descend to be inserted into the hyoid bone ("Weber's Hildebrandt," p. 369). 10. The tendon of intersection not unfrequently is continued from one side across to the other, forming a zone immediately above the hyoid bone, to the body of which it is tied down by fibrous tissue; from the upper surface of this the anterior bellies arise, either α. distinct from each other; or β. more rarely inseparable; γ. still more seldom I have found them with decussating fibres; this is by no means an uncommon arrangement among the Quadrumana, and has been described in man by Haller (Lib. IX., sect. 11), Sömmerring (p. 104), and myself. 11. Three fasciculi from the hyoid bone near the mesial line, and on both sides, passed up internal to the anterior belly to be inserted with it (Hallett); this is an instance of the union of the digastric and the mento-hyoid to be referred to hereafter; 12. The stylo-hyoid has been found inserted into its tendon; 13. The mesial tendon, which usually passes through the stylo-hyoid, I have found passing in front of the entire stylo-hyoid (Hallett); 14. This may pass behind it, but this is much rarer (M.); 15. The cleft in the stylo-hyoid I have never seen strong enough to act as a pulley, as Boerhaave taught; and it is probable that, as Laborie showed, no such pulley arrangement can ever occur in this muscle (Laborie, quoted by Portal, p. 98); 16. The anterior belly was found fused with mylo-hyoid by Casserius, (vol. i. Tab. 1, fig. 1E,); 17. The tendon I have found sending distinct slips of insertion into the lesser cornu of the os hyoides; 18. A case described by Mr. Lovegrove is given in a letter by Mr. Pittard ("Med. Times and Gazette," N. S., vol. xviii., p. 198), in which the digastric of the left side sent an expansion into the mylo-hyoid of the opposite side.

From the body of the os hyoides to the chin a muscle has been found passing, for which, in 1867, I proposed the name mento-hyoid. This may be a single mesial slip, or may consist of two parallel bands. Variety No. 11 of the

digastric seems to be a case of the fusion of this muscle and the digastric. The most remarkable form of it which I have yet found was one in which it formed a median triangle, whose apex was at the chin, and its base at the hyoid bone. From its frequently superficial position, and its occasional connexion to the platysma, I think it may be regarded as a differentiated position of that muscle, as I have frequently found it on a plane superficial to the digastrics. I have also seen it appearing like a mental continuation of the sterno-hyoid. Instances of this muscle are described by Schwegl (loc. cit.).

The mylo-hyoid has been found—1. Inseparably connected with the anterior belly of the digastric (Casserius); this occurrence was found by Hallett as often as once in every five subjects, a most unusual degree of frequency; 2. Almost completely absent, and replaced by the anterior belly of the digastric; this I have once seen, and Hallett has noted its occurrence; 3. Having a slip of sterno-hyoid continued into it (Haller, vol. vii., p. 248; and M'Whinnie); 4. Or of the stylo-hyoid (M.); 5. Having some of its fibres inserted into the tendon of the digastric (M'Whinnie, Horner, and myself); 6. Fasciculation of this muscle has been noticed by M'Whinnie, and I have seen it split into two parts by the Whartonian duct, which in this case perforated the muscle; 7. I have seen a slip from the omo-hyoid continuous into it; 8. The hyoid insertion I have seen (α.) reduced to the breadth of the tenth of an inch; or, (β.) extending from one lesser cornu, along the entire upper margin of the body of the hyoid bone to the other; 9. I have several times failed to separate it without tearing, from the genio-hyoid, at its hyoidean attachment; 10. The stylo-hyoid has been seen passing into the outer border of the muscle; 11. And it has also been found with a large bundle continuous into the mylo-pharyngeus.

The genio-hyoidei I have found—1. Blended and inseparable in the mesial line, as described by Hallett and Theile; 2. Each cleft into two, making thus four in all; this is also described by Meyer ("Beschreibung des Mensch. Körpers," Bd. iii., p. 547); 3. Horner saw this muscle receiving an accessory slip from the great cornu of the hyoid bone; 4. I have seen the mento-hyoid in a case of partial deficiency of the mylo-hyoid inserted into this muscle; in this instance the mento-hyoid was triangular; 5. They have also been found by myself inseparable from the genio-hyo-glossus, and joined by its outer edge to the hyo-glossus in another instance.

Genio-hyo-glossus occasionally has been seen—1. Sending a fascicle to the epiglottis; this is described by Prof. Luschka as the genio-epiglotticus ("Reichert's Archiv," 1868, p. 224); 2. Sending a fasciculus to the pharynx which is described by Winslow ("Tr. de la Tete," p. 480), as the genio-pharyngeus; in Todd's Cyclop., vol. iii., p. 365, it is confounded by Mr. Simon with the lingual origin of the superior constrictor; but which, although several times present, I have never seen sufficiently separate to deserve a special name; 3. An accessory deep slip from the stylo-hyoid ligament I have seen blending with the fibres of the genio-hyo-glossus; 4. Or a slip of this muscle has been found by Sömmerring extending to be inserted into this ligament (p. 122); 5. It has been found triple (Meyer, vol. iii., p. 547); 6. Professor Luschka found a slip of the genio-epiglotticus continuous directly with the stylo-glossus.

Mylo-glossus is a muscle described by Rolfincius, Blancard, and Wood; it passes from the angle of the jaw to the tongue, usually close to the stylo-glossus, of which, indeed, it seems to be an aberrant slip. It has been seen taking the place of the stylo-glossus in cases when that muscle is absent, or forming a second head for that muscle (for these conditions, see stylo-glossus).

Stylo-glossus—1. has been found arising from the internal pterygoid muscle by Moser ("Meckel's Archiv," vol. vii., p. 226); 2. From the angle of the lower jaw (Riolan, p. 74; Winslow, p. 519); these authors describe it as arising by a fibrous band, and Douglas and Theile describe it as a fleshy band, a form which I have seen. 3. Cloquet states that he has found it frequently arising mainly from the stylo-maxillary ligament (p. 285). 4. Harrison assigns as its origin the inner side of the styloid process, near its point (p. 38). Gray states that it rises from the anterior and outer side (p. 222); 5. A second head may come from the stylo-maxillary ligament; 6. This second head, either ligamentous or maxillary, may send some fibres into the stylo-hyoid, and some into the stylo-glossus; 7. Sometimes this slip from the angle of the jaw to the tongue is separate for its whole length, forming a mylo-glossus of Hallett and Wood. 8. A double stylo-glossus composed of parallel bellies is described by Courcelles, Cheselden, Haller, Meckel, and M'Whinnie, and as two superposed laminæ, Fallopius ("Institutiones," p. 56). 9. Portal has seen it sending deep

fibres to join the genio-glossus ("Cours d'Anat. Medicale"); 10. Sandifort saw it sending one slip to its normal insertion and one into the pharynx (" Exercit. Anat. Lugd. Batav.," 1758, c. 6, p. 81); 11. Absence on one side alone was seen by Albinus ; 12 On both sides by Böhmer (loc. cit., p. 6). In this case the other styloid muscles were larger than usual, but in a case seen by Quain the other pair were of normal size. It has been seen by Hyrtl and Gruber, attached to the stylo-auricularis ; and in one case the latter author describes one slip as passing from the external meatus to the tongue ; this he calls auriculo-glossus (see " Henle u. Pfeufer's Zeitschrift, Reihe 3, Heft 3, Bd. x., p. 368). A list of the anomalies of this muscle I published in my paper on the varieties of the styloid muscles ("Journal of Anatomy and Physiology," vol. v., p. 28).

Stylo-hyoid is subject to a good many varieties, and is treated of in a small paper by myself in the " Journal of Anatomy and Physiology," vol. v., p. 28.—1. It has been found triple by Hyrtl (" Lehrbuch der Anatomie," p. 290), and Gruber ("Müller's Archiv," 1848, p. 424) ; 2. Absence has been described by Hallett and Böhmer (" Observationes Anatomicæ Rariorum, Præfatium Fasciculus alter" Halæ, Magdeburg, 1712, p. 5), the former author describes the occurrence as present in one in every two hundred cases ; 3. It has been found simply doubled, the two muscles being similar in attachments, by Eustachius, (Tab. xli., fig. 5), Albinus, Bidloo (Tab. xiv.), Santorini, (" Observ. Anat." Venet., 1727, p. 117), Wistar (" Anatomy," Pennsylvania, 1823, p. 189) ; 4. I have seen it in two parts, one over, and the other under the digastric tendon and the same has been noticed by M'Whinnie ; 5. The second stylo-hyoid, when this muscle is double, has been found arising from the tip of the styloid process, coexisting with the stylo-hyoid ligament, cases of this are described by Courcelles, Santorini, (loc. cit.), p. 117 ; and Boyer ("Trait. Complet d'Anat.," Paris, 1815, p. 87); 6. Or the supplemental muscle has been seen replacing the stylo-hyoid ligament by Drake (" Anthropographia," Book iii., ch. 17) ; Weitbrecht ("Comment Petropol.," ix., p. 256), Blandin ("Nouveaux Elemens. d'Anat. Descriptive," 1838, p. 374) ; and Gavarde (" Traité de Myologie suivant la Méthode de Desault." Paris, An. vii.), in all these cases the muscle was attached to the lesser cornu of the hyoid bone ; 7. A second stylo-hyoid, arising from the base of the styloid process, is described by Mr. Lawson Tait ("Journal of Anat.,"

vol. iv., p. 237; 8. One inserted into the cartilago-triticea is described by Petsche ("Haller's Disput. Anat. Selectæ," vi., 767); 9. A normal stylo-hyoid inserted into the digastric tendon by two slips is a rare form, described by Haller ("Element. Physiol.," iii., p. 416), M'Whinnie, and myself ("Journal of Anat.," vol. v., p. 29); 10. A normal stylo-hyoid may pass undivided behind the digastric (Hallett and myself); 11. Or, still rarer, in front of the digastric tendon (Sömmerring and myself); 12. A second head for a normal stylo-hyoid has been found from the angle of the jaw by Mr. Kelly and myself; 13. This angular slip I have also described as having been once nearly detached, forming a hyo-angularis muscle (loc. cit., p. 29); 14. I have also described a stylo-maxillary muscle in the form of an offshoot to the angle of the jaw from the stylo-hyoid: a similar case is described by Meyer ("Beschr. des Menschl. Körpers," iii., p. 547); this muscle lay on the stylo-maxillary ligament; 15. Sömmerring describes connecting slips between the omo-hyoid and the stylo-hyoid (vol. iii., p. 113); 16. And also a slip from the stylo-hyoid ligament to the genio-hyo-glossus; 17. A curious case of complexity has been described by Mr. Wood, in which a separate styloid fascicle of muscular fibres joined the posterior belly of the digastric, and this coexisted with an insertion of the stylo-hyoid into the digastric tendon; 18. A slip of the stylo-hyoid inserted into the tongue under the hyo-glossus.

Triticeo-glossus is a small muscle, first particularly described by Bochdalek, junior, of Prague ("Prager Vierteljahrschrift," Hft. ii., 1866), but which is very liable to be overlooked, unless carefully sought for; it passes from the corpus triticea in the posterior thyro-hyoid ligament upwards and forwards to enter the tongue along with the kerato-glossus muscle; it may be connected to the kerato-glossus, but more commonly is separate; the former condition only occurs when the kerato-glossus lies posterior to the lingual artery—a position which the triticeo-glossus, in my experience, always occupies. Bochdalek describes its frequency as being in the proportion of eight out of twenty-two subjects; but in the Dublin College of Surgeons' Dissecting Room I have not found it within the past two years more frequently than once in every six subjects, and not as a perfectly separate muscle oftener than about once in thirty. Sömmerring speaks of a fourth part of the hyo-glossus which may be this muscle (p. 122); it is always inserted with the hyo-glossus.

Hyo-glossus varies chiefly in the degrees of separation and of development of its three components, which were first distinguished from each other by Albinus. Mr. Kelly, Demonstrator of Anatomy in the Carmichael School of Medicine, Dublin, has communicated to me an instance in which a slip of this muscle lay superficial to the ninth nerve. Sömmerring found it four-cleft, as above described. It has been seen connected to the genio-hyoid, or with the insertion of the stylo-glossus blended with it. A slip from the thyro-hyoid has also been continuous with its origin.

Linguales. Of this series of muscles Cloquet describes the following irregularity. A small bundle ascending towards the frænum of the epiglottis to be inserted into the dorsal surface of the epiglottis where the genio-glossi touch each other (Knox, Tr., p. 285). Something similar is the central muscle of Bochdalek, a small muscle in the posterior fourth of the tongue, where the middle bundles of the genio-glossus are close together, in the middle line ("Anatomische Beiträge, Reichert's Archiv," 1866, p. 744). The presence of vertical linguales as normal constituents in the tongue is denied by Prof. Theile; and he considers the fibres usually described as such as detached portions of the genio-hyo-glossus, with which indeed they are usually connected. The division of the other intrinsic fibres is purely arbitrary.

Genio-glossus accessorius: under this name Professor Luschka describes a separated slip of the genio-glossus from the lowest point of the genial tubercle, separated by areolar tissue from the rest of the muscle.

The cephalo-pharyngeus muscle has been described by Sandifort ("Exercit. Acad. Lugd. Batav.," 1783, lib. 2, cap. 6, p. 61); and by Albinus ("Annot. Acad.," lib. 4, cap. 4, p. 26); and Rolfincius ("Dissert. Anat.," p. 534). It has been found to arise—1. from the vaginal process of the temporal bone, separated from the stylo-pharyngeus by the glosso-pharyngeal nerve, and lost below on the inferior constrictor of the pharynx; 2. On one occasion Meckel found this muscle double, presenting the appearance of two accessory fascicles to the stylo-pharyngeus; 3. I have found it to arise from the spine of the sphenoid, and from the pterygo-spinous ligament of Civinini, blending at its insertion with the inferior constrictor; 4. From the angle of the petrous portion of the temporal bone, external to the attachment of the pterygo-pharyngeal aponeurosis, as well as from the vaginal process I have found it arising; 5. I

have seen it inserted into the mucous membrane of the pharynx, passing between the middle and superior constrictions, as described by Albinus; 6. I have seen a strong fibrous band from the spine of the sphenoid taking its place; 7. It has been found attached to the cartilage of the Eustachian tube by Sandifort and myself; 8. In general this muscle is a flat ribbon, more rarely a round bundle; a third form was seen by Professor Luchtmans (quoted by Sandifort) in which the muscle arose outside the petrosal canal, and was inserted into the pharynx by radiating fibres. I have found the former methods of arrangement, but not the latter. It sometimes is fused with the superior constrictor of the pharynx, of which it appears as a supernumerary origin. A specimen is described as occipito-staphylinus by Sappey ("Tr. d'Anat. Descriptive," Paris, 1857, tom. iii., p. 38).

The stylo-pharyngeus alter of Böhmer has been referred to under the title stylo-pharyngeus, as it is nothing but a variety of that muscle by duplicity.

Azygos-pharyngis is a small slip, which overlies the superior and middle constrictus, arising from the pharyngeal spine of the basilar process of the occipital bone, and inserted into the raphe of the pharynx; this was described by Meckel, and I have met with it not unfrequently. It is also referred to by Santorini ("Obs. Anat.," Venetiis, 1724, p. 121); it is always single and central, and usually overlies, though occasionally it is inseparable from the superior constrictor; it rarely reaches a greater length than from a quarter to half an inch, but I have seen and figured it once twice this length.

Constrictor pharyngis superior is a composite muscle, according to Prof. Luschka, and consists of pterygo-, bucco-, mylo- and glosso-pharyngeus, separate parts. Of these the bucco-pharyngeus was described and figured as a separate muscle by Rowley ("Schola. Med. Universalis Nova," London, 1794, vol. i., Tab. 18, fig. 4 k); and the glosso-pharyngeus and mylo-pharyngeus by Santorini ("Obs. Anat.," Venetiis, 1824, p. 123), and Albinus and Winslow ("Exp. Anatomique," Amster., 1793, p. 350). The pterygoid head I have seen nearly attached to the root of the hamular process, attached for little more than half the process (the commonest), or for its whole length; it is to this portion that the cephalo-pharyngeus is most frequently attached, especially when in the form

of the petro-pharyngeus (Winslow). The bucco-pharyngeus I have described with muscular fibres continued into the buccinator.

The mylo-pharyngeus has been seen continuous with the outer fibres of the mylo-hyoid, and the glosso-pharyngeus may be continuous with the hyo-glossus, genio-glossus, or lingualis transversus superior. Winslow describes under the name genio-pharyngeus a slip distinct from the genio-glossus, and passing to the side of the pharynx; the bucco-pharyngeus is the weakest, and it has been found nearly deficient.

Constrictor pharyngeus medius—1. May receive a band from the vaginal process of the temporal bone to the middle constrictor; 2. Or a slip from the thyro-hyoid muscle; 3. An extension of this muscle as far as the base of the skull, along the stylo-hyoid ligament; 4. Or a slip may spring from the corpus triticea; 5. Or the muscle may be tripartite, one from the cornu, one from the corpus triticea, and one from the thyro-hyoid ligament, Sömmerring (p. 130), Hasse ("Myotomiæ Specimen," Leipsic, 1784, p. 28); 6. The central part is often weaker than the upper and lower, giving the appearance of a bipartite muscle; 7. I have seen it with difficulty separable from the succeeding muscle.

Constrictor pharyngeus inferior has been found—1. With an origin from the trachea; 2. Or a slip from the thyro-hyoid muscle; 3. Or from the crico-thyroid; 4. Or the sterno-thyroid.

Levator palati varies in the following ways:—1. I have found it split for its whole length—one part arising from the Eustachian tube, and the other from the petrous bone; the former part has been described as a separate muscle under the name salpingo-staphylinus; 2. The whole muscle may be salpingeal in origin; 3. Or the portion from the Eustachian tube may be suppressed; 4. The petrosal part may be split; 5. Or its origin single, and its insertion double.

The tensor palati—1. May be split for its whole length; 2. I have seen it receive a slip from the internal pterygoid muscle; 3. Another accessory slip may exist from the outer border of the scaphoid fossa, which joins the rest of the muscle about midway; 4. On one occasion I found a fasciculus arising as described (No. 3), passing downwards, lying on the pharyngeal aponeurosis and tensor palati, passing beneath the levator palati, and below the level of the pa-

late process of the palate bone; 5. A part of this muscle may be inserted into the root of the hamular process, Albinus ("Historia Musculorum," p. 247); 6. A fascicle of the muscle I have traced, arising from the hamular process and inserted into the palate bone; 7. I once saw a few muscular fibres in the tendon after it had passed the hamular process, giving the muscle a digastric arrangement; 8. It has rarely some fibres attached to the cellular tissue and wall of Gerlach's bursa (figured first in his "Dissertatio de Bursis Mucosis in Capite et Col.," repet., Viteb., 1793, p. 31, Tab. ii. x., x.); 9. A slip to the buccinator (Sömmerring, p. 133).

Motor uvulæ I have only found to vary in the degree of separation of the twin muscles. I have seen them in all stages, from perfect fusion to perfect distinctness. While referring to this muscle, I might mention a singular mode of its action which I noticed in a patient under my care in the Adelaide Hospital Dispensary, two years ago. The man was a baker, and suffered from chronic enlargement of the tonsils; his uvula was not enlarged, and was natural in appearance; on directing him to draw in a long breath when his mouth was opened widely, the uvula retracted in a remarkable manner, the fundus of it being drawn up, making the organ look like the inverted finger of a glove; this action he could perform at pleasure; in a case of this kind the motor uvulæ must have been inserted into the extreme tip of the uvula, and have been free from the rest of the organ.

The palato-glossus is not subject to much variety. I have found some of its fibres traceable back beneath the tonsil towards the palato-pharyngeus. The two muscles are often joined by a slip across the mesial line of the velum.

Palato-pharyngeus I have found split into two parts. Winslow describes it as divided into three parts—peristaphylo-pharyngeus, pharyngo-staphylinus, and thyro-staphylinus, passing, as their names imply, from the velum to the pharynx, from the uvula to the pharynx, and from the uvula to the hinder margin of the thyroid cartilage. Professor Luschka divides it into a pars thyreo-palatina, and pars pharyngo-palatina ("Virchow's Archiv," xlii., p. 482), and ("Anat. des Menschlichen Halses," Tübingen, 1862, p. 200).

Salpingo-pharyngeus. A slip behind the palato-pharyngeus, passing from the Eustachian tube to the pharynx, is described by Sandifort ("Historia Musculo-

rum," Lugd. Batav., 1781) and Santorini ("Obs. Anat.," Venetiis, 1727, p. 122), but I have never met with it; it is a detached slip of the last named.

In a work called "Bibliotheca Anatomica, containing a Description of the several parts of the Human Body," London, 1712, a muscle is described under the name of peristaphylinus externus as arising from the upper jaw bone under the first molar tooth, and inserted into the side of the uvula!

Two new palate muscles have been described by Tourtual ("Müller's Archiv," 1841, p. 452), under the name of pterygo-palatinus, or levator palati mollis anterior and petro-salpingo-staphylinus; they are deep portions of the levator palati.

The varieties of the scalenus anticus are the following:—1. It may be absent, as I found on one occasion. Isenflamm found all the scaleni deficient in a hunchback ("Abh. der Knocher u. der Krankh.," 1782); 2. It may receive a large slip across the subclavian artery from the scalenus medius, as I have also noticed; and it has also been described by Mr. Wood; 3. I have seen the converse of this, or a band from the scalenus anticus passing behind the artery to be inserted into the tubercle of the first rib; 4. The vertebral attachments are liable to variation in point of number from the second to the sixth, but they are most commonly third, fourth, and fifth; 5. Its origin has been seen to be continuous with some fibres of the rectus capitis anticus major; 6. I have often found the fibres of origin inseparable from the intertransversales; 7. I have also seen it split into two parallel slips at its origin; 8. Moser found it pierced by the phrenic nerve ("Meckel's Archiv," vol. vii., p. 226); 9. I have seen its lower third tendinous.

Scalenus medius—1. May have a thick bundle of fibres covering the first intertransversalis, from the transverse process of the atlas (Wood); 2. Or it may receive a large fasciculus from the scalenus anticus (M.); 3. Mr. Wood has seen it giving a bundle of fibres from the insertion of this muscle to the levator anguli scapulæ; 4. Its absence has also been noticed by Isenflamm (loc. cit.); 5. its entire insertion has been seen into the second rib; 6. I have seen it attached to six cervical transverse processes, and the number of vertebral attachments I have found to vary from this to two, I have seen it from all but the seventh, from all but the atlas, from the second, third, fourth, and fifth, from the second, third, and fourth, from the second and third, from the third, fourth, and fifth, or the third and fourth alone; unfortunately I have preserved

no statistics of the relative frequency of these varieties of origin; 7. I have seen it inseparably united to the scalenus posticus not uncommonly; 8. Or at its origin to the multifidus spinæ or semispinalis colli; 9. Or pierced by some branches of the brachial plexus.

Scalenus posticus may be—1. Absent, Isenflamm (loc. cit.); 2. May send a slip to the levator anguli scapulæ; 3. May be split into three separate muscles going to the first, second, and third ribs respectively; 4. May entirely go to the third rib; 5. Or entirely to the first. All these varieties I have seen.

Scalenus minimus is described by Albinus, Sömmerring (p. 170), and Meckel; it lies behind the scalenus anticus and arises from the transverse process of the fifth, sixth, and seventh cervical vertebræ, and is inserted into the second rib. Meckel has seen it double, and I have once seen the same; its occurrence may be reckoned at about three times in seven subjects, but it is oftener present inseparably united to the other scalenes.

Scalenus lateralis of Albinus and Sömmerring (loc. cit.), lies outside the scalenus posticus and is inserted into the second rib from the lower cervical vertebræ. In one subject Meckel describes having found three accessory scaleni, two minimi, and one lateralis; it looks like a differentiated portion of the scalenus posticus.

Scalenus accessorius is a small muscle which I have found lying under cover of the scalenus medius, arising from the posterior tubercle of the fourth, fifth, and sixth cervical vertebræ, and inserted into the first rib, separated from the scalenus medius by a plane of cellular tissue, and part of the brachial plexus; this muscle has the same relation to the scalenus medius that the two former have to the anticus and posticus.

Rectus capitis anticus major I have seen arising from the second to the fifth cervical transverse process, or from the third to the fifth, more rarely from the third to the sixth, or from the second, third, and fourth: when a third occipital condyle (Halbertsma) existed, its insertion was immediately in front of it. I have seen it partly bilaminar. A few fascicles I have found to be continuous with the scalenus anticus. I have seen it also split into fascicles, or joined inseparably with the longus colli. Meckel has found it to arise from the first and second vertebræ. I have found its origin continuous with the intertransversales.

Rectus capitis anticus minor I have found double, or connected to the last, or running into the anterior occipito-atlantoid ligament; some fibres may spring from the anterior half arch of the atlas, and these may be separated from the rest of the muscle. A slip may also come from the axis (Henle). I have found it to be deficient twice.

Rectus capitis lateralis I have found double, fanshaped, deficient, (Henle); the second when present is the rectus capitis lateralis accessorius of Winslow, In case where the lateral process of the occipital bone is "pneumatic" (Hyrtl, "Wiener Med. Wochenschrift," 1860, No. 45), this muscle is widened in its insertion.

Longus colli (rectus colli of Luschka) may vary in the number of its tendons, may be thin and nearly absent, or may multiply its slips from each vertebra. A good description and figure of this muscle are given by Professor Luschka ("Müller's Archiv," 1854, p. 103, and Pl. 7).

Obliquus colli inferior may have a slip from the head of the first rib, may extend to the fourth dorsal vertebræ, and may even have a slip to the third rib; it may join the intertransverse muscle, and is often joined to the last by fibres.

Longus atlantis or obliquus colli superior (Luschka) may have origin from the third and fourth, fourth and fifth, or third, fourth, and fifth cervical vertebræ. I have seen it joined to the rectus colli or rectus capitis anticus major until near its insertion, sometimes it is continuous with the scalenus anticus.

Transversalis cervicis anterior is a muscle described by Retzius and closely connected to the longus colli; it arose from the oblique process of the sixth, fifth, and fourth cervical vertebræ, and was inserted into the third, second, and first; I have never met with an instance of this anomaly ("Forhandlinger, ved de Skandinavisk Naturforsk.," 1841, p. 767).

Of the pectoralis major the following anomalies have been seen and described—1. absence of all but the clavicular part of the left muscle ("Guy's Hospital Reports," Poland, 1841, p. 191); 2. Absence of the sternal part of the right great pectoral I once noticed in a thin subject; 3. Absence of the clavicular origin is described by Nuhn ("Untersuchung u. Beobacht.," Heft 1. p. 19), Quain p. 317, Barkow ("Monstra. Duplicia.," Lips. 1828, p. 21),

Gruber ("Virchow's Archiv," vol. xi., p. 42); 4. The lower half of the costal portion is absent occasionally ("Quain," p. 233); 5. On the other hand the entire muscle has been found doubled in an athlete by Tiedemann ("Journal Complementaire du Dict. des Sc. Med.," tom. 6, Paris, 1830, p. 272); in this case the deep portion had no clavicular origin; 6. I have found the sterno-costal fibres bilaminar, while the clavicular were undivided; this occurred in a subject of no unusual degree of muscularity; it seems to be a parallel case to the one described by Tiedemann; the deeper stratum gave origin to the anterior suspensory frænum of the capsular ligament; 7. Mr. Wood describes finding the clavicular fibres split into two, only united at their insertion; 8. I have found the three parts of the muscle perfectly separate, clavicular, sternal, and costal; 9. The pectoral and deltoid I have found completely fused, the cephalic vein either passing between them low down or passing over the clavicle, Cheselden (p. 85), Otto ("Path. Anat.," 1830, p. 249), Seiler ("Obser. Anat.," 1808, fascic., i.), Wood and Koster ("Nederl. Archief," 1864, p. 369), have also described cases of this kind; 10. The origin may extend down as far as the seventh or eighth rib; 11. Or fibres may be continuous with those of the external oblique; 12. A slip from the fifth rib cartilage connected with the lower costal fibres of the great pectoral passed upwards to be inserted along with the pectoralis minor into the coracoid process, sending some fibres into the coraco-brachialis (Wood); a similar case occurred in my experience; 13. In another instance seen by Mr. Galton, and published by Mr. Wood, the slip was inserted lower down into the fascia over the coraco-brachialis, an inch or so below the coracoid process. Cruveilhier describes a case of this kind in which the slip passed nearer to the coracoid process (p. 149); 14. I have seen a slip continuously extended into the biceps' short head; 15. and once I saw the whole short head arising from the great pectoral tendon; 16. A slip from this tendon extending to the supraspinatus was described by Horner; 17. The tendon of the great pectoral I found once split in the axilla by the long tendon of the biceps, the sternal and costal fibres passing behind while the clavicular fibres were in front; 18. A partially separate slip inserted into the capsule of the shoulder seems to be the representative of a muscle described by Gruber to be referred to afterwards; 19. The upper margin of the tendon sends up a frænum of fibres to the capsule of the shoulder; this was first described by Boyer ("Traité Complète d'Anat.,"

Paris, 1815, p. 116). In the Cyclopædia of Anatomy and Physiology (Art. Shoulder, vol. iv., p. 574,) this is mentioned as the suspensory frænum of Winslow, but it is plain that the only frænum mentioned by the last named author is that "ligamentary frænum" from the teres major and latissimus dorsi tendon ("Winslow's Anat." Douglas' Translation, London, 1776, p. 181, ss. 198, 204). Sömmerring mentions a slip from the pectoral tendon passing upward to join the insertion of the supraspinatus and the capsule of the shoulder (p. 220); 20. It is very often united to the next muscle when it is present; 21. Muscular fibres pass occasionally across the sternum from one side to another; 22. Union of the upper fibres of the rectus abdominis with the lower border of this muscle is a condition which has also been observed; 23. and the rectus sternalis has also been observed inserted into the pectoralis major; 24. The clavicular fibres have been seen joined to the deltoid, while they were separated as usual from the sternal portion.

Chondro-epitrochlearis. By this name Mr. Wood and other authors recognise that slip usually connected with the great pectoral which, arising low down from the ribs is inserted on the inside of the arm. Various forms of it are described: Gruber found it in three bands, the chief one of which passed over the vessels and nerves to the middle of the inner condyle; Struthers found it passing directly to the inner condyle (Struther's "Anat. and Phys. Observ.," Edinburgh, p. 199); Sömmerring found a slip from the great pectoral going to the brachialis anticus, which is evidently of this nature (p. 219); Caldani has described a slip like that referred to by Struthers ("Institutiones Anatom.," tom. I. p. 239, note, Leipsic). A separate chondro-epitrochlearis to the brachial aponeurosis is described by me, and a description of a similar specimen has been forwarded to me by Dr. Kelly, Demonstrator of Anatomy, Carmichael School of Medicine, Dublin. A still more extended slip is described by Blandin, stretching as far as the fascia of the forearm. A digastric chondro-epitrochlearis has been once met with likewise. The origin of this muscle is usually from the cartilages of the lower true and upper false ribs, and its insertion either into the fascia of the arm or into the condyle. When fascial it is usually connected to the internal brachial ligament from two to six inches above the condyle. Wood has described fibres with the characteristic origin of this muscle stopping short at the bicipital groove, distinct from the great pectoral.

A similar band is described by Bell and Wood inserted into the upper border of the pectoral tendon and continuous with Boyer's frænum. Another flat band arising by two slips, one from the abdominal aponeurosis and the other from the cartilage of the sixth rib, passes upwards to be inserted into the fascia over the coraco-brachialis extending as far as the coracoid process (Wood); this may be an example of the next muscle. A similar case I have found arising farther out, and Cruveilhier (vol. ii. p. 85, Paris, 1834,) saw it inserted into the short head of the biceps ; fibres of this muscle often stop short in the axillary fascia, and occasionally are connected to the achselbogen (described sub voce latissimus dorsi). It is also when present not uncommonly continuous by some fibres with the obliquus externus.

Pectoralis quartus is a muscle whose indefinite nature and position in lower animals have led to its receiving a multiplicity of names ("Ann. Nat. Hist." 1869), it is rarely present in man but the last example but one given above under the title chondro-epitrochlearis should possibly belong to this. Its origin is from the lower ribs and lateral thoracic fascia, its insertion usually with or below the pectoralis major, or else purely fascial; the muscle described by Betz (Froriep's "Tagesberichte," No. 211,) probably is of this nature, it arises from the fascia over the serratus magnus and is inserted into the lower border of the pectoral tendon in the arm; from the relations of this muscle in lower animals it is not improbable that it is represented in the connecting band so commonly noticed between the pectoralis major and the latissimus dorsi (the achselbogen of Langer).

Pectoralis minor presents a good many varieties—1. Poland describes its absence in the case of deficient great pectoral ("Guy's Hosp. Reports," loc. cit.), and the same has been noticed by Barkow ("Monstra. Animalium Duplicia," Lips., p. 21) ; 2. Tiedemann describes its being double in a subject with double great pectoral muscles (which see); in this case the deep one arose from the second and third ribs (op. cit., p. 272) ; 3. This muscle was seen by Mr. Wood split into four parts, the first passed from the second rib and intercostal fascia, over the coracoid process, giving off a slip to the coraco-acromial ligament and was inserted into the glenoid and capsular ligaments ; the second portion from the third and fourth ribs was inserted by tendon into the coracoid process ; the third arose with the fourth from the fifth rib, and was inserted

into the coracoid process, while the fourth passes over the coracoid muscles, pierced the capsular ligament above the subscapularis, and joined the tendon of the first as it was inserted into the glenoid ligament; 4. the tendon of this muscle is prolonged over the coracoid process not unfrequently, seventeen times out of a hundred and six subjects; according to Mr. Wood these fibres may go to the supraspinatus tendon, to the capsular ligament, to the great tuberosity, or to the glenoid ligament; when the fibres are not continued over the process this portion of the tendon exists as the coraco-glenoid ligament; 5. Clason describes an instance in which the tendon wound over the coracoid process and trifurcated, sending slips to the root of the coracoid, to the neck of the scapula and the glenoid ligament ("Om. Muskel. Anom.," Upsala, II. Bd., p. 417); 6. the continued fibres may be lost in the coraco-acromial ligament (Wood); 7. the same author found, in a case in which the origin of this muscle extended as high as the first intercostal aponeurosis, that its upper fibres were inserted into the costo-coracoid membrane and not into the clavicle itself; 8. or into the costo-coracoid membrane and not into the clavicle (Wood); 9. or into the clavicle by a separate portion from the second rib, forming an imperfect sterno-clavicularis; 10. its origin may be from the second, third, fourth, and fifth, or the third, fourth, and fifth, or from the second third, and fourth ribs, and it may even receive a slip from the sixth rib (Blandin); 11. as noticed under the pectoralis major this muscle may receive a slip from that at its insertion; 12. the lowest fibres missing the coracoid process may go into the coraco-brachialis, as I have seen, and is described by Sömmerring (p. 221); 13. or into the fascia over it; 14. or into the short head of the biceps, or into both coraco-brachialis and biceps, (Winslow, p. 175); 15. the entire tendon may go into the capsule of the shoulder (De Souza, "Gazette Med. de Paris," 1855, No. 12), or to the great tuberosity and capsule (Gruber).

Supra-clavicularis proprius is a unique muscle described by Gruber which passes above the clavicle from the sternal to the acromial end in front of the sterno-cleido-mastoid and trapezius ("Reichert's Archiv," 1865, p. 703); this muscle is fleshy in the middle, and tendinous at each end, and enclosed in a sheath of cervical fascia; functionally this muscle is regarded by Professor Gruber as a tensor fascia colli. It has never been described by any other author.

Præ-clavicularis lateralis or acromio-clavicularis (Reichert's "Archiv," 1865, p. 714) is a small muscle from the outer third of the clavicle which passes over the deltoid to be inserted into the tip of the acromion. It has not been described by any other author but Professor Gruber, but from its position it is extremely likely to be overlooked, and hence its remaining unnoticed by other anatomists.

Tensor semivaginæ articulationis humero-scapularis is a muscle which lies between the pectoralis major and minor, and arises from the manubrium sterni and cartilage of the first rib; it crosses the pectoralis minor a short distance below the coracoid process, and is inserted into the superficial layer or semivagina of the shoulder joint; this muscle was first noticed by Gantzer, but more fully described and named by Professor Gruber ("Mém. de l'Académie Imp. de St. Petersbourg," 1860, No. 2, tom. iii., p. 10). I found this muscle on the left side of a female Chimpanzee, but in this case it had an additional second-rib-cartilage origin. An instance of its existence in a subject in which the radius was deficient is given by Gruber (Virchow's "Archiv," Band 40, p. 427).

Scapulo-clavicularis is a small muscular band described and named by Mr. Wood, which arises from the outer side of the origin of the omo-hyoid and joined to that muscle, or as a fleshy slip, from the base of the coracoid process, and is inserted into the clavicle connected with the outer fibres of the subclavius. I have never met with this muscle. A somewhat similar slip is the coraco-clavicularis of Gruber and Calori which passed from the coracoid process to the outer third of the clavicle ("Mem. della Accad. delle Scienze, dell' Istituto di Bologna," 2 Ser., vol. 6, p. 137, and Reichert's "Archiv," 1863, p. 404); another coraco-clavicularis described by Koster ("Nederlandsch Archief," 2nd deel, afl. 2, 1864, p. 31).

Cervico-costo-humeralis (Gruber, "Mém. de l'Acad. Imp. de St. Petersbourg," 1860,) was found in a sailor as a complex muscular band passing from the oblique process of the sixth cervical vertebræ, and from the anterior end of the first rib, it extends between the latissimus dorsi and the subscapularis to be inserted into the humerus; this specimen seems to be unique, and its nature and homology seem very obscure.

Præ-clavicularis medialis, or sterno-clavicularis anticus; this name is given by Wenzel Gruber to a muscle either band-like, quadrilateral, or fusiform,

which he has described ("Mém. de l'Acad." &c., 1860, tom. iii., No. 2, and "Reichert's Archiv," 1865, p. 706) arising from the cartilage of the first rib or from the anterior sterno-clavicular ligament, either by a fleshy or tendinous band sometimes connected with the sternal fibres of the great pectoral (Gruber), passing superficial to the subclavius and internal to it, to be inserted into the front of the lower border of the clavicle for the middle third, extending sometimes as far as the outer third; sometimes the muscles of both sides are united in the middle, forming a digastric muscle ("Reichert's Archiv," 1865, p. 710, and Virchow, Bd. xl., p. 430); and in one instance it was inserted into the coracoid process (Schwegl loc. cit.); cases of this kind have been seen by Mr. Wood (Proc. R. Soc. 1866), by Mr. Berkeley Hill, and by E. Clason ("Om Muskel Anomalien, Upsala Läkareforenings Forhandlingar," Bd. II, p. 417). In Wood's case it arose from the lowest part of the manubrium; Clason's was on both sides. Dr. Stoker, Demonstrator of Anatomy, Royal College of Surgeons, Ireland, has communicated a case to me in which this muscle co-existed with an absence of the clavicular fibres of the great pectoral; it arose from the manubrium, and was inserted into the middle and part of the outer third of the clavicle; it was separated from the subclavius by the costo-coracoid membrane. A similar deficiency of the great pectoral was seen by Gruber (loc. cit. p. 708).

Supra-clavicularis (sterno-clavicularis, Hyrtl) was described by Haller ("Elem. Phys." Lausanne, vol. III., 1766, p. 46), Luschka ("Müller's Archiv," 1856, p. 282, and "Anat. des Menschen," Bd. Abth. 2, 1863, p. 173), Retzius ("Hygeia," 1856, Bd. 18, p. 649), Gruber ("Mém. de l'Acad. de St. Petersbourg," s. VII., vol. iii., No. 2, p. 4), Hyrtl ("Sitzungsberichte der k. k. Akad. in Wien. math. phys. Klasse," 1858, Bd. XXIX., p. 265), and Hellema ("Geneeskundig Tijdschrift," 5, Jahrg. 1st Afd). This muscle arises from the manubrium sterni or with a flat tendon from the medial tendon of the sterno-cleido-mastoideus; passing over the sterno-clavicular joint, it ends by being inserted into the clavicle. I found it in a subject in 1868, co-existing with a rectus sternalis, a levator claviculæ, a chondro-epitrochlearis, and an achselbogen. In Retzius' subject it went to the outer third of the clavicle on the left side, and partly arose from the sterno-clavicular capsule; in some cases the muscles of both sides were united by a median slip, and the origins often were connected to the inter-clavicular ligament. Gruber found its frequency to be five times

out of every hundred subjects; of these two were on the right, one on the left, and two on both sides. Hyrtl found it present six times out of eighty-three subjects; of these three were on both sides, one was on the left side, and two right. One of Hyrtl's varieties he names impar-bicaudatus. A case of it is described by Mr. Bucknill, and given in a letter from Mr. Pittard ("Med. Times and Gazette, N. S.," vol. 18, p. 198), under the name sterno-omoid.

Retro-clavicularis was described by Weber ("Handbuch," 1839, Bd. 1, p. 560); it arose from the back of the manubrium, and was inserted into the posterior surface of the sternal end of the clavicle. A similar muscle is described by Mr. Lawson Tait ("Journal of Anatomy and Physiology," vol. VI., p. 237), which arose by two heads, one from the posterior surface of the manubrium sterni at its junction with the cartilage of the first rib, the other was from the posterior edge of the rib itself; it lay on the brachio-cephalic trunk, on the lower thyroid veins, the scalenus anticus, where it had a tendon over the subclavian artery and the brachial plexus. The supra-costal muscle lay behind it and the omo-hyoid crossed over it, and here it had another tendon. It was inserted into the clavicle at the inner margin of the insertion of the trapezius. This is much rarer than the foregoing, and it is probably only a form of the supra-clavicularis given above.

Subclavius proper principally varies by being doubled; this variety may arise from simple splitting in the muscle, or apparently from the co-existence either of the previous or the succeeding muscle; this doubling is described by Sömmerring (p. 222), Haller (in "Boerhaave's Prælect. Acad. Instit. Med.," tom. 5, pt. 1, p. 95), Böhmer describes it as split into a coracoid and clavicular part and a costo-scapular part ("Obs. Anat. rarior. Halæ. Magdeb.," Preface p. 9, 1752); Rosenmüller describes the cleft normal subclavius ("Isenflamm u. Rosenmüller's Beiträge," Band. 1, Heft. 3, p. 375): the same arrangement as that referred to by Böhmer is mentioned by Sandifort ("Exercitationes Acad. Lugd. Batav.," 1783, p. 88). A normal single subclavius may be inserted into the coracoid process, into the acromion process, into the conoid or trapezoid ligament (Cloquet). A bursa may exist between the costal tendon and the first rib (Koch, "Dissertatio de Bursis Mucosis," Lipsiæ, 1789, p. 37). Its absence was described by Gruber ("Virchow's Archiv," vol. XL., p. 430).

Pectoralis minimus is described by Professor Gruber ("Mém. de l'Acad.

Imp. de St. Petersbourg," Series VII., tom. iii.), but was previously remarked by Böhmer (loc. cit.); it lies superficial to the costo-coracoid membrane, and is attached internally to the first rib, to the external border of the rhomboid ligament and by a few fibres to the manubrium sterni; it passes outwards, lying superior and superficial to the pectoralis minor to be inserted into the coracoid process; this is also noticed by Rosenmüller ("De nonnullis Musculorum Varietatibus," p. 6); it seems like an outward extension of the præ-clavicularis medialis before described.

Levator tendinis musculi latissimi dorsi is another small muscle described by Professor Gruber passing from the coracoid process and shoulder capsule to the tendon of the latissimus; it has been noticed and described by Wood and others as an anomaly of the latissimus dorsi, or of the coraco-capsular, under which heads we will refer to it again.

Scapulo-costalis minor (Gruber, "Neue Anomalien," Berlin, 1849, p. 19) is described as the subclavius posticus by Rosenmüller ("Beiträge für die Zergliederungskunst," Bd. 1, Hft. 3, 1809, p. 375, Leipzig), and Wagner ("Heusinger's Zeitschrift, für Org. Physik, Eisenach," Bd. 3, 1833, p. 335); it is described as an anomaly of the omo-hyoid by Sömmerring (p. 112), and as sterno-scapular by Ehlers ("Henle u. Pfeufer's Zeitschrift," 3te, Reihe. XXI., p. 247); it may exist as a separate part of the subclavius inserted into the root of the coracoid process (Wood); this occurs about once in every fifteen subjects. It always lies underneath the costo-coraco-clavicular membrane.

Supra-costalis is a small muscle which has been found under different forms—1. It may arise from the third rib and pass in front of the serratus magnus to be attached to the first rib and the cervical fascia (Wood. M.); 2. Or arising from the same place it may lie on the serratus magnus in front of the respiratory nerve, behind the axillary vein to be inserted into the first rib, or into the cervical fascia (Ibid.); 3. A slip from the third to the sixth rib is given as an anomaly of the great pectoral by Sömmerring; 4. A supra-costalis on both sides passing from the upper border of the fourth and third ribs, and from the outside of the second rib to the first, beside the insertion of the scalenus-anticus, from which it is separated by the subclavian vein; this muscle lay internal to the serratus magnus (Turner, "Journal of Anatomy and Physiology," vol. IV., p. 300); 5. A singular form has been described by Mr. Lawson Tait

("Journal of Anatomy." vol. IV., p. 236), passing from the fourth rib upwards, attached to the upper edges of the first and fourth rib and to the lower edge of the second and first rib, then crossing the first rib it is inserted into the transverse processes of the upper six cervical vertebræ; this seems to be a fusion of one of the scalenes with the supra-costalis, and reminds one closely of the prolonged scalenus posticus of the quadrumana; 6. From the fifth rib to the first slip of serratus magnus it may extend, beginning and ending in a flat tendon, and this may co-exist with a second origin from the fourth rib near the serratus magnus, in which case its insertion is into the first rib external to the subclavius tendon ("Roberts, Liverpool Medical and Surgical Reports," Oct., 1867); 7. The form described by Sömmerring was seen by Bonn ("Fabricius, Idea Anatomiæ Practicæ, Wetzlar," 1741, p. 17, and Bourrienne in "Roux' Journal de Médecine," tom. XXXIX., 1773, p. 45). Another case of this is described by Pye Smith ("Virchow's Archiv," vol. 43, p. 142). Frequently when no muscular fibres exist in this place a flat tendinous or cellular band takes its place. This is always a deep muscle and never overlies the pectorals.

The muscle just described is liable to be confounded with the rectus sternalis (sternalis brutorum of some authors, supra-costalis anticus of Bochdalek junior): this muscle has been found in the following conditions—1. Passing from the third costal cartilage to the sternum, behind the pectoralis major and on the right side (Bochdalek in "Virchow's Archiv," XLI., p. 257); 2. From the fourth, fifth, and sixth costal cartilage to the lower border of the manubrium; this I have seen from three to six inches long, and from half an inch to two inches wide; 3. From the fifth rib to the sternum (Haller, "Elem. Phys.," tom. III., p. 72); 4. From the cartilage of the seventh rib (Turner, "Journal of Anatomy," vol. I., p. 247); 5. Or from the cartilage of the fifth and sixth ribs (Ibid. M.); 6. Bergmann describes it as an oblique band crossing the sternum from the sixth rib on one side to the third on the other; 7. Teichmann also describes a decussating muscular band passing from the cartilage of the third rib to the platysma myoides; similar decussating bands have been described by Turner (loc. cit.); 8. I have found a few vertical fibres from the sternal origin of the sterno-mastoid descending, and evidently a rudiment of the rectus; 9. It has been found passing from the fifth, sixth, or seventh rib or

from the external oblique and inserted into the tendon of the sterno-mastoid (Wilde, "Comment. Acad. Petropol.," vol. II., 1740, p. 320, Tab. 8, fig. 5); instances of this connexion with the sterno-mastoid are described by Turner, and I have met with it several times; Bourrienne ("Roux' Journal," vol. 39, p. 45, 1773), describes it as a downward prolongation of the sterno-mastoid, and he states that he has found it mingling with the fibres of the rectus abdominis; Marjolin ("Manuel d'Anatomie," Paris, 1815, p. 95) believes in the same doctrine of the nature of this muscle, and describes corresponding attachments; 10. Kelch and Theile describe this muscle as being represented by oblique tendinous slips, an arrangement I have often noticed; 11. Actual tendinous intersections in this muscle have been seen by Portal ("Cours d'Anat. Médicale," vol. 2, 1803, p. 156), G. Meckel ("De Monstrosa Duplicitate," Halle, 1815, p. 35), and Hallett ("Edin. Med. Journal," vol. 69, p. 11, 1848); 12. It has been described as attached to the clavicle by Galen (see letter of M. Juppin, "Roux' Journal," loc. cit., p. 312), and Weitbrecht ("Comment. Acad. Petropol.," 1729, vol. 4, p. 258); 13. It has been found continuous with the external oblique (Turner, loc. cit.); 14. Halbertsma, who has written a monograph on this muscle ("Verslagen en Mededeelingen der Koninklijke Akad. van Wetenschappen," vol. 12, 1861, Amsterdam), at p. 169 describes this muscle as co-existing with and separate from the continued rectus (Accessorius ad rectum) and he supports the theory that this muscle is a part of the platysma, a theory which is also supported by Wilde, Hallett, and Turner; 15. A curious variety was described to me by Dr. Kelly, Demonstrator in the Carmichael Medical School, Dublin, in which the muscle arose from the anterior surface of the sheath of the rectus, and was inserted immediately below the tendon of the sterno-mastoid; this muscle was five and a half inches long and half an inch broad.

The frequency of occurrence of this muscle has been differently stated; Gruber ("Mém. de l'Acad. Imp. de St. Petersbourg," tom. III., p. 12, 1860) describes its frequency as five out of ninety-five, and of these three were on both sides, one right, and one left; Turner found it in twenty-one out of six hundred and fifty subjects, twelve times single and nine times double, seven times in males and eleven times in females. I found it eleven times present in three hundred and fifty subjects; this gives its frequency as once in twenty-seven or thirty-seven in one thousand and ninety-five. The other anatomists in whose works

special mention of this muscle occurs are—Cabrolius ("Anatom. Elenchus Accuratissimus," Observ. 8, p. 96, 1604); Huber ("Acta Phys.-Med.-Acad. Curios.," vol. 10, Noribergæ, 1754, p. 112); Haller ("Icones, Fascic. 6," Göttingen, 1753, p. 47); Sabatier ("Exercit. Acad. Lugd. Batav.," 1783, pp. 83, 88); Lauth ("Elém. de Myologie," Paris, 1778, p. 56); Bergmann (quoted by Henle); Otto ("Selt. Beobacht.," Heft. 1, p. 90; 2, p. 39); Loschge ("Abhandl. d. Phys.-Med. Soc. in Erlangen," Bd. 1, p. 94); Kaau Boerhaave ("Comment. Nov. Petropol.," 1749, II., p. 257); Sandifort ("Exercitationes Acad. Lugd. Batav.," 1783, p. 83); Du Puy ("Hist. de l'Acad. Royale des Sciences," 1726, p. 26); De la Faye (Ibid., 1736, p. 59); Albinus ("Hist. Musculorum," Paris, 1734, p. 291); Gantzer ("Dissertatio Anat. Muscul. Var. Sistens.," Berolin. 1831); Budge ("Henle u. Pfeufer's Zeitschrift," vol. 7, 1859, p. 276; [from the sixth to the third rib continuous with pectoralis major fibres]). It has been seen passing from the second to the seventh rib in a monster by Barkow ("Monstra Duplicia," Leipsic, 1828, p. 10).

Intercostales externi vary—1. By extending as far as the sternum, that of the fourth interspace is the most liable to this variety or the first (Sömmerring); 2. One may be absent, the last is the most prone to this variety; 3. Supernumerary, as in case of a cervical rib; instances of this are described by Fischer ("Wiener med. Wochenschrift," 30, 1858, quoted in Schmidt's Jahrbuch, vol. 110, p. 157), from the oblique process of the seventh cervical vertebræ and lower border of new rib to the upper border of the usual first rib; 4. The fibres of the posterior part of the first intercostalis externus may be united to those of the internus; 5. Fibres from this muscle are occasionally inserted into the obliquus externus; 6. or the serratus magnus.

Intercostales interni may be—1. United to infracostalis; 2. The first and second, and third often extend to the spine; 3. One or two of the inter cartilagines ("Hamberger, de Respirationis Mechan.," Jenæ, 1788, p. 11) I have seen nearly separate from the proper intercostales interni, especially in the middle intercostal spaces, and the same is described by Schoemaker ("Hollandsch Archief," Bd. 11, p. 196); 4. Meissner saw a second more vertically ascending layer of this muscle, and I have noticed the same arrangement as he has described, see the "Jahresberichte," 1856, p. 440; 5. The internal intercostal may extend to the spine in the first intercostal space; 6. Those of the eighth,

ninth, tenth, eleventh, and twelfth interspaces, or some of them, are continuous often with the internal oblique (which see); 7. Deficiencies here and there in these muscles have occurred in at least one instance, so that the external intercostals lay on the pleura in several places, especially in the middles of the sixth and seventh intercostal spaces.

Of the levatores costarum, besides the varying degrees in the separation of levator longus from the brevis and of either from the external intercostal in its course, and the occasional suppression of one of them, or their union into a sheet, I have seen no anomalies. Sömmerring saw them connected to the iliocostalis dorsalis and the uppermost to the scalenus medius posticus.

The infra-costales or intra-costalis or transversus thoracis posterior first particularly described by Verheyen ("Anatomie," 1705, p. 495), called by Kelch the serratus internus, not uncommonly arises in a sheet continuous from the twelfth to the third ribs, and the insertion extends from the tenth to the fourth rib; Petsche describes finding this muscle as a thick lamina extending to the spine ("Syllogis. Muscul. Observ. Anat. Select.," p. 769 in "Haller's Disput. Anat. Select.," vol. 6, Göttingen), and I have seen it deficient entirely. Very frequently in females it is only represented by scattered fibres.

The infra-costalis anterior or transversus thoracis anterior or triangularis sterni may be—1. Absent; 2. Separated into distinct fascicles (Sömmerring); 3. A slip from the second rib over the third to the fourth rib cartilage seems to be either a displaced slip of this muscle or a supra-costalis ("Camper, Verhandl. over Kankerwording," Tab. XIII.); a corresponding slip from third to second was found by Tarin ("Myographie," 1753, p. 32); 4. The number of teeth may vary from six to two ("Portal," p. 138), or only one (Blandin); 5. It may be continuous with the transversus abdominus ("Rosenmüller, de nonnullis Musc. Var.," Leipzig, 1814). I have twice seen a separate upper slip from the third piece of the mesosternum to the second rib-cartilage.

Serratus magnus—1. Its central part may be absent (Meckel, Henle, M.); 2. Or the upper part may be absent; 3. No slips may arise from the first rib; 4. The three parts I have seen quite separate from each other; 5. Only one slip may exist from the second rib; 6. A deep layer may be present in the upper part of the muscle; 7. It may only extend to the seventh rib, and one tooth in its centre may fail, Meckel describes a deficiency of two; 8. Two

slips sundered from the rest of the muscle may have a special insertion into the inferior angle; these arise from the ninth and tenth ribs or from the eighth and ninth; 9. In one case of multiple muscle-suppression described by Mr. Poland only the first and second slips were present; 10. Its upper fibres may extend as far forward on the superior costa as the notch; Theile saw a forward extension in an instance in which the omo-hyoid was absent; 11. Ten teeth to the serratus magnus (Horner, p. 384), the tenth arising from the ninth rib; 12. Or an eleventh slip from the tenth rib (Sömmerring); 13. The first and second rib portion has been seen quite separate from the rest (Ibid.); 14. Continuity of some of its fibres with those of the external oblique has been seen; 15. Or with the supra-costalis; 16. Or external intercostals.

Levator anguli scapulæ has been found—1. With a slip of origin from the squama temporis ("Meckel, Descriptio nonnullorum Monstrorum," Leipzig, 1826, p. 22); 2. Or from the mastoid process (Blandin, p. 349); 3. Or from the trapezius; 4. With five separate vertebral origins, and one from the oblique process of the sixth cervical vertebra; 5. With a head from the second to the fourth dorsal spines (Meckel's "Archiv," Band 5, p. 115); 6. Cleft into two, the upper coming from the first and second, the lower from the third and fourth cervical transverse process (Wood, M.); this splitting is found about once in every nine subjects, it is figured by Cuvier and Laurillard in their "Myologie Comparée," Pl. 7, in a negro; 7. Kelch found it cleft into three, and of these the middle was inserted into the scapulo-thoracic fascia ("Beiträge für path. Anat.," 1813, p. 33); 8. Rosenmüller found an aberrant slip of this muscle passing from the first cervical transverse process into the first tooth of the serratus magnus ("De nonnullis Musc. Var.," Leipsic, 1814, p. 5); 9. A union of this muscle and the serratus like that in the quadrumana is described by Henle; 10. A slip of the levator was found by Flower and Murie in a Bushwoman to be inserted into the middle of the serratus ("Journal of Anatomy and Physiology," vol. i., p. 194); 11. Wood found it split into six slips; 12. I have seen it divided into three; 13. In one case of fission into six Mr. Wood found it to take one origin from the fascia and fibres of the transversalis colli, the three upper and part of the fourth formed a normal muscle, the rest of the fourth and nearly all the fifth were inserted into the vertebral border of the scapula as far as the spine, intimately connected to the serratus magnus,

the lowest slip was at its insertion blended with the rhomboideus minor (Wood); 14. A slip into the scapular spine was seen by M'Whinnie and Blandin (loc. cit.); 15. I have seen a separate slip lying deeper at first than the rest of the levator, finally crossing its insertion, to be inserted into the base of the triangular space at the root of the spine of the scapula; 16. A slip missing the scapula, and inserted between it and the spinal column, was seen by Otto and Meckel: a similar slip inserted into the second rib was seen by Blandin, Meckel, and Theile ("Archiv," vol. viii., p. 585); and one from the lower part of the levator, inserted into the first and second ribs anterior to the serratus magnus, was found by E. Clason ("Om Muskel Anomalien, Upsala Läkaref.," 2 Band, p. 517); 17. It has been found sending a slip to be inserted with the serratus posticus superior (Theile, Kelch, p. 23); 18. Or to the scalenus posticus (Ibid. and Wood); 19. Or scalenus medius (Wood); 20. to the splenius capitis (Wood); 21. Or to the complexus.

Trapezius exhibits many varieties. 1. It may not extend above the fifth cervical vertebra, both the upper cervical and the cephalic portion being obsolete; 2. Zagorsky found it to arise from the four lower cervical and three upper dorsal spines only ("Mémoires de l'Acad. de St. Petersbourg," T. 1, p. 359); 3. Fleischmann found it attached to all the vertebræ between the axis and the fourth dorsal ("loc. cit.," Bd. i., p. 25); 4. Sömmerring found the central part of the muscle deficient; 5. Quain describes a case in which there was no clavicular attachment; 6. It has been found extending only to the eighth and ninth, or tenth dorsal spines by Sömmerring, Fleischmann, Zagorsky, Theile, Meckel, Wood, and myself; according to Wood, this is present about four times in seventy subjects, and I have seen it more frequently in females than in males; it occurs in about the proportion of once in every fifteen females, and in every nineteen males; 7. The cervical and dorsal portions may be separated as in rodents and some other mammals. This has been noticed by Zagorsky and Fleischmann as well as by Wood and myself; 8. Sömmerring found the lowest part separate from the rest (p. 225); 9. The clavicular fibres have been found divided into separate occipito-clavicular and cervico-clavicular parts, the first of these is usually separate from the cleido-occipital proper; 10. A singular variety is described by Meckel ("Descriptio nonnullorum Monstrorum," Leipsic, 1826, p. 22), in which the left trapezius had no scapular insertion and

was all clavicular and in two parts—one from the fourth dorsal, and one from the second dorsal up to the occipital bone. On the right side it was purely occipito-clavicular, and had an additional slip from the mastoid process (a displaced cleido-occipital); 11. I have found the clavicular attachment reduced to a very rudimentary slip; 12. Or, prolonged as far as the sterno-cleido-mastoid over one-half or two-thirds of the clavicle, and sometimes united to that muscle (Blandin, "Nouveau Élém. d'Anat.," Paris, 1838, p. 345). This is also described by Gruber ("Vier Abhandlungen aus dem Gebiete d. medic. chirusgisch. Anatomie," Berlin, 1847, p. 17), and by M'Whinnie, Hallett, and Wood. In some of these cases the external jugular vein and the supra-clavicular nerve passed through small tendinous openings, or the muscle formed a tendinous arch, under which these parts passed (Gruber, Schwegl, "Ueber Muskelvärietaten," Wien, 1861, p. 8); 13. The cleido-occipital (Wood) has been found occasionally fused with the trapezius at its insertion, but separate at origin; 14. An aponeurotic slip, from the tendon of insertion opposite the scapular spine, has been found to pass downwards and outwards to the lower angle of the scapula (Ibid.); 15. Bands of fibres not unfrequently join this muscle at its insertion to the origin of the deltoid, either at the posterior margin of the spine of the scapula or across it; 16. I have found the occipital and cervical parts of this muscle split up into a number of separate fasciculi; 17. From its anterior margin a cylindrical tendinous slip has been traced passing behind the omo-hyoid and inserted into the breast bone (Gruber, loc. cit., p. 22). This seems to be a modified sterno-scapularis; 18. A similar slip to the sterno-cleido-mastoid over the subclavian artery was seen by Quain ("Anat. of the Arteries," p. 186); 19. Potter saw a similar case in which the slip pierced the omo-hyoid (quoted by Quain); Budge ("Henle u. Pfeufer's Zeitschrift," Bd. vii., p. 273) saw a slip of this muscle passing to the levator anguli scapulæ; he also saw this muscle with no internal occipital origin but one from the posterior border of the sterno-mastoid (Ibid.).

The latissimus dorsi is a muscle which is very liable to vary. 1. It may have an origin from the ninth rib (Cloquet, Paxton, "Anatomy," p. 217); 2. Or from the eight lower dorsal spines (Winslow); 3. Or from the ninth intercostal fascia (Wood); 4. The iliac origin may be suppressed; 5. Or reduced to a rudiment; 6. I have found it extending so far forward as to overlap the

external oblique ; 7. It frequently has a slip from the inferior margin of the scapula, as is described by the majority of anatomists ; 8. I have seen it rising no higher, in point of origin, than the first lumbar spine ; 9. I have seen the twelfth rib slip deficient, the muscle having but two costal origins. This is also described by Sömmerring ; 10. Various forms of communication or connexion are found between this muscle and the insertion of the pectoralis major ; these unions are very expressively designated by the name "Achselbogen" (Langer, "Œsterreich. medicin. Wochenschrift," 1846, Nos. 15 and 16) : it usually goes to the lower part of the insertion tendon of the pectoral muscle, passing in front of the axillary vessels and nerves. Various forms of this achselbogen are enumerated by Ramsay, its first discoverer, who described it in 1795 ("Ed. Med. and Surg. Jour." vol. viii., 1812, p. 281); Meckel, Kelch (op. cit., p. 34); Langer, Gruber ("Neue Anomalien als Beiträge der phys. chirurg. u. path. Anat.," Berlin, 1844, p. 31, taf. vi.) ; Struthers ("Anat. and Phys. Observ.," Pt. i., Edinburgh, 1854, p. 193) ; Calori ("Mem. della Accad. delle Scienze dell' Istituto di Bologna," 2 serie, t. 6, p. 187) ; Wood, and myself. Its frequency is stated variously by authors—one in thirty by Meckel ; eight in one hundred and five by Struthers ; six in one hundred and six by Wood, which gives us as the average out of two hundred and forty-one subjects a rate of once in sixteen as its frequency. Struthers says it is commoner in females than in males, but my own observations do not bear out this observation as a rule ; 11. The achselbogen from the latissimus may be attached to the lower border of the lesser pectoral tendon (Wood); 12. Or to the fascia over the biceps (Ibid.) ; 13. Or to the fascia over the coraco-brachialis (Ibid.) 14. In one singular case this coexisted with a second slip to the fascia over the fifth digitation of the serratus magnus, in the interval between the great pectoral and latissimus dorsi, which was inserted along with it ; and a third, arising with the first from the latissimus, passed horizontally forward to the axillary fascia over the pectoralis major (Fritsch, Reichert and Du Bois Reymonds' "Archiv.," 1869, p. 367); 15. A prolongation from the costal origin of the latissimus to the coracoid process is not unfrequently met with (three times in one hundred and two cases, Wood). This may be perfectly separate from the rest of the latissimus (Sömmerring, p. 226), in which form it is the chondro-coracoid of Wood and Rosenmüller ("Var. Musc.," p. 5 ; Kelch, p. 35), and may be inserted

(α) into the tip of the coracoid process with the pectoralis minor; or, (β) into the lower surface of the process; or, (γ) partly into the lower border of the process, and partly into the capsule of the shoulder with the tendon of the supraspinatus. In this case Wood found it to separate the posterior from the lateral cords of the brachial plexus. As a form of the chondro-coracoid may be reckoned a muscle described by Theile, which passed from the fifth, sixth, and seventh ribs to the muscles connected with the coracoid process; another allied variety is the levator tendinis latissimi dorsi of Gruber, already mentioned, which, with variety β, may be displaced examples of the coraco-capsularis; 16. A slip, tendinous (most commonly) or muscular (rarely), passes from the muscle into the long head of the triceps (Halbertsma), which occasionally runs for a short distance separate from the triceps: this is described by many anatomists, and its frequency has been variously stated as once in eighteen, or once in ten. The long head has been seen deriving a very large accession of fibres from this tendon, or from this and the teres major; these forms are rudimental of the dorsi-epitrochlearis; 17. Varieties are found in the development of the frænum described by Winslow (p. 181); it is sometimes attached to the teres major, sometimes to the latissimus dorsi. This is the proper suspensory frænum; and although there is very often a similar process from the pectoralis major, yet it is a mistake to apply to it the name frænum suspensorium of Winslow, as is done in the "Cyclopædia of Anatomy and Physiology," vol. iv., p. 574; 18. The insertion has been found bilaminar by me on one occasion: the upper layer was attached to the normal site, while the posterior and inferior plane was inserted into a tendinous arch or sling attached above and below and free in the middle. This arrangement coexisted with a deficiency of the teres major: the separation between the lamellæ only extended as far as the end of the fleshy part, which was indivisible. 19. The costal portion I have found severed from the rest as far as the insertion; 20. In the relations of this muscle to the teres major I have seen some variation: the most common arrangement is a union of the upper margin of the latissimus tendon and the teres muscle, while the actual insertions are separated by a bursa. I have seen the lower margins also united, and I have sometimes, though rarely, failed to separate one from the other; 21. A union of the tendon to the inner brachial ligament I have several times seen. Occasionally the ligament

arches over the tendon, and the abnormality described as No. 18 seems to be a case of the insertion of one-half of the tendon into this arch, while the rest passes underneath it; 22. The slip from the inferior angle of the scapula has sometimes been seen with a few of its fasciculi directly continuous with the fibres of the rhomboideus major, or doubled (Grüber "Mém. de l'Acad. Imp. de St. Pet.," 1857, No. iv., p. 3); 23. Langer found the costal fibres with a separate insertion into the axillary fascia; 24. The achselbogen has been found inserted into the sheath of the long head of the biceps in the bicipital groove; 25. Meckel found the latissimus arising from the eighth, ninth, tenth, and eleventh ribs ("Archiv," vol. viii., p. 585); 26. Meckel saw the achselbogen coexisting with a slip of the pectoralis major going to the pectoralis minor, and one to the latissimus dorsi (Henle u. Pfeufer's "Zeitschrift," Bd. xxix., p. 168). Rüdinger figures a remarkable and somewhat similar instance in which the pectoralis major sends a slip to the latissimus; and a slip from the latissimus going with this sent a fascicle of tendon to the pectoral ("Ueber die Muskeln des vordern Extrem.," &c., Haarlem, 1870 tab. 12, fig. xxxiv).

The rhomboideus occipitalis (Murie and Mivart), or occipito-scapularis (Wood), is a rather rarely present human anomaly, although present in a large number of animals: it has been found arising from immediately below the inner third of the superior curved line of the occipital bone, or from the bone above the attachment of the complexus, and internal to the insertion of the splenius capitis; its insertion is above that of the rhomboideus minor. It is occasionally connected to the occipital part of the trapezius.

Rhomboideus minor—1. May extend up to the fourth cervical spine: Bartholin (p. 564) says that it arises normally from the 5th; 2. It may be fused with the major inseparably (Albinus); 3. I have seen it bilaminar; 4. Or it may overlap the major (Sömmerring).

Rhomboideus major—1. May arise from only two, from three, or from five dorsal spines, four vertebral origins being its commonest arrangement; 2. It may be inserted only half way down the vertebral border of the scapula; 3. Mr. Kelly has communicated a case to me in which the muscle was inserted into the inferior angle of the scapula alone, by a tendon only half an inch broad; in both these cases the muscle was triangular in shape. In my specimen (No. 2) the muscle arose from four vertebræ and was easily separable from the minor,

and some of its fibres (the upper ones) crossed others at their insertion; 4. The insertion may be by means of a tendinous arch, attached above at the root of the spine, and below to the inferior angle; 5. I have found its lower part to be bilaminar; 6. The major may send its lowest fibres into the teres major (Henle, "Muskellehre," p. 27); 7. Or into the scapular origin of the latissimus dorsi (M.); 8. Sömmerring found it fused with the upper border of the latissimus dorsi (p. 227); 9. I have, in a weak female, found it divided into parallel fasciculi separate from each other, to the insertion; 10. With fascicles continued into the serratus magnus.

Serratus posticus superior—1. Has been described as absent in a hunchback (Isenflamm, "Abhandlung über die Knochen u. deren Krankheiten," Erlangen, 1752); 2. Sending a slip to the transverse process of the first cervical vertebra, from its upper part (Otto, Rosenmüller, op. cit., p. 5). This slip is a form of the rhombo-atloid to be hereafter described; 3. The muscle may have three to six teeth (Henle), may be attached to the second, third, fourth, and fifth ribs (Cloquet), or to the second, third, and fourth, or to the upper five ribs.

The serratus posticus inferior may be absent as in the case described by Isenflamm (loc. cit.), or may be attached to two, three, or four ribs (Cloquet says that it may be attached to two dorsal and three lumbar spines, p. 247), may skip the last rib, or may be fused with the latissimus dorsi, or may arise from one, or from two lumbar spines alone, with no dorsi-spinal origin (I have found all these varieties except the first).

Splenius capitis—1. May receive a slip from the levator anguli scapulæ (Cloquet, Wood); 2. The slip of splenius colli to the atlas may be joined to the S. capitis, and separate from the rest of the S. colli (Henle, M.); 3. The two may be inseparable (M.); 4. Mr. Wood found the splenius colli lying superficial to the origin of the serratus posticus superior, while the S. capitis was beneath it; 5. I have found once the splenius capitis sending a slip to the occipital protuberance; 6. Moser found it split into two—one part from the spines of the 2–6 cervical vertebræ, and inserted as usual; the second from the last cervical and upper two dorsal spines to the mastoid portion of the temporal bone (Meckel's "Archiv," vol. vii., p. 224).

Rhombo-atloideus is a name which I proposed in 1866 for a small muscle

passing from the deep surface of the origin of the rhomboideus minor or major to the transverse process of the atlas lying superficial to the serratus posticus superior, and behind the rhomboideus minor. I have found it attached to the tendon of the rhomboideus minor, or major, or to the upper two dorsal spines, or to the second and third dorsal spines, or to the lower two cervical spines, or to the spine of the vertebra prominens alone; Walther describes it as adjutor splenii, or muscular singularis splenii accessorius (Haller's "Disput. Anat. Selectæ," 1733, vol. vi., p. 589). Mr. Wood has found it three times in thirty-six subjects. Henle has found it going to the spines of the sixth and seventh cervical vertebræ; it is also described by Sandifort (op. cit., lib. ii., cap. vi., p. 62); and another case of it is more recently published by Clason, passing to the first and second dorsal vertebræ (Upsala "Läkaref., Bd. ii., p. 417); other two cases have been seen by Budge ("Henle u Pfeufer's Zeitschrift," Bd. vii., p. 273, 3te Reihe).

Sacrospinalis (erector spinæ) I have never found to present any noteworthy anomaly, except some slight variations in extent of attachment or individual development which are not deserving of special record.

Ilio-costalis varies chiefly in the number of its tendons. Most commonly those for the twelfth, or for the eleventh and twelfth ribs are obsolete, and occasionally that for the first rib. The musculi accessorii of Diemerbröck vary in number, often the lower ones are suppressed, and occasionally some of them skip a rib and are thus longer than usual. Two accessory slips are often attached to each of the middle ribs.

Longissimus dorsi may receive bundles from some of the lower ribs outside the levatores costarum. Morgagni and Walther describe a prolonged slip of this muscle extending to the head ("Adver. Anat.," xi., p. 38, and Walther, "De Artic. Ligamentis et Musculis Hominis," Leipsic, 1728, p. 70), or may send a slip to the complexus, or splenius (Cloquet).

Transversalis colli may have its fifth tendon absent, or may be joined to the ilio-costalis.

Cervicalis ascendens has been known to have slips as low as the tenth rib, and to be inserted as far up as the third cervical transverse processes. Within these limits its number of attachments are capable of variation.

Longissimus capitis (trachelo-mastoid) I have once failed to find. I have

found it attached to the two lowest cervical processes. I have seen the cervical origins as numerous as five and the dorsal as five, and these I found several times forming two separate heads for the muscle. I have seen it receiving a band from the middle of the longissimus dorsi, and very commonly I have found it with one or two tendinous inscriptions. The middle part may be absent (Sömmerring, op. cit., p. 183), or it may be united to the complexus. Its origin may extend to the eighth dorsal transverse process, or may be joined to the ilio-costalis.

Spinalis dorsi I have seen with only three origins, Henle has noticed the same, ("Muskellehre," p. 39). Its upper and lower insertions I have found deficient, or two tendons may arise from one or more spines. Its insertion is often inseparable from that of the semispinalis (Henle).

Spinalis colli has been seen sending a special bundle to the superior curved line of the occiput under the complexus (Henle, p. 41), or a slip with the rectus capitis posticus major (Theile, p. 167). Some of its tendons may be deficient. A good description of this muscle and nineteen of its varieties of form and attachment are given by Henle ("Müller's Archiv," 1837, p. 297).

Complexus major may have from two to seven origins, connected with the longissimus dorsi by the lateral head, or to the trachelo-mastoid by the median. A slip from its tendon may be inserted into the ligamentum nuchæ (Theile). A special deeper muscle has been found passing from the oblique process of the second dorsal vertebra to the skull under the complexus, and inserted below it (Henle, p. 44), or the complexus itself may receive a slip from the oblique process of the last cervical and first dorsal vertebra.

Biventer cervicis I have found several times to be a trigastric muscle.

Semispinalis dorsi. The origins may vary from two to seven, the insertion, from two to eight (Albinus). I have not found the former fewer than three, nor the latter fewer than four; they oftener reach the maximum than the minimum.

Semispinalis colli. Seven to four origins, from the eighth dorsal to the seventh cervical (Albinus). I have often found it joined to the last: so frequently indeed is this the case, that Krause regards them as one muscle.

Multifidus spinæ may have one or more of its slips absent: that from the seventh cervical was described as suppressed by Sömmerring (p. 195) and Albi-

nus. Theile found the first one cleft at its origin, and attached by one slip to the spine, and by the other to the posterior half arch of the axis. Calori saw a slip of the muscle from the second and third rib going to the sixth and fifth cervical vertebræ ("Mem. della Acad. delle Scienze dell' Istituto di Bologna," t. vi., p. 137).

The rotatores spinæ of Prof. F. W. Theile (Müller's "Archiv," 1839, p. 103), described also by Weber, Günther and Milde, and described, but not named, by Cloquet, vary in the distinctness of their long and short elements; they are often absent in one or more intervertebral spaces; sometimes they are split into fasciculi.

Intertransversalis may skip a vertebra in the neck or abdomen: in the former place it constitutes the musculus singularis colli of Sandifort from the lower part of the second and third to that of the fifth cervical transverse process ("Exercit. Anat.," B. 1, cap. vi., p. 93). One or more may be doubled or suppressed.

Rectus capitis posticus major is often doubled (Douglas and Albinus, "Hist. Muscul.," p. 385). It has been seen with a slip from the spinalis or complexus (Theile, Jourdain's tr. p. 150).

Rectus posticus minor doubled (Grüber, "Abhandl. aus dir Mensch. u. vegl. Anat.," St. Petersburg, 1852, p. 125). Douglas, Hallett, Sandifort, Flower, Murie, Wood, and myself, have also described this.

Obliquus inferior has been found overlapping the rectus capitis posticus major (Asch, "De Primo Pars Nervium," Tab. ix., in Ludwig's "Coll. Scrip. Neurol. Minor.," vol. i., p. 334). It may be double (M.).

Obliquus superior was found bilaminar in a bushwoman (Flower and Murie, "Journal of Anatomy," vol. i., p. 200). I have also seen it several times in ordinary subjects. Professor Dursy saw a second obliquus superior from the spine of the axis to the mastoid process (Henle u. Pfeufer "Zeitschrift," vol. xxxiii., p. 49).

A rectus capitis lateralis longus is described by Otto ("Path. Anat.," 1830) passing from the transverse process of the second cervical vertebra to the occipital bone.

Sacro-coccygeus posticus is not generally described in anatomical manuals, although it is a tolerably constant muscle; it is described by Sandifort ("Exercit. Anat.," Lugd. Batav., 1783, Book i., ch. vi., p. 89) as curvator coccygis, and by Morgagni ("Adversaria Anat.," iii., Animadversio xlv., p. 94), as

levator coccygis. Sömmerring found this muscle all tendinous. The pre-coccygeal portion is described by Von Behr as curvator, and the post-coccygeal as extensor coccygis (p. 139).

These muscles are also described by Günther ("Chirurgische Muskellehre," Taf. 34, fig. ii., 19.

The extensor coccygis is described by Theile, and by Luschka in his paper on the pelvic fascia, and is a rare muscle.

The coccygeus has been found attached to the sacrum and not at all to the coccyx (Sömmerring), double (ibid.), treble (ibid.), or joined to the levator ani (ibid., p. 212).

Of the levator ani a good description is given by Professor Luschka in "Henle u. Pfeufer's Zeitschrift," vol. v., p. 113; but I have noted no anomalies.

Obliquus externus; 1. I have found to vary in the number of its costal slips; they have been from six to nine in number; 2. I have also seen two slips from one rib; and this, when it does occur, is more commonly seen in the case of the eighth and ninth ribs; 3. Occasionally, no slip arises from the twelfth rib, or scarcely any from the eleventh: these deficiencies are more common in males; 4. I have found some fibres continuous with those of the serratus magnus; 5. Occasionally the slip constituting the pectoralis quartus is inseparably attached to this muscle, giving the appearance of a band of the external oblique rising to the axilla, or it may be connected to the serratus magnus. Budge saw the ninth rib fascicle continuous with the latter, so that a slip ran from the scapula to the ilium; 6. I have seen this inserted into the coracoid process; 7. Absence of the decussating bands of Camper I have seen once; 8. and the partial deficiency of these I have seen also, producing a large hole in the oblique tendon; 9. On one occasion I found this muscle double on the left side, the deeper muscle passing from the three lowest ribs (except the twelfth) to the crest of the ilium, precisely parallel to the normal muscle, and separated from it by a cellular interval. A similar case was described to me by Mr. Kelly of the Carmichael School of Medicine; another symmetrical instance of this is described by Dr. Baker ("North American Archives of Medical and Surgical Science," 1835, p. 306). In this case a muscle like the sterno-hyoid ran from the eleventh rib to the crest of the ilium, one inch and a half behind the spine, less oblique than the external oblique; 10. I have noticed a considerable variety in the

degrees of development of the *adminiculum lineæ albæ;* 11. A considerable amount of variety also exists with regard to the backward extension of this muscle, which causes the size of Petit's triangle to be very variable; 12. Mr. Poland records a remarkable case in which this muscle was perfectly separate from the linea semilunaris ("Guy's Hosp. Reports," 1841, p. 192).

Obliquus internus I have found to arise—1. From four ribs only; 2. I have found it fused with the transversalis; 3. or perforated by the spermatic cord at its lower border; 4. with the plica semilunaris of the posterior wall of the sheath of the rectus on the level of the umbilicus; 5. with deficiencies of its anterior and upper part; 6. Where this muscle is continuous with the tenth intercostal muscle, the line of the tenth rib is often prolonged inwards by a tendinous inscription, "so dass vom M. obliquus internus, eine Art Intercostalmuskel abgegrenzt wird" (Henle, p. 67). A slender cartilaginous slip, perfectly separate from that of the rib, has been seen lying in this inscription (Henle's "Muskellehre," p. 65, fig. 28). This inscription at the tenth rib is described by Sömmerring (p. 146). I have seen it at the eleventh, never at the twelfth, nor have I seen the tenth and eleventh slips co-existing, though the separate existence of one or the other is by no means uncommon.

Transversalis—1. I once found deficient, without any other anomaly of structure; 2. It has been seen attached to seven ribs (Morgenbesser, "Dissertatio de Vomitu," in Haller's Disp. Anat. Select., p. 273). 3. Hoffmann and Diemerbrœck describe it as normally attached to the six lower ribs; 4. Galen ("De Administr. Anatom.") and de Marchetti ("Anatomia," cap. 2, p. 17) give five as the normal number; 5. I have seen it fused with the preceding muscle, and the same is described by Sömmerring, p. 146); 6. I have also seen, as Guthrie describes, the spermatic cord piercing its lower border; 7. It has been seen with tendinous intersection in some of its fibres (Schwegl); 8. Horner describes it as having been seen doubled, the second or deep part attached to the twelfth rib (p. 390); 9. Its absence in Ectopia vesicæ has been described by Charvet ("Gazette des Hopitaux," No. 8, 1857); 10. With no attachment to Poupart's ligament (Hargrave, "Operative Surgery," p. 487).

Rectus abdominis has been seen—1. ascending to the third and fourth ribs (Meckel); 2. to the sternum, (Harrison and myself); 3. to the fourth rib alone (Kaauw, "Nov. Comment. Petropol.," Tom. ii., p. 559); 4. Nicaise

("Archives Génerales de la Medicine," in his Notes sur l'Anatomie de la region inguino-crurale, p. 44, 1866), describes that some fibres of the inner part of the muscle have been seen by him inserted into the linea alba; 5. Charvet ("Gazette des Hopitaux," No. 8, 1857) has noticed its absence in a case of Ectopia vesicæ. The umbilical part was found deficient in one instance, and the entire muscle in another by Barkow ("Monstra Anomalia," Lipsiæ, 1828, pp. 9–20); 6. I have seen it equal and flattened, so as to extend for nearly four inches out from the umbilicus. 7. I have also found it weak and fasciculated; 8. The number of lineæ transversæ is liable to variation; the inscription at the umbilicus is constant, the one at the ensiform cartilage is, of the usual three, the most frequently absent: when there are four, one is infra-umbilical, and when five, two are infra-umbilical, (Cloquet); 9. It was found double by Otto ("Path. Anat.," p. 244); 10. A supplemental rectus lateralis is described by Kelch, ("Beiträge," p. 42) lying between the internal and external oblique, arising by a short tendon from the middle of the lower border of the tenth rib, and inserted into the crest of the ilium. A similar case is described by Gunz ("De Herniis," Lipsiæ, 1744, p. 18); 11. Hargrave describes it as separated from the pyramidalis by the conjoined tendons (op. cit., p. 485).

Pyramidalis is frequently absent; 2. Still more rarely only one is present (Rolfincius, "Dissert. Anat." Noribergiæ, 1656, p. 580), and when this is the case, the muscle present is almost invariably the right (ibid.). When both are absent the internal oblique is said to extend farther down along Poupart's ligament (Crooke, "Microcosmographia," London, 1650, p. 603); 3. When two are present, they may vary in height—one may be higher than another; one has been seen inserted by a tendon near the umbilicus (Spigelius and Hoffmann), or fleshy or tendinous as high as the umbilicus (Rolfincius, Diemerbrœck, and Verheyen, p. 60): the latter author has seen it with a tendinous intersection. I have seen it to vary in breadth, either being a narrow slip, half an inch wide, or occupying the entire pubic crest. In the latter case I have found the rectus to arise by a single tendon alone. Riolan has found the left smaller than the right; 4. Three pyramidales were described as present by Winslow (p. 168), that on one side being doubled. The same is described by Ruysch ("Thesaurus Anatomicus," iv., No. 83); 5. Four pyramidales were found by Sabatier ("Anatomie," t. i., p. 263) and Poland. Horner (p. 391) says two, three, or four have been seen on a side. (?)

Cremaster I have seen arising in whole or in part from the fascia transversalis (Gruber), or from the transversalis muscle (Hargrave, op. cit., p. 484), with a special spino-pubal slip, or doubled, or in separate fasciculi.

Pubio-peritonealis is a small muscle, which I described ("Med. Press," 1866, p. 94), arising from the ilio-pectineal line behind Gimbernat's ligament, ascending on the transversalis fascia under the muscle of the same name, and, crossing the epigastric artery, it was inserted into the fascia transversalis at a point situated about two-thirds of the interval between the pubis and the umbilicus. A more superficial vertical slip is described by Gunz ("De Herniis," Lipsiæ, 1774, p. 18); but this is more of the nature of the lateral rectus of Kelch. A similar but much shorter muscle is described by Luschka ("Reichert u. du Bois Reymond's Archiv," 1870, p. 227), from the horizontal ramus of the pubis, ascending behind the fascia transversalis, on the inner side of the internal abdominal ring.

Quadratus lumborum (Scalenus lumborum of Meyer) varies in the number of lumbar transverse processes to which it is attached, and in the relative thicknesses of its fibres: slips may pass from it to the bodies of the tenth and eleventh dorsal vertebræ, to the body of the eleventh dorsal, or to the eleventh rib (Sömmerring, p. 178, Horner). Oblique fibres may cross the face of the muscle, from the transverse processes to the rib (Blandin).*

The superficial and deep sphincters of the anus I have found united; the former I have seen sending wandering slips forwards into the dartos, and backwards to the tip of the coccyx. A third deep sphincter (Sphincter ani tertius) has been described by Nuhn ("Verhandlungen, der nat.-hist. Med. Verein in Heidelberg," ii., p. 225); but it is not by any means constant.

The diaphragm is liable to few varieties, and is the subject of a very excellent description by Luschka ("Die Brustorgane des Menschen," p. 6, Tübingen, 1857). Leaving out of account such deviations, not uncommon in monsters, as fissure (Dönitz in "Reichert u. du B. Reymond's Archiv.," 1865, p. 132), or absence of the diaphragm, it may have—1. oblique fibres around, in front of, and behind the aorta (Sömmerring); 2. or across the central tendon (Huber, quoted by Sömmerring, vol. iii., p. 162); 3. from near the border of the œsophageal opening

* The three components of this muscle may vary in their relative development, and in their degrees of union; the pars ilio-costalis may be separate from the pars ilio-lumbaris, or the latter from the lumbo-costalis, (Luschka, "Anat.," p. 109).

to the liver* (Knox, "Lond. Med. Gazette," vol. ii., New Series, 1842, p. 531); 4. A slip from the diaphragm to the psoas; 5. or to the quadratus lumborum, is described in Haller's "Opuscula Anatomica," Gottingae, 1751, p. 10; 6. the left crus may be attached to from one to three vertebræ (Haller); the right to two (Luschka), three, or four (Drelincourt "Epistola ad Mangetum, Bibliotheca Anatomica," tom. i., p. 812); 7. it may be attached as high as the sixth rib ("Weber-Hildebrandt," p. 411); 8. a remarkable slip, from the ninth and seventh ribs, passed under the origin of the rest of the muscle, to be attached to the margin of the sternal fascicle of the diaphragm (Henle, p. 83). This is considered by Bourgery (Pl. 75–2) as a detached slip of the transversus thoracis anterior. Moser found the right crus twice the size of the left ("Meckel's Archiv," vol. vii., p. 224). Dursy found in one case two oblique slips, one on each side, passing forwards and inwards, to be inserted into the central tendon ("Henle u. Pfeufer's Zeitschrift," vol. xxxiii., p. 45).

Scapulo-humeralis digastricus singularis of Gruber is a muscle whose description has been given by that indefatigable anatomist ("Virchow's Archiv," vol. xxxii., p. 218). It arises from the root of the acromial process, and from the two-thirds of the spine of the scapula. It passes between the deltoid and the infraspinatus, has a tendinous intersection, and is inserted into the humerus, between the deltoid and the triceps. This muscle I never have had the opportunity of seeing.

The deltoid muscle exhibits the following varieties:—1. Suppression of the clavicular head: this I have never met with (Otto., "Path. Anat.," 1830, p. 247); 2. Absence of the acromial fibres I have seen. In this case the muscle consisted of an anterior or clavicular, and a posterior or scapular part; 3. I have seen a great degree of fasciculation in a thin female; so that at least six of the seven wedges of the muscle, described by Sömmerring, were nearly separate. But I have not met with any degree of segmentation equal to that described by Jenty, who describes the deltoid as being made of from eighteen to twenty distinct muscles ("Course of Anatomico-Physiological Lectures," vol. iii., p. 232, 1757); 4. I have found the spinal or scapular part separate from the acro-

* Knox has named this as hepatico-diaphragmaticus, "London Medical Gazette," 1842, p. 531.

mio-clavicular ; 5. As described under the head of the great pectoral, fusion has been met with of the contiguous clavicular fibres of these two muscles (Koster, "Nederlandsch Archief.," 1869, p. 395): this I have twice seen; 6. I have found this muscle to arise from the infraspinatus fascia, near the inferior angle of the scapula, forming a small second head. A head like this, attached as well to the axillary margin of the scapula, was seen by Meckel ; 7. An origin from the lateral edge of the shoulder blade, between the infraspinatus and teres minor (Albinus, p. 422, and Henle, p. 168) ; 8. I have seen it inserted into the origin of the supinator longus, so that a single long muscular ribbon extended from the acromion process to the lower end of the radius : this is evidently homoplastic with the tensor plicæ alaris of the bird ; 9. Haller found it inserted into the brachialis anticus in a monster (Op. " Anat. Argument." vol. iii., Lausanne, 1768, p. 23), and I have once found in a male fibres traceable from the front of the deltoid into this muscle ; 10. The clavicular fibres of the pectoral may be so fused with those of the deltoid as to give the appearance of the extension to the sternum of the clavicular deltoid (Seiler, Obs. Anat. Fascic. I., Wurtemburg, 1808). This case is quoted by Theile (Muskellehre, p. 230) ; 11. I have seen fibres from the trapezius crossing the spine of the scapula, and continuous into the deltoid ; 12. In a case of Phocomelia, Dumeril found this muscle, with the great pectoral and the teres major, forming a species of sac-like insertion, together by a common tendon ("Bulletin de la Soc. Philomathique," t. iii., p. 122) ; 13. A slip from the inferior border of the scapula was found by Calori similar to No. 7, but arising between the teres minor and teres major ("Mem. della Accad. delle Scienze dell' Istituto di Bologna," 2 series, vol. vi., p. 137).

Supraspinatus I have found singularly invariable. Sometimes the pectoralis minor is inserted into its tendon (see pect. minor). It varies in strength and development. I have seen it with a slip from the suprascapular ligament, or from the pectoralis major.

Infraspinatus.—1. May send a few fibres into the deltoid; 2. I have seen it split into two partly overlapping laminæ; 3. It may vary in its extent of scapular attachment ; 4. I have three times, at least, seen it inseparable from the teres minor; 5. The bursa of Hagen, beneath its tendon I have found very inconstant, oftener absent than present; 6. It may receive a slip from the Deltoid (Theile).

Teres minor.—1. I have seen inseparable from the infraspinatus (as above); 2. It has been found absent by Petit ("Mémoires de l'Acad. Royale des Sciences," Paris, 1733).*

Teres major I have once found deficient ("Journal of Anat.," vol. i., p. 316). I have seen it closely joined to the scapular head of the latissimus dorsi, or continuous with the lower fibres of the rhomboideus major. I have also seen it attached to the long head of the triceps, and, by a frænum suspensorium, to the capsule of the shoulder. A slip to the latissimus dorsi is described by Meckel. A still more interesting anomaly is the continuation of a slip from this muscle to the brachial aponeurosis, described by Blandin, as " cette varieté etablit au bras un muscle analogue au tenseur du fascia lata à la cuisse" (Blandin " Nouveaux Élémens," vol. i., p. 461, note).

Subscapularis.—The commonest variety in this muscle seems to be its being cleft into distinct parts. In one case I found it split, corresponding to a line from the lesser tuberosity of the humerus to the junction of the middle and lower thirds of the vertebral border of the scapula. In this instance, of the five superficial bundles of fibres described by Sömmerring (p. 234), three were in the upper and two in the lower part; and of the deep, two portions were above and the rest below. In a second instance of split subscapularis, the two lower superficial fasciculi were separated from the rest by the circumflex nerve; these arose by two separate tendons from the posterior inferior margin of the scapula, and this portion did not pierce the capsular ligament. A connecting band from the pectoralis major to this muscle has been described; and I have seen the first and the first and second of the superficial bundles suppressed.

Subscapulo-capsularis is a small muscle, first described by Professor Wenzel Gruber ("Abhandl. der aus menschl. und. vergl. Anat.," 1854, p. 109, St. Petersburg), but described independently by myself in 1866; it has, however, been referred to, though not named, by other authorities. Cruveilhier (p. 155) described it as inserted into the inner lip of the bicipital groove.

* This muscle is singularly constant in its nervous supply, which is invariably drawn from the circumflex nerve. This is the case in all the mammals which I have examined. I regret that I have no notes as to the existence or position of the gangliated cord of the circumflex in the cases of fusion with the infraspinatus. The latter muscle is equally constant in receiving its nerve supply from the suprascapular.

Blandin ("Nouveaux Élém. d'Anat. Descriptive, Paris," 1838, p. 458) describes it as a detached internal slip of the subscapularis, and Henle mentions the occurrence of a detached cylindrical muscle, from the lateral border of the scapula, in front of the long-head of the biceps, over the joint-capsule. This muscle is usually present as a slip of the subscapularis; when separate it is marginal, and continues fleshy to its insertion; its origin is usually by a flat tendon, and I have seen a tuck of the subscapular fascia intervening between it and the rest of the subscapular muscle; it never pierces the joint-capsule, and is inserted low down on the lower tuberosity, near to the inner lip of the bicipital groove. Since I called attention to it, Mr. Wood has described its presence in several subjects.

Gleno-brachialis is a small muscle described by Professor Gruber, arising along with the long-head of the biceps, piercing the capsular ligament, and inserted into the humerus. I have never met with this muscle.

Coraco-capsularis, or coraco-brachialis brevis. This muscle was described by Prof. Wood, of King's College Hospital ("Journal of Anatomy and Physiology," vol. I., p. 45), but has been noticed by several other authors; it exists in several forms, the principal of which are as follows: 1. As a small muscle, arising from the coracoid process, beneath the ordinary coraco-brachialis, and inserted into the neck of the humerus, below the insertion of the subscapularis; 2. As a slip from the fascia over the subscapularis tendon, beneath the coracoid process, passing down to the fascia derived from the tendon of the latissimus dorsi, covering the long tendon of the triceps (Wood); 3. When the lower part of this muscle is strong and attached to the humerus, and the upper attached to the subscapular tendon, it constitutes the depressor tendinis subscapularis majoris, or the retinaculum musculare tendinis subscapularis majoris of Wenzel Gruber; 4. Another form, whose name speaks for its course, is that described by Gruber, as the capsularis-humero-scapularis superior, which that indefatigable anatomist has found three times in three hundred and eighty subjects; 5. Another variety, also described by Gruber, is the tensor-fasciæ, et cutis foveæ axillaris, which consists of a slip passing from near the humeral attachment of the depressor tendinis-subscapularis into the skin and fascia of the axilla. I have met with all these varieties, and think their nature very plain as short coraco-brachiales, con-

sequently it cannot but be a matter of regret that an anatomist of such eminence as Professor Gruber should have sought to affix distinct names to such varieties as Nos. 3 and 4. Other notices of this muscle will be found in the works of Cruveilhier (p. 164) ; M'Whinnie ; (inserted into the internal brachial ligament) ; Theile ("Lehrbuch," p. 290), Otto ("Neue selt. Beobacht.," Bd. ii., 1824, p. 40), Gruber (Müller's "Archiv.," 1848, p. 425). Henle saw it twice, and other anatomists have mentioned it. There are a few other forms which I have seen, and which are worthy of record. 6. I have found the muscle arising from the coracoid-process by a tendon, and inserted into the fascia of the subscapularis-tendons ; 7. or interlacing with the fibres of the subscapularis ; or 8. Inserted into the capsular ligament of the shoulder. This muscle, in some of its varieties, has singularly enough been described as an anomaly of the deltoid, and some forms are regarded by Henle as varieties of the subscapular. Another case of this muscle is figured by Rüdinger, ("Die Muskeln der vordern Extremitäten der Reptilien, &c." Haarlem, 1868, Tab. xv. fig. xxxvi.)

Coraco-brachialis-medius (Wood) ; 1. May be pierced by the musculo-cutaneous nerve. This is said by Mr. Wood to occur not so frequently as described, but out of thirty subjects I have not found the nerve to occupy any other position ; 2. It may have its origin connected to the insertion of the lesser pectoral—this I have frequently seen ; 3. Its insertion may be into the intermuscular septum, and in a large number of cases, in which the upper fibres are inserted into this structure, the vertical fibres of the septum (the internal brachial ligament of Struthers) form a sling which may arch over the insertion of the latissimus dorsi and teres major. This sling insertion is figured by Henle in his "Muskellehre" (fig. 86, and in Henle u. Pfeufer's "Zeitschrift," vol. 8., p. 223), as the normal attachment of this muscle. A singular muscular variety of this sling is described by Clason ("Muskel Anomalier, &c., Upsala Läkareförenings-Förhandlingar," Bd. III., Hft. 2), but not connected apparently to the coraco-brachialis, and a similar case joined to the coraco-brachial by Calori, (loc. cit., p. 137,) from the tuberculum minus. A slip from the lower border of the latissimus dorsi, and teres major tendons, I have found ending in the coraco-brachialis tendon, crossing the brachial artery. The same is described by Struthers ("Anat. and Phys. Obs.," Edin. 1854, p. 195). A nearly similar

slip from the coraco-brachialis, to the inner head of the triceps, and internal brachial ligament, crossing the artery in the opposite direction, I have also met with. These are aberrant slips of the coraco-brachialis-medius, and are described by the same author. Suppression of the entire coraco-brachialis in a monster is described by Barkow (op. cit., p. 21).

Coraco-brachialis longus (Wood) is more seldom present as a separate muscle than the medius, but very often as a slip united to the medius. Its insertion is into the external brachial ligament, and it may extend as far as immediately over the inner condyle. It was recognized by Theile (Jourdan's tr., p. 215), who describes it as continuous with the brachialis anticus ; it may cross the brachial artery and median nerve (Meckel, Gruber, " Neue selt. Anom.," p. 28, Taf. 1, fig. 1). This often co-exists with the supra-condyloid process, but I have seen it distinctly developed without this process.

Brachialis anticus is a variable muscle. It has been seen—1. Cleft into two, and both parts normal (Wood, M.) ; 2. Cleft, the outer part sending fibres to the supinator longus and semilunar fascia, its inner normal (ibid.) ; 3. Cleft, one part being a purely brachio-fascial muscle—this part passes from its outer head to the fascia (Hyrtl, Sömmering, M'Whinnie, Theile) ; 4. With separate lateral slips (Winslow, Douglas' Tr., p. 187) ; 5. A slip from its inner part sending fibres into the semilunar fascia over the median nerve and brachial artery (Sömmerring, Weber-Hildebrandt, Wood, and myself) ; 6. Continuity of its outer fibres with the supinator longus, in which case the musculo-spiral nerve passed through a tunnel ; this I have seen, and it is described by Mr. Wood ; 7. Continuous with the pronator radii teres ; this also I have seen ; 8. I have seen it receiving large accession of fibres from the coraco-brachialis, and the same is described by Theile, Hyrtl (" Lehrbuch der Anatomie," p. 863), M'Whinnie, and Wood ; 9. A part of this muscle may cross the brachial artery, or may overly the ulno-interosseal artery (Quain, p. 270, and Pl. 37, fig. 384) ; 10. A fusiform muscle arising from the outer border of this muscle has been seen by Mr. Wood and myself to pass into the pronator teres, near its insertion: a similar slip to supinator longus is described by Theile ; 11. A separate brachio-fascialis has been described by Struthers arising from the external bicipital ridge by a tendon, and being inserted over the pronator teres,—this muscle crosses the brachial

artery obliquely ; 12. A slip from the brachialis to the biceps directly was described by Horner (p. 419) ; 13. A three-headed brachialis is described by Gruber, in his monograph on this muscle, in the "Melanges Biologiques," xii., p. 363 ; 14. He also describes a biventral arrangement ; 15. With an attachment over the radial tuberosity (Führer) ; 16. or beneath it (Henle) ; 17. With a slip beneath the tuber ulnæ, which crosses and overlies a tendon from the ulna, which ends in the middle-finger-head of the flexor sublimis ; 18. A supernumerary internal brachialis lateralis minor, to the tuber ulnæ (Dawson, "Ed. Med. Journal," vol. 18, p. 82, and Hildebrandt) ; 19. A medial brachialis internus minor, inserted into the ulna (Meckel) ; 20. Into the tubercle of the radius (Gruber) ; 21. Two separate bellies, one radial and one ulnar (Gruber and myself) ; 22. A quadripartite arrangement was found by Clason ("Muskel Anom. observerade på Anatomisalen," Upsala Läk.-för. Förhandl. Bd. iii., Hft. 2), in this accessory brachialis, one slip arched over the ulnar artery and joined the pronator teres, a second ran to beneath the tubercle of the radius, a third in front of the biceps to the ulna, and a fourth was inserted with the normal brachialis behind the biceps tendon ; 23. A slip of the normal brachialis to the radius I have found ; 24. And still more rarely, one to the middle finger-head of the sublime flexor of the fingers ; 25. Suppression of the entire brachialis was described in a case deficient of radius by Voigt ("Archiv für Heilkunde," 1863, Leipzig, p. 27) ; 26. In Dawson's case, above described, one of the supernumerary muscles had a radial insertion ; 27. Another case of two separate slips is described by Gruber (Müller's "Archiv," 1848, p. 427), and another by Hyrtl (p. 326, "Lehrbuch.")

Capsularis-sub-brachialis, described by Portal ("Cours D'Anatomie Medicale," Paris, 1806, p. 251), consists of two slips lying under the brachialis anticus, sometimes contrary to it in direction, inserted into the capsule of the elbow ; these are in reality parts of the last muscle, these are really only detached slips of the last muscle, like some of the above-mentioned cases. A very good example of it was shown to me in the dissecting room of the College of Surgeons, Dublin, by Dr. D. B. Hewitt, formerly Demonstrator there. A muscle like this is described by Dawson ("Ed. Med. and Sur. Journal," vol. 18, p. 82).

Epitrochleo-anconeus is a little muscle frequently overlooked, which was described by Professor Wenzel Gruber, of St. Petersburg ("Mémoires de l'Acad. Impér. de St. Petersb.," 1867, tom. x.), and by Mr. Wood ("P. R. S.," 1867, p. 522). The former author found it existing in thirty-four subjects out of a hundred; nineteen times on both sides (fifteen in men, four times in women); three times on the left side alone (twice in men, and once in a woman); twelve times on the left side alone (nine in males, and three in females). Mr. Wood (1868) found it four times in thirty-six subjects (three times in both arms, and once in the left arm only). The same author seems to have met with it once in 1867 (out of thirty-six subjects), and once in 1866 (out of thirty-four). This would give an average frequency of existence of once in every five subjects. During my last year of office as Demonstrator of Anatomy in the Royal College of Surgeons, I endeavoured to determine the average of its relative frequency, and found it present rather oftener—more nearly once in four than once in five subjects—sixteen times in sixty-three. This gives it a frequency of about forty-eight in two hundred. Its origin is from the back of the inner condyle of the humerus, and may be (α) narrow; (β) wide and fleshy; or (γ) tendinous. It is usually placed on a level rather above and behind the condyloid origin of the flexor carpi ulnaris. Its insertion is into the olecranon process, and into its subjacent fascia. Professor Gruber found this muscle (δ) with two heads one above, and the other below. (ϵ) In another instance the muscle was doubled. (ζ) It is sometimes triangular; (μ) but more commonly and typically it is a flat, rectangular band. (θ) The same eminent anatomist has found it fleshy in the middle, and tendinous at both ends. It usually runs backwards, with a slight downward obliquity, and always covers over the ulnar nerve. Professor Gruber, in the work above cited, enters largely into a detailed description of this muscle, and its relations in other animals. Mr. Wood comes to the conclusion, from the comparison between his statistics, and those of Professor Gruber, that it is more common in Sclavonic than in Anglo-Saxon races (in the proportion of one in three to one in seventeen); but I have found it fully once in every four subjects during the last Session in which I held the office of Demonstrator in the Royal College of Surgeons, Ireland, as noted above. I have seen it, but small

and rudimentary, in a female chimpanzee. Its existence is noticed by Koster, under the name of Anconeus quintus, in the "Nederlandsch Archief." ii., p. 462.

Triceps extensor cubiti has been found to exhibit the following varieties: —1. It may have a fourth head from the inner part of the humerus below the head, by a long, slender tendon, and by an aponeurotic expansion from the capsule of the shoulder—this head blended with the inner head of the muscle (Blumenthal, Henle u. Pfeufer's "Zeitschrift," 3 Reihe, xxxvi., p. 1); 2. I have frequently seen the long head split, one attached to the capsule, and the other to the tricipital spine, or axillary border, or the first slip I found splitting the capsular ligament, like the curved head of the rectus femoris; 3. The existence of a slip from the tendon of the latissimus dorsi has been several times noticed. It is described by Bergman ("Müller's Archiv.," 1855, p. 347); and it is also mentioned by the late Professor Halbertsma ("Verslag. en Mededeelingen der Koninklijke Akad. van Wetenschappen." F. 4, p. 238), under the name of the Anconeus quintus; this may occasionally come from the teres major. This slip is the remains of the tricipiti accessorius, or the dorsi-epitrochlearis, which is present, in some form or other, in all quadrumana. I have never found this in any other forms, except as—1. A tendon of union for the lower border of the latissimus dorsi to the long head; 2. A fleshy slip of connexion from the costal fibres of the latissimus into the same part of the triceps; 3. Harrison describes the triceps as attached occasionally to the anconeus: this I have also seen; 4. A slip from the long head up to the coracoid process, as an additional head, was found by Professor Gruber; 5. A slip from the tendon of the subscapularis; 6. A small slip connected to the capsule of the shoulder has been seen continued up to the upper portion of the glenoid cavity; 7. A double fourth head, one from the coracoid process, and one from the capsule of the shoulder: all these three remarkable cases I have met with; 8. Fusion of the external head with the origin of the extensor carpi ulnaris, is described by Jenty (op. cit., p. 267); A second external head (posterior, inferior, and external) is described by Meckel ("Archiv," 1826, p. 39). Professor Theile has given a short monograph on this muscle ("Müller's Archiv," 1839, p. 420). I agree with Henle and Weber relative to the bursa of Munro.

Subanconeus is very variable, and indeed by no means constantly, or even frequently, present. It is the subject of a memoir by Kulæwsky; and the only varieties of it which I have met with have been absence, fasciculation, or fusion with the triceps ("Reichert u. du Bois Reymond's Archiv," 1869, p. 410).

The biceps shares with the palmaris longus the notoriety of being the most variable of all the muscles in the body, as the following list will show:—1. It has been found entirely suppressed; 2. The short head has been found absent by Meckel ("Archiv," Bd. 8, p. 587); 3. The long head was found deficient by Otto ("N. selt. Beobacht., p. 40, Heft 1, p. 899), Lauth ("Nouveau Manuel de l'Anat.," p. 144), Hyrtl (p. 362), Henle (Meckel "Archiv," 1826, p. 39), and myself; 4. The fascial insertion I have also found to be absent in one arm, but I have not found any record of such; 5. The radial insertion has been found absent by Petit ("Mémoires de l'Acad., 1733, p. 21), and Haller ("Op. Anatomica Argument. Minor.," vol. iii., Lausanne, 1768, p. 23); 6. The same, with an insertion into the elbow capsule and coronoid process, was found in a case of congenital want of the radius, by Gruber ("Virchow's Archiv," vol. xxiii., 212); 7. The two heads I have seen, as stated in one of my former papers, separate for the whole length of the muscle; and similar cases have been seen by Riverius ("De Dissectione Partium Humani Corporis," Paris, 1545, p. 314); in another subject the muscle was split for three-fourths its course; 8. The long head was found double by Moser ("Archiv," vol. viii., p. 227); 9. I have found the long head arising from the capsular ligament; 10. or from the bicipital groove; 11. A double coracoid head is described by Mr. Wood, and I have found it once present; 12. This anomalous portion may join the main body of the muscle, or else it may unite with the normal coracoid head before that portion of the biceps joins the long head. This was the case in the instance which occurred to me. The former seems to have been the arrangement in Mr. Wood's case; 13. An extension of the coracoid head from the coraco-acromial ligament I have met with six times; it has likewise been mentioned by Wood and other authors; 14. I have also seen three times a slip to the short head from the insertion-tendon of the pectoralis minor; 15. The most common of all the varieties of this muscle is the occurrence of a third head, and the

most frequently present third head is that from the interval between the coraco-brachialis and the brachialis anticus : the frequency of this has been differently stated by authors : Theile (" Encyclopædie Anatomique," vol. iii., p. 217) found it once in nine ; Hallett once in fifteen ; Wood eighteen out of one hundred and seventy-five subjects ; and I have found it in five sessions to occur as often as one in ten—which, I think, is about the proper average of its occurrence ; 16. I have also found a third head directly from the brachialis anticus, but this is not as common as No. 15 ; Hyrtl, however, considers the third head to be nothing more than an aberrant slip of the brachialis anticus, split by the external cutaneous nerve—a view which, I think, will scarcely find acceptance with most anatomists (" Osterreich. Zeitschrift für praktisch. Heilkunde," vi., No. 28) : this third head I have found crossing the brachial artery, as described by Quain, Struthers, and others, but more commonly it is not so : in Struthers' case the muscle covered the artery by a tendinous arch (p. 196) ; 17. A separate slip from the coraco-brachialis continuous with the short head is described by Mr. Wood, and I have seen it once to exist ; 18. A humeral head starting from the insertion of the coraco-brachialis I have seen, but not so frequently as connected with the brachialis anticus ; 19. A third head I have found also from the outside of the humerus, between the insertion of the deltoid and the origin of the supinator longus ; 20. This has been found as a separate muscle, distinct for its whole length, as far as its insertion from the biceps, and is described under the name of brachio-radialis (Gruber, in " Müller's Archiv," 1848, p., 428 ; Theile, " Myologie," Jourdain's trans. p. 217 ; Hyrtl, " Lehrbuch der Anatomie" ; Wagner, in " Heusinger's Zeitschrift," Bd. iii., Heft 3, p. 345) ; Wood and Pietsch describe it as imperfectly separate ; 21. A head like No. 19, but continuous from the deltoid, I have seen ; 22. I have also found this in a subject in which there was no long head co-existing ; 23. I have seen it likewise continuous with the supinator longus ; and an instance of this is likewise mentioned by Mr. Wood ; 24. A variety of the internal third head (No. 15), described by Hyrtl (loc. cit. supra), is that in which the accessory part arises from the internal intermuscular septum ; I have noticed this since reading Professor Hyrtl's description, but before I did not discriminate instances with a septal origin from those with a bony origin alone, and hence I have not observed

as to the frequency of occurrence of this variety; 25. An instance of the origin of the long tendon of the biceps from the tendon of the pectoralis major is described by Koster ("Nederlandsch Archief." ii., p. 371), and I have in my notes a record of a single case of the kind, and in the description of the pectoralis major I noticed a step towards this irregularity in the instance in which the biceps long tendon split the great pectoral tendon; 26. A third head from the outer tuberosity is described by Meckel: this I have met with; 27. The long head I once found attached to the great tuberosity, but there were some signs in this case of chronic rheumatic arthritis; 28. Haller describes a third head from the outer margin of the bicipital groove arising by a tendon; 29. I have found an accessory third head present, derived from the upper border of the pronator teres, inserted with the biceps; 30. A series of four heads I once found present, the usual third humeral origin co-existing with a head from the great tuberosity similar to No. 26; 31. Mr. Wood found a four-headed muscle, in which the inner and outer humeral origins co-existed (15 and 19); 32. Another quadricipital arrangement has been seen by myself, where the short head was doubled, and the inner humeral head, (15), co-existed with the great-tuberosity-head (26): in this case the long tendon was absent; 33. Henle notices a quinqueceps, the normal heads and the third humeral head co-existing with one from the deltoid, and a fifth from the middle of the bicipital groove as high as the tendon of the pectoralis major; 34. In another four-headed biceps which I met with, the two normal heads, and the inner humeral co-existed with an origin from the outer lip of the bicipital groove; 35. Meckel describes a biceps consisting of a short and a humeral head ("Archiv," 1826, p. 39); 36. Of unusual insertions, I have met with some cases worthy of note: a slip from the outer head inserted separately into the radius is one of the rarest of these. Cases of it have been noticed by Sels ("Dissertatio Anat. Musc. Var. Sistens," 1815, p. 12), and Rudolphi ("Blumenbach's Med. Bibl.," Bd. 1, p. 176); 37. A normally-arising biceps was seen by Mr. Wood to have three insertions—an outer normal portion, giving off a radial tendon and a superficial semilunar fascia; a middle smallest portion was inserted into the fascia, over the supinator brevis, and the bursa of the radial tuberosity; the inner division, which ended in a strong trifurcating tendon, sent a slip to the coronoid

origin of the pronator teres, one into the coronoid insertion of the brachialis anticus, and one into the coronoid attachment of the flexor sublimis; 38. Mr. Kelly has communicated to me an instance in which there was a small tendon detached from the lower end of the muscle, giving origin to part of the flexor carpi radialis; 39. A slip from the biceps to the internal intermuscular septum is described by Quain (p. 270); 40. I have received from Mr. Kelly notes of another interesting case, seen during the past winter session (1869–70), in the Royal College of Surgeons' Dissecting-room, in which there arose from the long head of an otherwise normal biceps a tendinous slip, which becoming fleshy, was inserted at the lowest part of the antecubital fossa, by three tendons, one into the fascia over the supinator longus, a second into the semilunar fascia, and the third dipped into the bottom of the fossa between the pronator and supinator muscles, but Mr. Kelly did not follow it to its termination; 41. A somewhat similar case has been noticed by Hyrtl ("Œsterreich Zeitschrift," viii., No. 20, 1862), in which a similarly-arising muscle was inserted by two slips—one into the bursa over the tubercle of the radius under the normal tendon, and one into the coronoid process; 42. Another, and larger accessory biceps muscle is described by Pietsch, which arose by three heads—one from the outside of the humerus, one from the inner, and one from the short head of the biceps itself; these three were inserted by a separate tendon into the radius at its tubercle behind the ordinary biceps tendon ("Journal de Médecine de Roux," Tom. 31, p. 245); 43. I once found a slip from the biceps to the brachialis anticus, running downwards and a little inwards: this I think is not recorded by any other writer; 44. Theile describes a tricipital muscle in which there were two coracoid heads, and a slip from the capsular ligament with no long head; 45. I have found in one instance the muscle inserted by four slips, besides the normal insertion, to the internal brachial ligament, to the capsule of the elbow joint, to the coronoid process, and to the pronator teres; 46. The slip which, arising from the biceps and crossing the brachial artery, is sometimes traceable down as far as the fascia over the pronator teres, is described by Professor Struthers, of Aberdeen, as a "brachio-fascialis;" it is also figured by Quain (Pl. 37, fig. 5); 48. A simple fascicle of the biceps into the origin of the pronator teres I have three times seen: it is described also by Clason ("Upsala Läkareforenings Förhandlingar," Bd. iii., Hft. 2);

48. Gruber found in a case of deficient radius the biceps to possess a weak coracoid head, two humeral origins, and two insertions—one radial and one ulnar.

Pronator radii teres I have found: 1. Split into separate coronoid and condyloid muscles for its whole length, as described by Albinus; 2. The coronoid head absent: this has been recorded by several authors; 3. The two heads separate as far as the insertion, and inserted in common; 4. A double coronoid head, the second lying posterior to the artery: this I met with in 1868. Mr. Kelly has informed me that he met with another instance of this in 1870, both heads being posterior to the median nerve; 5. The coronoid head I found twice attached directly to the brachialis anticus; 6. Or going, not to the pronator, but to the palmaris longus: a case of this I described in the "Journal of Anatomy and Physiology," vol. ii., p. 9; 7. A single slip from the coronoid process giving origin to two slips, one of which goes into the pronator, the other to the flexor carpi radialis; 8. An accessory medio-ulnar origin was found by Brugnone; 9. The condyloid origin has been found with a sesamoid bone in it (Schenzer, quoted by Sömmerring, p. 259); 10. With an accessory origin from the semilunar fascia of the biceps; 11. With an accessory slip from the brachialis anticus; 12. A treble origin I have found on one occasion (1866): one head from the biceps tendon, between its ordinary tendon and its semilunar fascia, one from the humerus from about three inches and a half above the condyle to about an inch and a half above it, and one (a short narrow tendon) from the tip of the inner condyle; 13. A high origin, like the second head of the preceding, was seen by Prof. Struthers, nearly separate for its whole length from the rest of the muscle ("Anat. and Phys. Observations," Edin. 1854, p. 201); 14. A case of this humeral head not detached has been described by Nuhn, ("Untersuchungen u. Beobacht. aus dem Gebiete der Anat.," p. 20); 15. A more complex case of high origin is described by Hyrtl, in which there existed one origin from the inter-muscular ligament, one from the inner condyle, and from the humerus, at the insertion of the coraco-brachialis, lying over the brachialis-anticus ("Œsterreich. Zeitschrift," vol. 8, No. 20, 1862); 16. A high humeral origin has been found co-existing with a supra-condyloid process of the humerus by Gruber ("Neue Anomalien," p. 28); Struthers, (loc. cit.), Schwegl; and by Voss of

Christiania ("Norsk Magazin for Lœgevidenskaben," Bd. 10, Hft. 11); in the latter case an additional humeral head lay above the supra-condyloid process; 17. The insertion of the pronator has been found by me once sending a slip into the radial origin of the flexor sublimis, and the same is described by Otto; 18. Koster, in a case of congenital deformity, found this muscle inserted into the lower end of the radius ("Nederlandsch Weekblad voor Geneeskunde," 1856, p. 125); 19. Another instance of cleft pronator is figured by Prof. Rüdinger ("Ueber die Muskeln der vordern Extremitäten," &c., Haarlem, 1868, Taf. 15, fig. 37.)

Flexor carpi radialis I have found—1. With a triple origin—one head from a slip of the biceps tendon and from the deep surface of the semilunar fascia; this head being separated from the second or normal head by the median nerve: the third head was from the ulna, below the coronoid process, and this was likewise separated from the foregoing by the same nerve; 2. With a radial head joined to the radial origin of the flexor sublimis; 3. With a coronoid head in a case in which the pronator teres had no such slip; 4. And, as described under the foregoing muscle, in one instance these two had a coronoid head in common; in both these cases (3 and 4), the coronoid was separated from the condyloid origin by the median nerve, and in the former case by the brachial artery: in another example of No. 3, which occurred to me in 1868, the nerve alone separated the parts; 5. A case of second head, from a special slip of the biceps tendon, was communicated to me by Mr. Kelly; 6. It has been found inserted in whole or in part into the trapezium by Albinus, Fleischmann, Loschge, Hyrtl, Wood, Theile, Henle, and myself; 7. Into the annular ligament and scaphoid by Fleischmann (p. 25); 8. Into the basis of the third and fourth metacarpals; 9. Into the lower end of the radius, in case of a congenital deformity by Koster ("Nederlandsch Weekblad," 1856, p. 125); 10. Into the scapho-trapezium bone, in a case in which only a thumb and two fingers were present, by Friedlowsky ("Sitzungsberichte der K. Akad. der Wissensch. mathematisch-naturwissensch. Classe," Band. lix., Wien, 1869, p. 532).

Flexor carpi ulnaris, has been found—1. With an origin from the triceps (Jenty); 2. With a slip to the fourth metacarpal bone (Wood and myself);

3. Having a slip to the annular ligament, without or with a coexisting palmaris longus; the latter form by Harrison and myself; the former was found by Winslow ("Anatomy," p. 193,) and myself; 4. In the case quoted under the preceding muscle by Koster ("Nederlandsch Weekblad," p. 125, 1856) it was inserted into the interosseous membrane; 5. In the case described by Friedlowsky (loc. cit.) this muscle sent a slip to the annular ligament, and the palmaris longus was suppressed.

The palmaris longus forms the subject of a monograph by Prof. Gruber of St. Petersburg, and is the most frequently varying muscle in the body, for while we find the biceps to present some form of variety, at least once in six subjects, we find this muscle exhibiting some deviation from the normal arrangement once in every four instances. The varieties which I have seen or which have been hitherto described are as follows: 1. Absence, which occurs in the proportion of once in ten subjects, (9 in 102, according to Wood); this is most commonly symmetrical, but I have found it present on one side, usually the right, and absent on the opposite; cases of symmetrical deficiency are, however, three times more frequent than those of unsymmetrical. Absence was first described by Columbus in the case of some thieves ("De Re Anatomica," 1593, Lib. v., p. 288); 2. Doubling of the ordinary muscle, the two being the same size, or the ulnar being a little the smaller, I have seen several times; I have never found the ulnar muscle larger than the radial in cases of this kind; 3. Doubling of the muscle, the outer one coming from the coronoid process (Meckel); 4. Doubling of the muscle, the ulnar muscle having an intermediate belly and a tendinous origin and insertion (this I have at least twice noted); 5. Doubling, the ulnar muscle being inverted, and having a long tendinous origin and a fleshy insertion into the annular ligament and fascia over the hypothenar eminence; 6. Triplicity has been found by Wenzel Gruber, the three bellies being condyloid in origin, the middle being the smallest, and their tendons being separate from each other, ("Mémoires de l'Acad. Imp. de St. Pétersbourg," 1868, Jan. 23); 7. Under the head of double palmaris, I would put all those cases in which accessory superficial muscles lie at the middle and lower part of the forearm, passing into some of the muscles of the hypothenar eminence. The commonest form of this is as a fan-shaped muscle

from the lower third of the fascia of the ulnar side of the forearm, inserted by a tendon into the abductor minimi digiti. This is figured in Gunther's "Chirurgische Muskellehre," Taf. xx. fig. 5, and is described by several authors; it usually coexists with a normal palmaris; 8. I have found it arising by two heads—one from the tendon of a normal palmaris longus, and one fleshy from the fascia over the flexor carpi ulnaris; these two heads united above the wrist to form a fleshy belly, which ended in a tendon that ran parallel to the abductor minimi digiti, to be inserted into that muscle immediately before its insertion; 9. It has been found by M'Whinnie arising from the sheath of the flexor carpi radialis, and passing to the abductor of the little finger: to this he gives the name "accessorius ad flexorem carpi radialem;" 10. As rudiments of this, even when the muscle is not developed, we not unfrequently find slips of fibrous tissue or fascia from the normal palmaris longus tendon to the abductor minimi digiti; 11. I found it once in a left arm arising from the fascia over the ulnar artery by the inner side of the normal palmaris longus; the muscle was two inches in length and the tendon one inch; it ended by being inserted into the annular ligament: this variety is more unequivocally palmaris than the others; 12. The palmaris I have found single, and inverted, with a fascial band of tendon as origin; 13. Or the same, with a round tendon of origin, separate from the fascia. I have seen two examples of this form for every one of the preceding; 14. A single muscle, with an intermediate belly and a tendon of origin and insertion, is by no means a rare variety. I have seen the fleshy part of this varying from two to seven inches in length. This commonly has a flat tendon of origin and a round insertion tendon, but I have seen much more rarely a round origin tendon and a flat insertion tendon. Once I saw both rounded, and once both flattened. This form of the muscle is described by very many authors. This and No. 12 were first noticed by Petsche ("Syllog. Muscul. Obs. Anat. Select.," p. 770, in Haller's "Disput. Anat. Select.," Göttingen, vol. vi.), and by Winslow (p. 193); 15. I have seen this muscle, on two occasions, fleshy for its whole length (an approach to this was seen by Boyer, p. 282); 16. Or replaced by a tendon for its entire length. This form I have also twice met with, and it has also been seen by Mr. Wood and Columbus; 17. I once saw it arising from the internal inter-

muscular ridge an inch higher than usual; 18. And I have also, on one occasion, seen it under cover of the condyloid origin of the flexor sublimis digitorum; this was also seen by Harrison; 19. I have seen it also attached to the coronoid process of the ulna—as has also been seen by Meckel; 20. Or with two heads, one coronoid, and one condyloid—the former lying internal to the coronoid head of the pronator teres; 21. A second head from the tubercle of the radius (Janser "Nederlandsch Lancet," 1850, Jan., p. 431); 22. A form arising from the radius, in common with the upper border of the radial origin of the flexor sublimis digitorum. This I saw in 1867, and it is described by Henle; 23. A very large bicipital variety of the palmaris I saw, with one head from the inner edge of the ulna, under the flexor carpi ulnaris, extending for the lower two-thirds of that bone. The ulnar nerve separated these heads. No ulnar artery co-existed with this. A slip of the tendon went into the abductor pollicis; 24. I once saw it as a tendon from the flexor sublimis digitorum; this variety has been seen in an idiot by Mr. Carver ("Journal of Anat. and Phys.," vol. iii., p. 260); 25. Mr. Wood describes a tendinous slip to the annular ligament from the flexor sublimis digitorum as co-existing with the normal palmaris longus; 26. Fleischmann saw it as a slip from the flexor profundus ("Abhandlungen der Physic. Med. Soc. in Erlangen," Bd. i., s. 25). A case of this I have also met with and described ("Proc. R. I. A." 1867); 27. I found its substitute twice as a slip from the flexor carpi ulnaris, and Friedlowsky describes the same; 28. Mr. Wood has seen it as a band from the superficial fibres of the flexor carpi radialis; 29. Mr. Wood has described a curious case of double palmaris, in which the second head arose with the condyloid head of the flexor sublimis; 30. Mr. Wood describes another interesting case, in which two tendons arose from one muscular belly; 31. The same author has also seen a median bellied muscle whose tendon gave a slip to the abductor pollicis; 32. Professor Gruber saw it inserted into the pisiform bone—an anomaly which I observed two years ago; 33. I have seen it entirely inserted into the origin of abductor pollicis in 1868; 34. In a case of double, not reversed, palmaris, I found that the second muscle was inserted into the abductor minimi digiti, and into the annular ligament and palmar fascia; 35. I have also seen a normal palmaris tendon

giving off a large fasciculus, which gave origin to the inner head of the abductor pollicis brevis; 36. Fleischmann has seen the tendon of this muscle inserted into the scaphoid bone, as also has Winslow (p. 193), and Jenty ("Anat. Phys. Lectures," 1757, p. 281); 37. It has been seen inserted into the interosseous membrane by Koster ("Nederlandsch Weekblad voor Geneeskunde," 1856, p. 125).

Flexor digitorum sublimis has been seen presenting the following anomalies:—1. Absence of the tendon for the little finger, seen by Mr. Wood and myself; 2. suppression of the radial origin (Wood); 3. A deep slip to the annular ligament, coexisting with the palmaris longus (ibid.); 4. A double coronoid origin—the upper slip fleshy, the lower tendinous (ibid.); 5. Otto has seen a slip from the insertion of the pronator teres to the radial origin ("Neue seltene. Beobachtungen," Hft. i., p. 90; 6. From the outside of the coronoid origin a slip arose, which, becoming tendinous, became the sole origin of the first lumbricalis (Wood). The same author describes a similar case in which the lumbricalis had a second normal origin, and a third instance in which a muscular slip joined this tendon to the indicial tendon of the profundus; 7. A connecting muscular band sometimes exists between the origins of the flexor sublimis and the flexor pollicis longus—this I have seen, and it is described by Mr. Wood; 8. Professor Turner has noticed the variation in the length of the radial origin of this muscle—I have found it to vary from half an inch to four inches in length; 9. In a case in which this muscle sent off no tendon to the little finger, Mr. Kelly found its representative to arise from the front of the synovial sheath of the fourth perforating tendon; 10. I have found the index perforated flexor as a separate muscle arising from the coronoid process, this has also been seen by Mr. Wood; 11. We have both met with cases of this kind, in which the separate muscle was digastric; 12. A similar digastric muscle, supplying both the index and middle fingers, I have met with; 13. The entire muscle was seen to be digastric by Dursy of Heidelberg ("Henle u. Pfeufer's Zeitschrift," vol. 33, p. 46); 14. A separate coronoid tendon joining the radial origin was seen by Mr. Wood; and 15. The same author saw a separate fusiform muscle for the little finger arising from the inner condyle of the humerus; 16. I found

the tendons for the index and little finger springing from a separate digastric muscle, with a condyloid and coronoid origin; 17. Mr. Wood found all the tendons connected to separate bellies—the first being digastric and condyloid, the second radial and coronoid, the third and fourth condyloid; 18. Another variety of differentiated flexor I met with in 1868, in which the radial head and part of the condyloid went to the middle finger, a tendon from the muscle, a few inches below the condyle, becoming fleshy, supplied the ring finger, and the remainder supplied the index and little fingers; 19. A slip from this muscle to the palmar fascia coexisting with a feeble palmaris longus, has been met with by Rosenmüller (l. c. p. 6), Wood, and myself; 20. Mr. Wood has seen a double tendon for the middle digit; 21. In a case of absence of the little finger tendon, the same author has seen the perforated flexor for this finger supplied by the fourth lumbricalis; 22. I saw the little finger slip of the flexor sublimis coalescing with, and not split by, the corresponding tendon of the flexor profundus; 23. Cross slips joining the muscle-bundles for separate digits were seen by Professor Turner; 24. The same author saw a slip from the flexor sublimis to the inner side of the palmaris longus, crossing under that muscle, and joining the tendon of the supinator longus at the lower end of the forearm.

Radio-carpus is a muscle which has attracted a considerable amount of attention of late. It was first described by M. Fano ("Bulletin de la Société Anatomique de Paris," November, 1851, p. 375), then by Professor Wenzel Gruber, of St. Petersburg ("Bulletin de l'Acad. Impériale des Sciences," St. Petersbourg, 1859, No. xxviii., p. 439; "Mélanges Biologiques," tom. iii., livr. 2, 1859, p. 184, fig. 1 and 2). It was afterwards described by Mr. Wood ("Proc. R. Soc.," 1866–7–8; and "Journal of Anat. and Physiol.," vol. i., p. 55). It has since been described by Professor Zaaijer ("Nederlandsch Tijdschrift voor Geneeskunde," 1869), and by Friedlowsky ("Sitzungsberichte der Mathematisch-naturwissen. Classe der k. Akad.," Bd. 59, 1st Abth., Wien, 1869, p. 533). It has also been seen by Professor Theile, of Weimar (Schmidt's "Jahrbücher," vol. civ., p. 155). According to Mr. Wood, this muscle has been found four times in thirty-four subjects (1866), twice in thirty-six (1867), and twice again in thirty-six (1868). I found it twice in fifty-three

(1866), once in sixty (1867), and four times in sixty-four (1868). These observations give its average frequency as fifteen in 283—almost one in nineteen. Its origin has been found to be—1st. From the outside of the front of the radius, above the pronator quadratus, and external to the flexor pollicis longus; 2. Or from a lunated aponeurosis as high as the oblique ridge on the radius, under the radial origin of the flexor sublimis; 3. Or it may arise by two heads—one the ordinary radial, and the second by a flat aponeurotic band from the inner condyle of the humerus. This variety I found in 1867, and communicated it to Mr. Wood, by whom it was published ("Proc. R. Soc.," 1868); 4. I found it arising by one head, internal, to the flexor pollicis; 5. I have also seen it to arise from the oblique radial ridge alone; 6. In Fano's specimen, the muscle arose from the lower third of the anterior surface of the radius; 7. Its insertion is far more variable. In the first described specimen, or that of M. Fano, it was inserted into the trapezium, as it was also in the case described by Professor Theile; 8. Mr. Wood has seen it inserted into the head of the os magnum; 9. And, in another case, into the deep process of the annular ligament which encloses the groove from the flexor carpi radialis; 10. It has also been inserted into the base of the middle metacarpal bone and into the os magnum, connected to a slip of the flexor carpi radialis tendon, and giving origin to some of the fibres of the flexor brevis pollicis; 11. Into the ulnar side of the base of the second metacarpal; 12. Professor Zaaijer's first specimen was inserted into the transverse ligament of the carpus, and into the carpal ends of the second, third, and fourth metacarpal bones, and into the trapezium; 13. Mr. Wood and I found it inserted into the deep surface of the annular ligament, coexisting with the ordinary palmaris. A muscle similar to this, but not co-existing with the ordinary palmaris, was seen by Professor Theile ("Muskellehre, Jourdan's trans.," p. 237); 14. It has been seen by Messrs. Norton, Wood, and myself, inserted into the middle metacarpal bone alone; or 15. Into the bone, and into the trapezoidal portion of the annular ligament (Wood); 16. It has been found inserted by three slips, by one tendon into the deep trapezoidal part of the annular ligament, and by two into the deep portion of the ligament itself; 17. The muscle described by Friedlowsky, and called by him the abductor pollicis

accessorius, is inserted by a short tendon into the eminentia carpi radialis superior, sending a fascicle to the base of the thumb metacarpal.

In his recent papers on this muscle ("Mélanges Biologiques," tom. vi., livr. 4, 1868, p. 493 ; and tom. vii., livr. 4, p. 439, 1870 ; "Bulletin de l'Acad. Imp. de St. Petersbourg," tom. xv., p. 245) Professor Gruber remarks that he has found three varieties of this muscle—1. A radio-carpal, uncommon ; 2. Radio-metacarpal ; and 3. Radio-carpo-metacarpal ; of the first, Professor Luschka has described an instance (1863).

In the seven years between 1859 and 1866, Professor Gruber met with eight examples—six in males, two in females ; two symmetrical, two on the right, and one on the left, were radio carpals, and one of each of the other varieties was found. Mr. Wood originally described this muscle as the flexor carpi radialis brevis, or profundus, and Professor Gruber as the radialis internus brevis, or minor ; but Fano's name has the priority. Another instance of this muscle is recorded by Calori, in the Memoirs of the Academy of Science of Bologna (vol. vi., second series, p. 137).

Flexor digitorum profundus has been found—1. Cleft, the flexor indicis profundis being separate ("Weber-Hildebrandt," p. 454; Wood); 2. An origin from the radius along the entire border of the flexor pollicis (Kelly). I have seen a separate slip of origin from the radius above the pronator quadratus, not going specially to the index ; 3. A coronoid fasciculus of origin with the coronoid head of the sublimis (Wood); 4. In one subject a separate muscle arose from the coronoid process, in the same situation as the last, and received fibres from the inner part of the front of the radius. It passed beneath the annular ligament, and divided into two—one to join the tendon of the flexor pollicis longus, and one to the tendon of the index perforans (Wood) ; 5. Some of the fibres to the index finger arose from the radius (Wood) ; 6. I have seen a detached muscular slip of the profundus becoming tendinous, and joining that of the sublimis, or the index, in the palm, the same is described by Mr. Wood ; 7. Mr. Wood has seen the fourth lumbricalis arising from a special tendon of this muscle ; 8. Sömmerring found a similar slip which expanded on the synovial sheath of the tendons, giving origin to the first lumbricalis ; 9. A distinct pyriform muscle, arising from the coronoid process, constitutes the accessorius ad

flexorem profundum of Gantzer. This may form a common coronoid origin for the flexor sublimis and profundus, or a special one for the latter. It has been found joining the perforating tendon of the index finger (six times out of seventy-two, Wood), or the perforating tendon of the middle (eight out of seventy-two, ibid), or that of the ring-finger alone (Turner), or those of the ring and little (one out of thirty-six, Wood), or that of the little finger alone (Turner), or those of the index or ring fingers, or the index and little —these two I have seen—or the index, middle, and ring (Wood), or the middle, ring, and little (Turner). All these nine varieties I have met with from time to time, but I have preserved no record of their frequency of occurrence ; 10. I once saw a pear-shaped muscle arising from the inner condyle, under the flexor sublimis, which ended in a long tendon, and united with the little finger slip of the flexor profundus, opposite the metacarpo-phalangeal articulations ; 11. Methods of union between this muscle and the flexor pollicis longus are not very uncommon. I have seen a tendon joining the indicial part of the profundus with the pollicis, or a muscular slip. (See flexor pollicis) ; 12. I have seen a slip from the ulnar side of the flexor profundus running to join the little finger tendon of the sublimis ; 13. Professor Turner has found the tendons for the middle, ring, and little fingers joined by intertendinous bands, or the middle and ring-finger tendons so joined, or those of the little and ring ; 14. The tendon has been seen divided into five slips, two of which united to form the tendon for the ring-finger ; 15. A special head from the inner condyle (Theile) ; 16. An origin from the radius has been seen by Theile, Hallett, Henle, and myself ; 17. In one subject the palmaris longus arose from it, as described also by Fleischmann (op. cit, p. 25).

Lumbricales have presented the following varieties—1. The first lumbricalis I have seen double, one being normal, the other arising from the flexor sublimis. A case of this kind is described by Mr. Wood, in which the origin of the second flexor was a fine tendon which sprang from the outside of the coronoid origin of the flexor sublimis (Wood). In another case of this anomaly, Mr. Wood saw the abnormal lumbricalis split into two parts, the innermost being implanted into the tendon of the perforatus, near its division in the sheath of the index finger, the outer joined the normal lumbricalis, near

its usual place of insertion; 2. I have seen a slip from the flexor pollicis giving a tendon to the first lumbricalis (Wood, who described the occurrence of this). The same author has seen the first lumbricalis arising from the sublimis and pollicis alone. In the preceding case the muscle was bipenniform; 3. A supplementary head for the first lumbricalis, from the origin of the first palmar interosseous, was met with by Mr. Wood. I have never seen this variety; 4. The first has been seen inserted into the second finger by Moser ("Meckel's Archiv," vii., p. 230); 5. I have seen the first absent; 6. The second has been found also absent by myself; 7. Mr. Wood has seen it doubled at its insertion, sending one slip to the radial side of the middle finger, and one to the ulnar side of the index; 8. The same author has also found this muscle bipenniform in origin, taking rise from the tendons of the index and middle fingers; 9. A similar division of the origin of the third I have seen; and it is described by Froment ("Recherches sur plusieurs points d'Anatomie," Paris, 1853), Wood, and others; 10. The third has been found bifurcated at insertion—one half going to the ulnar side of the middle, and the other to the radial side of the ring fingers (Wood, Froment, myself); 11. The entire of the third has been seen going to the ulnar side of the middle finger. This I have met with, and it was found by Mr. Wood in a subject in which the fourth was absent; 12. The fourth has been noticed as absent by Sömmerring, Meckel, Weber-Hildebrandt (p. 453), Wood, and myself; 13. I found all absent in one subject—a female; 14. The fourth has been found inserted into the extensor aponeurosis of the ring finger by Theile, Wood, and myself; 15. It has been found bifurcated, supplying the ulnar side of the ring, and the radial side of the little finger (Wood); 16. The same author found the fourth giving off the perforated flexor for the little finger in a case of absence of this tendon from the flexor sublimis; 17. The first and fourth have been found springing from special tendons of the profundus; 18. The third and fourth I have seen bifurcated at their insertion; and a case of this kind has been communicated to me by Mr. Kelly, and the same is described by Gegenbaur ("Virchow's Archiv," vol. xxi., p. 376); other cases of this are noticed by Froment and Wood; 19. Five lumbricales have been seen by Walther ("Tenerorum Musculorum Anatome repetita," in

Haller's "Disput. Anat. Select.," vol. vi., p. 598), by Petsche (ibid., p. 770), and by Böhmer ("Observationes Anat. Rarior., Halæ., 1702, præfat., p. 8). In the first of these two cases, two were for the middle, two for the ring, and none for the little finger; in the second, two were for the middle, and the rest were normal. Another instance of five lumbricales is described by Mr. Carver as occurring in an idiot ("Journal of Anatomy and Physiology," vol. iii., p. 260). In this instance, the entire group formed a muscular mass, which clothed the front and the sides of the flexor profundus tendon, concealing them. This ended in a series of five tendons, four of which were normal, and one went to the radial side of the perforated tendon for the ring finger. The varieties of these muscles have been investigated by Froment (op. cit.), who has found these anomalies in 45 per cent. of subjects—a far greater percentage than I have found in the dissecting-room of the Royal College of Surgeons in Dublin. Mr. Wood finds them to vary in about $18\frac{3}{5}$ per cent. of subjects, or nineteen in 102. I have found varieties present only in about fifty subjects in 400—that is, in the proportion of $12\frac{1}{2}$ per cent.; 20. The lumbricalis for the fourth finger was found by Gruber to be a bundle of fibres from the third external interosseous ("Virchow's Archiv," vol. xxxii., p. 219).

Flexor pollicis longus has been seen—1. With an accessorius ad pollicem (Gantzer), present in forty out of 102 subjects (Wood). This may be distinct from the coronoid head of the sublimis, or may be blended with it; and I have occasionally found it receiving an accession from the condyloid fibres of the sublimis. I have found this to exist in the proportion of twice in every five subjects; 2. It has been seen by Mr. Wood and myself giving a tendon to the first lumbricalis; 3. A common slip from the coronoid process has been seen by me, giving off—1st, the coronoid head of the pronator teres; 2nd, a fleshy belly to the flexor pollicis, internal to which a slip for the flexor sublimis arose; 4. Another common coronoid slip I found in another instance, trifurcating and terminating in the three—flexor pollicis, profundus and sublimis; 5. A slip from the internal condyle, forming an accessory head; 6. A coronoid head I saw arising from the place where the second head of the pronator teres usually springs. (This head, however, was absent in this case.)

The slip was shortly joined by a fasciculus from the flexor sublimis, which passed beneath the ulnar artery, and ended in the flexor pollicis ; 7. I have also found a head from the brachialis anticus ; 8. Or from the middle of the superficial layer of the pronator teres (Moser. "Meckel's Archiv," vii., p. 230) ; 9. Sömmerring saw it doubled (p. 266) ; 10. Mr. Kelly has communicated to me a remarkable case in which a common condyloid head gave off a slip to the flexor pollicis, and to the indicial slip of the flexor profundus ; 11. I have also seen a slip from the coronoid process ending in two parts—one to the pronator teres, the second forming a belly, three inches long, which ran to the flexor pollicis ; in this case a separate slip went to the profundus ; 12. It has been seen sending a tendon to the index finger, which does not coalesce with the indicial tendon of the flexor profundus, but, running separately in the sheath, was inserted into the ungual phalanx ; 13. Cases of slips, sent from this muscle to the flexor profundus, are described by Loschge, Wood, and myself; 14. Mr. Carver describes a remarkable accessory muscle which he met with in an idiot ("Journal of Anat.," vol. iii., p. 260) ; in this case there arose, partly from the profundus and partly from the sublimis, a small muscle, which became tendinous, and, just above the annular ligament, divided into two—one for the flexor pollicis, and one for the flexor profundus of the index finger ; 15. Its absence is described by Professor Gegenbaur ("Virchow's Archiv," vol. xxi., p. 376) ; 16. In a case of double thumb, Professor Gruber found it sending a slip to both digits (ibid., vol. xxxii., p. 226).

Of the pronator quadratus few varieties have ever been described besides those which I have noticed in my paper on this subject, in the "Journal of Anatomy and Physiology" for 1870, vol. v., p. 32. 1. It has been seen split by Krause (p. 264). In cases of deficient radius, as described by Grüber ("Ueber congenitalen unvollständigen Radiusmangel," in "Virchow's Archiv, vol. xl., p. 427). It is either absent (Silvester, "Med. Times and Gazette," 1837, p. 673), or rudimentary (Gruber). This author describes a muscle, which he calls the ulno-carpalis-singularis anterior, arising from the lower third of the ulna, and inserted into the volar side of the carpus. In this instance, there was no trace of the pronator quadratus ("Virchow's Archiv," vol. xxxii., p. 218). It has been seen absent when the radius was present by Otto and Meckel. It has been seen triangular by Blancard ("Anatomia

Reformata Lugd. Batav.," 1695, p. 654), and by Riverius (" De Dissectione partium Corporis Humani," Paris, 1545, p. 316). This I have also found with and without a tendinous radial insertion. In all cases the apex is radial, and the ulna is normal. I have seen it bilaminar, as is described by Meckel. I have also found it composed of two triangles—one with an ulnar base and a radial apex, and the second with a radial base and an ulnar apex. A case like this was seen by Dr. J. R. Barton, of the Philadelphia Hospital ("Horner's Special Anatomy," vol. i., p. 426). A trifid variety I found with a triangular upper slip, having its base at the radius and its apex at the ulna; the middle portion was on a deeper plane, and more nearly transverse in the direction of its fibres, while the inferior was also triangular, with an ulnar base and a radial apex. In another form of this trifid muscle, the upper triangle completely overlay the middle deeper part. In another, the upper part had a radial base and an ulnar apex; the middle was deeper and transverse, and the inferior triangular, with an ulnar origin, was attached by a tendon at its apex into the deep portion of the annular ligament at its trapezial attachment. A still more remarkable variety I found, and have described ("Journal of Anatomy and Physiology," vol. v., p. 33), in which the whole muscle arose from the ulna, and was inserted into the scaphoid trapezium and annular ligament.* I have preserved the specimen in the Museum of the Dublin College of Surgeons. In another instance, a small detached slip from the lower border of the muscle was attached to the anterior ligament and synovial membrane of the inferior radio-ulnar articulation. It was found extending unusually high on the forearm by Friedlowsky (" Sitzungsberichte der k. Akad. der Wissenschaften ("Math. Naturwiss Sch. Classe,") Wien., 1869, p. 533).

The palmaris brevis I have seen presenting the following irregularities:—1. I have found its absence to occur about once in every forty-five subjects, but certainly not more frequently. Mr. Wood has seen this twice; its absence

* When writing this I was not aware that Prof. Gruber had described a similar case ("Bull. de la Classe Phys. Math.," tom. xvii. No. 28. Mélanges Biologiques tom. iii., livr. 2, 1859, p. 190, under the name of Cubito-carpeus, and a radial similar slip (radio-carpeus). In a more recent paper which Prof. Gruber has kindly sent me (Mélanges Biologiques tom viii., p. 142), he describes two other cases—one cubito-carpal; the other a two-headed radio-cubito-carpal muscle. None of these cases were precisely like the above described variety, for they all coexisted with a normal part of the pronator.

is also described by Arnold, Cloquet, &c.; 2. I saw its lower fibres attached to the pisiform bone four times. This is described by Cheselden (p. 91); 3. I have seen it receiving a few fibres from the tendon of the flexor carpi ulnaris; 4. Its condition, in point of continuity, is very variable. Most frequently it exists as a series of separate fasciculi; but I have seen it as a continuous muscular sheet. Krause describes it as being most strongly developed when the palmaris longus is absent (p. 252); I have found, sometimes, a deep slip to the palmar fascia on its under surface; 6. In the "Bibliotheca Anatomia," p. 116, vol. ii., London, 1712, it is described as arising from the fifth metacarpal bone and pisiform; 7. Portal has seen it extending as far as the thenar eminence (p. 295). The absence of this muscle is often spoken of as not uncommon. Thus we find Krause saying of it, that it "nicht selten fehlte;" and Boyer, Cloquet, Arnold, and others, speak of its absence as not very uncommon. This, perhaps, arises from the fact that it is sometimes so feeble as to require very careful search before finding it. The proportion which I have given above is that which I have found in six hundred subjects.

Flexor brevis minimi digiti accessorius is a separate muscle, deeper than the ordinary flexor, but with similar attachments. It has been seen by M'Whinnie, by Messrs. Flower and Murie (in a Bushwoman, "Journal of Anat. and Phys.," vol. i., p. 202), and by myself on one occasion.

Abductor minimi digiti I have found—1. With three origins: one from the pisiform, one from the posterior annular ligament, and one from the tendon of the extensor carpi ulnaris: a similar case is described by Mr. Wood; 2. The same author found a second head arising from the anterior annular ligament, and separate from the rest of the muscle, as far as the insertion; 3. I have found it split into two, and, in a case of this kind, the inner had the accessory palmaris inserted into it, as represented in the plate in Günther's "Chirurgisch Muskellehre" (Taf. xx., fig. 5); 4. I have seen it with a double insertion, but otherwise single; 5. It has been absent in three cases; 6. I have failed to separate it four times from the flexor brevis; 7. I have seen it arising only from the pisiform bone.

Opponens minimi digiti I have seen—1. Split into an upper and lower part; 2. Absent; and, 3. Inseparably united with the last as far as its insertion.

Flexor brevis minimi digiti has been seen—1. Absent (Cloquet); 2. With an accessory palmaris slip joined to it; this is not common, but I have seen it, and so has Mr. Wood; 3. I found it with an unciform origin alone; 4. Or sending a slip to the head of the metacarpal bone; 5. Or united to the abductor.

A small special muscle was found by Calori passing from the unciform to the pisiform; this was a small pyramidal muscle, but it has not, to my knowledge, been seen by any other author. It is described in the Memoirs of the Bologna Academy of Science, second series, vol. vi., p. 140.

Abductor pollicis brevis I have often found double—a condition which Sömmerring regards as normal in this muscle, one part being internal and one external; into the latter of these the tendon of the extensor carpi radialis accessorius has been found to go, and the extensor ossis metacarpi pollicis is often attached to the same; 2. I have found a third deeper belly for this muscle from the surface of the opponens pollicis; 3. An accessory belly from the styloid process has been seen by Cruveilhier and Turner; 4. A slip from this muscle was seen by Mr. Kelly to extend as far forward as the insertion of the extensor primi internodii pollicis, with which it was united; 5. A slip of this muscle to the skin of the thenar eminence is described as frequently present by Lepine ("Dictionnaire Annuel des Progrès des Sciences et Instit. Medicales," 1864, p. 35).

The flexor brevis pollicis, deep head I found inseparably connected to the abductor pollicis; this muscle was found absent by Gegenbaur (Virchow's "Archiv," vol. xxi., p. 376), doubled by Moser (Meckel's "Archiv," vol. vii., p. 230). Of these muscles a good description is given by Dursy, in "Henle u. Pfeufer's Zeitschrift" (1853, p. 65).

In a case of double thumb, Professor Gruber found a remarkable muscle, which he calls the interpollicaris transversus, passing from the metacarpal bone of the proper thumb to the basal phalanx of the supernumerary digit (Virchow's "Archiv," p. 226, vol. 32).

Supinator longus is a muscle which exhibits few very marked varieties—1. I have found its origin joined to the brachialis anticus, a variety described by Professor Gruber ("Mélanges Biologiques," xii., p. 389); I have also seen

its origin continuous with the insertion of the deltoid in part, and the same is described by Professor Gruber in Virchow's "Archiv;" 3. I have seen it cleft into two for three-fourths of its extent; 4. Its tendon of insertion is not very unfrequently double (three times in thirty-six, Wood); in these cases they are usually inserted together, and have the radial nerve underneath both —more rarely between the two; in one case of this kind, Mr. Wood found the deeper tendon attached three inches above the other; 5. A trifurcated tendon, the lower one being normal, the middle inserted opposite the lower border of the pronator quadratus, and the upper into the middle of the radius, the radial nerve separated the second and third (Wood); 6. The tendon of this muscle received a slip from the tendon of the extensor carpi radialis longior; 7. Dursy describes a case in which its tendon was prolonged into the base of the third metacarpal bone (Henle u. Pfeufer's "Zeitschrift," 3te Reihe. xxxiii., p. 45); 8. This muscle was once digastric and small; its lower insertion being prolonged into the scaphoid and trapezium; 9. It was found absent in a case of deficient radius by Meckel ("Archiv," 1826, p. 36), and Silvester ("Med. Times and Gazette," 1857, p. 643). It has been found with a second higher muscle inserted an inch above the normal tendon (Gegenbaur, loc. cit.).

Tensor ligamenti annularis vel orbicularis posterior is a small muscle, described by Professor Gruber, arising from the side of the ulna, behind the lesser sigmoid cavity, and inserted into the orbicular ligament. Professor Gruber has found it present in 162 out of 220 extremities: it is a small, separate slip of the supinator brevis; it is very distinct in about one in every four subjects, and with difficulty separable from the supinator brevis in one in every three subjects, and nearly always symmetrical (Reichert's "Archiv," 1865, p. 381).

Supinator brevis is described as having been found double by Sandifort "Exercit. Acad. Lugd. Batav.," 1783, p. 94). A case of this variety described by Bonn is quoted by Sandifort. I have found it cleft perfectly in the place where the posterior interosseous nerve perforates it. A slip is described by Fleischmann going to the tubercle of the radius; a similar separate portion lower down has been seen joining the pronator teres by Clason ("Muskel-anomalien observerade på Anatomisalen in Upsala, "Upsala Läkareforenings

Förhandlingar," Bd. iii., p. 12). A second or accessory supinator brevis is also described by Professor Halbertsma ("Nederlandsch Tijdschrift," vol. vi., p. 609). Dursy ("Henle u. Pfeufer's Zeitschrift," Reihe iii., vol. xxxiii., p. 45) describes a slip of this muscle inserted into the biceps tendon, and a slip into the bursa tubercularis.

Tensor ligamenti orbicularis anterior of Gruber is a small muscle, first described by Cruveilhier (p. 189), but most particularly by the accomplished Professor of Anatomy in St. Petersburg. It is by no means so common as the tensor posterior—indeed, Professor Gruber has only found it present in fifteen cases out of 100: nine of these were symmetrical, four were on the right, and two on the left (Reichert and Du Bois Reymond's "Archiv," 1835, p. 381).

Supinator brevis accessorius of Halbertsma ("Verslagen en Mededeelingen der Koninkl. Akademie van Wetenschappen," xiii. Deel.) is only a slip from the lower border of the brachialis anticus inserted into the tubercle of the radius.

Extensor carpi radialis accessorius is the name given by Mr. Wood to a carpal extensor inserted into the base of the first metacarpal bone; its origin is generally below that of the extensor longior, but its tendon runs in the same groove in the annular ligament as that muscle; 1. Sometimes it is inserted into the outer part of the abductor pollicis brevis, which gives it a digastric appearance; and rarely it is inserted into both metacarpal and abductor; 2. Fleischmann and Wood saw this muscle arising from the fascia over the other radial extensors of the carpus, running down separately, and inserted into the metacarpal bone (op. cit., 1810, Bd. i. p. 28). Cruveilhier notices a similar variety. The second form above described is mentioned by Meckel; 3. This muscle is sometimes represented by a slip from the extensor longior, inserted into the first metacarpal and first dorsal interosseous (Wood); 4. Or a slip like the last sending a fasciculus to the interosseous primus volaris of Henle; 5. Or it may be continuous with the origin of the flexor brevis pollicis. Dr. Richardson, of Dublin, communicated to me also a case of the second variety noticed above. In one case in which a normal accessorius was present in the right arm, on the left a tendon arose from the supinator longus, two inches above the wrist, which ended in a belly half an inch above the styloid

process of the radius. This was inserted into the outer part of the abductor pollicis, and went to the first phalanx. This co-existed with a double abductor pollicis, and a double extensor ossis metacarpi. A case of the simple accessorius is described by Harrison (p. 174).

The existence of cross-slips between the long and short extensor has been supposed by Mr. Wood to indicate the presence of an extensor carpi radialis intermedius, but I think these cases can be easily interpreted without any such supposition. Of varieties of the extensor carpi radialis longior, the following are described:—1. Its tendon has been seen split into two: the first joined a slip from the brevior, and this compound tendon bifurcated, and each part was inserted under the normal tendons into the second and third metacarpals; 2. Its tendon has been found split into two, one of which went to the second and one to the third metacarpal (Bergmann, quoted by Otto); 3. I have frequently met with a single slip from the longior to the brevior. Slips like this have been met with by Mr. Wood six times in thirty-six subjects; 4. A fleshy slip from the longior, becoming tendinous, may pass to be inserted with the brevior, while a slip from the brevior passes in a similar manner to the longior. This occurred four times out of thirty-six subjects (Wood). Fusion of the longior and brevior I have seen, either one muscle giving off two tendons—or, in one case, three tendons—two into the third, and one into the second. In another instance a common muscle gave rise to four tendons—two into the second, and two into the third.

The extensor carpi radialis brevior has been found absent ("Salzmann Dissert. de sist. plurium pedis muscul. defecta," Strasburg, 1734, p. 11), and by myself once. I have also found it double, each part being normal (Dec. 1867). I have also seen it inserted, by two tendons, into the second and third metacarpals, as also is described by Meckel. I have seen it inserted into the third metacarpal by three tendons. Albinus has seen it attached to the third and fourth (p. 446). The method of insertion of this muscle by two tendons, one to the second and one to the third metacarpal, was found by Mr. Wood to exist four times out of thirty-six subjects. I have seen the brevior split into two parts, one normal, and the other with the double insertion just described into the second and third metacarpal. This is one of the cases supposed by Mr. Wood to represent the hypothetical extensor inter-

medius. Cases of this kind are described by Albinus, Meckel, Wood, and others.

Extensor communis digitorum—1. Has been seen without a little finger tendon, by myself; 2. With each tendon doubled; 3. With the ring finger tendon almost doubled; 4. With the index tendon only doubled; 5. With the middle finger alone doubled; 6. With the middle finger slip trebled; 7. With the middle and ring doubled, the contiguous portions being joined by a vinculum; 8. The index tendon being doubled, and the ulnar tendon being joined to the middle-finger tendon; 9. All the tendons united by vincula—this is not at all uncommon; 10. Mr. Wood describes the occurrence of this, one of the index tendons being the only one free; 11. Albinus saw a separate belly for each tendon, and the same has been noticed by Brugnone, Meckel, and Wood; 12. A special belly for the index finger tendon has been seen by Henle, Wood, and myself; 13. The little finger tendon doubled; 14. The ring and little finger tendons both doubled; 15. The index and little finger tendons doubled—these I have seen; 16. Mr. Wood saw, in one case, two tendons to the index and little fingers, and four to the ring; 17. I saw this muscle sending a slip, which ended in a tendon that passed downwards and outwards in the groove with the extensor secundi intermodii pollicis, which it joined at the base of the first phalanx of the thumb; this has also been seen by Mr. Wood; 18. The same author saw a similar slip going to the indicator; 19. I have found it sending one, two, or three slips to substitute the extensor minimi digiti, which was absent in the subject; 20. It has been found likewise deficient of a tendon for the second finger, the tendon for which arose from the extensor brevis carpi radialis; 21. I have seen it in two bellies, one supplying the second and third, and one to the fourth and fifth; 22. Wilde ("Comment. Acta. Petropol.," T. xii., p. 321) describes a splitting of the third and fourth bellies into two parts, each of which had a tendon inserted into their respective fingers; 23. Sömmerring saw it sending off a slip which joined the extensor minimi digiti; 24. Rüdinger, of Munich, figures a case of remarkable subdivision of the extensor muscles, in which there were eleven extensor tendons for the fingers between this muscle, the extensores medii, et tertii digiti, and the indicator ("Ueber die Muskeln der vordern Extremitäten der Vögeln," &c., Haarlem, 1868, tab. xv., fig. 38).

Extensor minimi digiti—1. Has been found absent by myself, and substi-

tuted by a slip of the extensor communis; 2. Or by a slip of the extensor carpi ulnaris; 3. Or with no substitute—this is described by Brugnone (p. 167); 4. With two tendons, both inserted into the little finger; this is a very common arrangement; 5. It has been found double, both muscle and tendon, by Wood and myself; 6. I have found it with a double tendon, the outer going to the ring finger, and one to the little finger; cases of this have been seen by Vesalius, Meckel, M'Whinnie, Hallett, Theile, and Wood. This has been found by the last author thirteen times in 106 cases—that is, about once in eight. I, however, have not found it to occur in Irish subjects more than once in fourteen; 7. I have seen it with a treble tendon, two going to the little finger, and one to the ring finger; 8. It may receive an accessory slip from the common extensor (Sömmerring).

Extensor carpi ulnaris—1. Has been found receiving a slip from the triceps; 2. Sending a tendon which replaced the extensor minimi digiti, which was suppressed; 3. Giving origin from its tendon to the abductor minimi digiti; 4. A slip inserted into the septum in the posterior annular ligament, between this and the last muscle (extensor minimi digiti); 5. A slip to the tendinous aponeurosis, on the back of the little finger (Henle, Meckel, Wood); this is described by the last-named as occurring in 12 per cent. of subjects; I have, however, not seen it nearly so often as these authors describe. This slip has been called ulnaris quinti.

Anconeus has been found—1. Joined to the triceps; 2. Sending a slip into the last muscle; 3. Or split into two parts.

Extensor brevis digitorum manus is a muscle whose rudiment is often seen on the back of the hand (at least, once in fifteen subjects—Mr. Wood has found it once in thirty-six). It has been described by Albinus under the name of extensor brevis digiti indicis ("Annotationes Acad.," lib. iv., cap. vi., p. 28). Gantzer describes this as a second head of a biceps extensor of the index, under the name of indicator biceps, and Otto as an indicator anomalus brevis. The same author has described an extensor brevis anomalus of the middle finger. This form I have found, and so has Mr. Wood. I have seen it, too, to the second and third fingers; to the ulnar side of the middle and ring finger; one to the middle, and two slips to the fifth finger; to the second, third, and fourth digits, from the corresponding metacarpal bones. Professor Humphry

("Journal of Anatomy and Physiology," vol. ii., p. 308) saw it arising from the lower margin of the radius, on the ulnar side of the groove for the extensor of the second internode of the thumb: this ended in a tendon to the ulnar side of the extensor tendon of the third digit. Otto ("Neue seltene Beobachtungen," Heft. i., p. 91) describes having found it arising from the base of the third metacarpal, and going to the index finger; or from the annular ligament. A radial origin is described by Albinus (loc. cit.). Mr. Kelly has contributed an instance to me, in which he found a small digastric extensor from the os magnum to the ring finger. Other cases of an extensor brevis digitorum have been published by Petsche ("Haller's Disp. Anat. Select.," vol. vi., p. 771). Dursy, Henle (p. 216), Sandifort ("Exercit. Acad.," p. 93, and "Observ. Anat. Path.," lib. iv., p. 39), Sömmerring (p. 254).

Extensor indicis I have found—1. Wanting; the same was found twice by Cheselden (p. 94), and Moser (Meckel's "Archiv," vii., p. 225); 2. With a double tendon, not uncommon; this has been seen by Mr. Wood and myself; 3. With a double muscle and tendon; 4. With a slip from the extensor communis digitorum (Wood); 5. With an origin from the radius, carpus, and interosseous ligament, and none from the ulna (Wood); 6. I found a double tendon coexisting with an accessory slip from the common extensor; 7. It has been found inserted by fascia into the back of the hand; 8. Or into the posterior annular ligament (Moser, loc. cit., p. 226); 9. A digastric indicator, whose second belly was on the back of the hand (Rosenmüller, "De Var. Musc.," p. 6); 10. It may have some union with the extensor medii digiti, or be inseparable from it; Meckel found it with tendons to the second, third, and fourth fingers; 11. Gantzer once saw it double, and one of these muscles arose from the radius.

The extensor medii digiti is often a differentiated portion of the extensor indicis; arising below that muscle from the back of the ulna, it varies indeed only in its greater or lesser amount of differentiation from its neighbour. Mr. Wood has found it eleven times in a hundred and two subjects; and the same author has seen it when separate from the extensor indicis, sending an additional tendon to the index finger; in one case this coalesced with the indicator before its insertion. He has also found it arising from the intermuscular septum between the extensor communis digitorum and the supinator

brevis, above the other deep muscles. It has also been seen sending a slip to be inserted partly into the indicator tendon, and partly into the middle metacarpal fascia ; cases of this muscle are described by Petsche (loc. cit., p. 771). Meckel has seen it arising from the radius (Meckel, Doane's trans., p. 130, note); and it has been found springing from the carpus (Brugnone, p. 168). Most commonly, however, it is only a second tendon connected to the indicator, and showing no sign whatever of being connected with a separate belly; sometimes, indeed, this tendon is an offshoot from the indicator tendon.

Extensor secundi internodii pollicis was found—1. Receiving a slip of the extensor communis digitorum which runs in the same groove in the radius (Wood) ; a similar case I described some years ago (Medical Press, 1866, p. 99). Blandin likewise mentions a similar instance; 2. I have found it double, both tendons normally inserted ; 3. Giving a slip to the base of the first phalanx in a case in which the extensor primi internodii was absent ; 4. A slip from the extensor secundi to the first phalanx co-existing with a slip from the extensor primi internodii to the second phalanx. Böhmer describes a case (loc. cit., p. 8), in which this muscle had four tendons, two to the ligamentum armillare and two to ungual phalanx. Moser found it very strong in a case in which the extensor primi internodii was suppressed (Meckel's "Archiv," vol. vii., p. 125).

Extensor pollicis et indicis is a rather remarkable muscle occasionally found, and one whose homologue is by no means uncommonly met with among some groups of the lower animals, the dog, fox, wolf, jackal, panther, and dingo all possessing it ; it has been described by Wood, Clason (Upsala Läkareforenings Förhandlingar, Bd. ii., p. 417), and myself; its commonest arrangement is an origin from the ulna between the extensor indicis and the extensor secundi internodii pollicis (with which it usually co-exists), it usually is also connected to the interosseous ligament and to the intermuscular septa ; its tendon passing downwards divides into two, one to the thumb, and the other to the index finger; sometimes the former runs into the indicial tendon of the extensor communis, while the latter passes to the first phalanx of the thumb, or one was inserted into the first phalanx of the index finger (Wood), or one has been seen joining the extensor primi internodii tendon, or substituted for it, or inserted into the second and third phalanges of the index finger, or into the common extensor aponeurosis of the index, or with the secundi internodii

pollicis. It has also been found arising from the intermuscular septum between the extensor digitorum communis and the extensor ossis metacarpi pollicis, forming a layer intermediate between them, and at first sight apparently connected with the common extensor; this case was inserted, with the tendon of the extensor of the last phalanx of the thumb; and with the outer of two tendons of a double indicator; this seems a further development of Blandin's muscle mentioned above. In Clason's case it co-existed with a special extensor annularis longus (from the extensor minimi digiti).

Extensor ossis metacarpi pollicis is a muscle very variable—1. It very often has a double tendon both inserted together; 2. Or one inserted into the trapezium connected to the tendon of the short abductor pollicis. (Berres, Anthropotomie, Wien, 1er Bd. 1835, p. 552; Wood, 1864). This is not uncommon, and occurs about once in every twelve subjects; 3. A triple tendon—two into the metacarpal and one into the trapezium; 4. A triple tendon, one into the metacarpal, one into the trapezium, and one into the opponens pollicis; 5. Or two into the metacarpal and one into the opponens pollicis; all these varieties I have met with; 6. Cloquet saw it sending a slip into the origin of the abductor pollicis brevis. This has been found by Mr. Wood seven times in thirty-six (1867); 7. A doubled muscle, one-half of which went exclusively to the abductor pollicis; 8. A triple tendon like No. 4, which sent a slip to the abductor pollicis; 9. An example with four tendons, three to the metacarpal bone, and one to the abductor indicis; these three I have also found; 10. It has been found fused with the extensor primi internodii; 11. A double muscle with normal insertion; 12. A double muscle, the inner segment of which was normal, the outer had four tendons, three of which go into the first metacarpal bone, and one gave origin to part of the opponens pollicis; 13. I have found it fused with extensor primi, and giving off two tendons; 14. Mr. Carver found it in an idiot, with three slips from a single tendon, one to the last phalanx of the thumb, one to the second phalanx, and one to the scaphoid bone ("Journal of Anat. and Phys.," vol. iii., p. 260); 15. Fleischmann describes it as digastric, but the case seems to be rather one of the examples of the extensor carpi radialis accessorius; a digastric example is also described by Sömmerring (p. 250); 16. I have seen it double; one muscle with four tendons, one to the trapezius, one to the external part of the abductor brevis pollicis, one to the opponens, and a fourth to the metacarpal;

the second went entirely into the metacarpal bone; this co-existed with a slip from the extensor secundi to the first phalanx; 17. I have seen a slip inserted into the annular ligament.

Extensor primi internodii pollicis has been found absent (once in thirty-six, Wood, 1867; three times in thirty-six, *ibid.* 1868); Sömmerring, Moser, and I have also seen this not unfrequently; 2. Theile and Henle have seen it fused with the extensor ossis metacarpi; the same has been found by Mr. Wood and myself; 3. The tendon I have found going to the ungual phalanx; 4. The tendon has been described as split (Böhmer, loc. cit., viii.); I have seen the whole muscle cleft, each part normally attached; 5. Mr. Wood found it replaced by a tendon from the styloid process of the radius to its normal site of insertion; 6. A slip to the tendon of the extensor secundi at the base of the ungual phalanx (Sömmerring); 7. The tendon entirely going to the ungual phalanx; 8. A slip has been seen going to the metacarpal bone at head; 9. Or it hasbeen noticed receiving a slip from the abductor which passed on to the second phalanx.

The interossei have been the subjects of several anomalies—1. The first dorsal has been cleft into two, one part going to the pollex and one to the index; this is a very rare anomaly, not occurring oftener than once in 120 subjects; 2. This muscle receives occasionally a slip from the extensor carpi radialis longior; 3. The third dorsal has received a slip from the extensor carpi radialis brevior; 4. I saw all the dorsals cleft once; 5. The dorsals of the second or third interspaces sending a slip or slips to the extensor brevis digitorum manus; 6. A first palmar interosseus, or interosseus primus volaris of Henle, is by no means uncommon; Mr. Wood has found it three times out of thirty-six (1867); I have found it in about the same proportion; it sometimes receives a slip from the extensor carpi radialis longior, or accessorius; 7. I have seen a double set of palmar interossei twice; 8. Two in the second intermetacarpal space; 9. A double third palmar; 10. The second dorsal attached to the ulnar side of the index, and the first palmar to the radial side of the middle finger; this is the arrangement ordinarily met with in the foot, and an instance of its being found as a hand arrangement is described by Meckel; 11. Two palmars in the fourth metacarpal, one to the little, and the other to the ring finger; 12. Two in the third interspace for the ring and middle fingers; 13. Two in the second for the index and

middle; 14. The indicial part of the first dorsal I have seen absent; 15. The second dorsal inserted into the ulnar side of the index; 16. And the first palmar to the radial side of the middle finger.

Tensor vaginæ femoris has been found—1. With some fibres arising from Poupart's ligament; 2. With an origin two inches in length; 3. With an origin, shorter than usual, falling short of the anterior superior spine; 4. I have seen its origin, thicker than usual, springing from a small triangular surface of the anterior dorsal border of the crest as in the racoon; 5. An additional slip from the abdominal aponeurosis, above the iliac crest; 6. I have seen it split into separate fasciculi; 7. I have likewise seen a firm tendinous band, continued from it, inserted into the posterior and inferior part of the root of the great trochanter; 8. Or cleft into two at its insertion.

Sartorius has been found—1. Absent (Meckel, p. 452); 2. Doubled (Otto; Rosenmüller, p. 7; Gantzer, p. 14; Sömmerring); in these cases, the two muscles resemble each other; 3. Doubled, the lower of the two representatives being inserted into the femur (Meckel); 4. Or into the tendon of the normal one ("Huber, Acta Helvetica," vol. x., p. 114); 4. Splitting of the lower border into two parts (Horner, p. 443); 6. It has been found much broader than usual in a negro by Horner; 7. Crossing the thigh more transversely than usual (Quain); 8. Ending in the fascia lata; 9. With two heads, one from the anterior superior spine, and another, perfectly separate, below it, from the incisura semilunaris; 10. With a central tendinous intersection (Kelch, p. 42; Hyrtl); 11. With its tendon inserted into the inner side of the capsule of the knee, and not extending any farther; 12. With some of its inner fibres arising from the splitting of the iliac portion of the fascia lata, which forms its sheath, as it forms the outer wall of the triangle of Scarpa.

Rectus femoris I have seen with—1. An origin from the anterior superior spinous process of the ilium joining the straight head; 2. A split curved head, part being outside, and part between the layers of the hip-joint capsule; 3. A double origin from the inferior spine; 4. A continuity of the acetabular and spinous heads; 5. With no acetabular origin; 6. Isenflamm describes a bursa between the lateral tendon and the brim of the acetabulum ("Anat. Untersuchungen," p. 83); 7. Diemerbröck describes a double long head, which I

have also found; 8. A large fasciculus of its tendon, passing across the front of the patella, was separated from the bone by the deepest part of the prepatellar bursa; 9. The relation of this muscle to the rest of the extensor is variable; sometimes at its insertion it is inseparable from the cruræus, at other times I have seen the two vasti uniting over the tendon of this muscle, thus forming a canal in which the rectus tendon ran to its insertion; or the vastus externus may be inserted into the outer border of its tendon, while the vastus internus and cruræus lie beneath.

Cruræus may arise as high as the anterior intertrochanteric line, or not within an inch of that line, and it may descend as far as half an inch to an inch above the knee ("Sandifort, Thesaurus Dissertationum," Roterodami, 1769, p. 249, § vi.), or more commonly it does not extend below a point three or four inches above the knee. This muscle is usually inseparable from the vastus internus, so much so that Nuhn and Cruveilhier regard them both as one. These authors call the extensor mass the triceps extensor, regarding the rectus and the vastus externus as the other two heads, and the vastus internus and cruræus as forming a third head. Weber uses the term triceps extensor for the extensor mass, minus the rectus, which he regards as a separate muscle; and Hyrtl calls the extensors quadriceps. Cruræus is not infrequently united to the sub-cruræus, or, when no sub-cruræus is present, deep fibres are, but rarely, attached to the synovial membrane.

Vastus internus varies only by being—1. Bilaminar; this I have five times seen; 2. Or by being united with the last; the first variety is interesting as being a common type of arrangement in birds; 3. It may have its lower fasciculi more or less separate at their insertion, passing into the inner condyle of the tibia by a separate tendon; 4. The vacant space between the upper part of the origin of this muscle and the cruræus is liable to considerable variation in size.

Vastus externus I have also seen bilaminar, and united to the vastus internus at its insertion, or to the cruræus, or to the rectus, or to the outer slip of the sub-cruræus, when that muscle consists of two lateral slips; but this is very rare, even in this condition of the latter muscle. Doubling of this muscle, as of the vastus internus, is the normal condition in many birds.

Sub-cruræus is only liable to atrophy, absence, splitting, or fusion, with the cruræus ; it was first described by Dupré, in his work, "Sur les Sources de la Synovie," published in 1699 ; also by Albinus ("Annot. Acad.," lib. iv., cap. 5, p. 27), and by Huber ("Acta Helvetica," vol. iii., p. 250), and has recently been made the subject of a paper by Kulaewsky of Kasan ("Reichert u. Du Bois Reymond's Archiv," 1869, p. 410). It may be inserted at either side of the knee, as described by South, or into the upper part of the synovial membrane (Platonoff) ; its lateral slips, when split, are usually equal.

Iliacus I have found cleft into two, or with a partly separate superficial lamina arising from the fascia iliaca, or with a slip from the psoas overleaping the anterior crural nerve, and continuous into it. A separate slip occasionally springs from the ilio-pectineal line, and sometimes another from the inner margin of the incisura semilunaris of the ilium. A few fibres from the ilio-lumbar part of the quadratus lumborum are continuous occasionally into its origin and its most posterior fibres, which frequently are attached to the ilio-sacral ligaments, may extend to the promontory of the sacrum itself.

Iliacus minor (Winslow, p. 205), or ilio-capsularis (Harrison), is a small detached portion of the iliacus, arising from the anterior inferior spine of ilium and inserted into the anterior intertrochanteric line. It is usually separated from the iliacus by arising either directly from the margin, or from the outer surface of it. Its insertion is always above the main iliacus tendon.

Psoas parvus has been found double, the second part lying internal. This is described by Winslow ("Exposition de la Structure de la Corps Humain," Paris, 1766, tom. ii., p. 146), Kelch, Cruveilhier, and myself. Theile saw an attempt at subdivision in the splitting of the tendon, one part of which went to the fifth lumbar and first sacral vertebræ, and the other to the ilio-pectineal line. This variety I have once noticed. Blandin saw this muscle replaced by a tendon. I have seen its insertion lost in the pelvic fascia, or directly into the iliac fascia, through which it was attached to the crural arch. This muscle is very often absent, being one of the five that are, of all muscles, the most frequently deficient (pyramidalis, psoas parvus, peoneus tertius, palmaris, and plantaris). When absent, I have seen an expansion from the inner edge of the psoas magnus taking its place. John Bell ("The Bones, Muscles, &c.," Edinburgh, 1793, p. 341) says it is more frequently absent in males than

females. Moser saw it present on the left and absent on the right. I have not been able to find any rule for its presence or absence, but do not think that sex has anything to say to it. It is a muscle specially developed in leaping animals, as the kangaroo, jerboa, Macroscelides, &c.

Psoas magnus—The two or three lower vertebral origins, or the upper oblique-process-origin of the medial head, may give a slip separate from the rest of the muscle (Albinus, Meckel). This may receive a slip from the psoas parvus tendon. A bundle of the lateral head may leap over one or more roots of the crural nerve. A second or lesser psoas, lying between psoas magnus and the iliacus, and going to the lesser trochanter, is described by Meckel (p. 148). Mr. Horner, of Philadelphia, has seen several slips for the psoas magnus, and the same author has seen this muscle perfectly separated from the iliacus (p. 400), an anomaly of very rare occurrence, as they are nearly always inserted at least by a common tendon. I have, however, seen similar instances, and so has Lieutaud (quoted by Sömmerring, p. 294). I have traced a band of fibres of the psoas, springing from the ligamentum arcuatum internum, and one continuous with some of the posterior fleshy fibres of the diaphragm, or with fibres continuous with those of the right crus. It may have an origin from the last lumbar vertebræ, but most commonly is either not at all, or but slightly connected to that bone.

Pectineus I have found—1. Receiving fibres from the adductor longus; 2. Sending a slip to the obturator externus; 3. Attached to the femur as far up as the lesser trochanter; 4. Cleft into two parts (also seen by Sömmerring); 5. Harrison describes having found it attached to the capsular ligament of the hip joint; 6. A bilaminar pectineus was seen by Winslow ("Esp. Anat.," vol. i., p. 117), the deeper being smaller; 7. Sandifort found it inserted by a tendon into the lesser trochanter; 8. Receiving a small slip from the iliacus; 9. With the insertion into the capsular ligament of the hip joint.

Adductor brevis I have found—1. Inseparable from the adductor magnus; 2. Split into two, or into three parts; 3. Inserted by a tendinous sling.

Adductor longus I found—1. Bilaminar; 2. Split into two; 3. Receiving a slip from the pectineus over the profunda artery; this variety has been seen by

M. Wood; 4. Extending down as far as the knee; 5. With its lower tendon of insertion perfectly inseparable from the insertion of adductor magnus; 6. Sending a small muscular fascicle over the femoral artery in the anterior wall of the Hunterian canal.

Adductor magnus varies in its degrees of segmentation, its condyloid part I have found quite distinct, and a similar slip has been met with by Weber, Hildebrandt ("Handbuch," p. 480). A separate slip into the lower pillar of the tendinous arch of insertion is described by Henle; I have found it inseparable from the quadratus femoris, so that the terminal branch of the internal circumflex artery appeared to pierce through the upper part of this muscle. The following muscle is often regarded as a part of this, with which indeed it is not unfrequently united.

Adductor minimus (Henle, Günther, p. 159), is a separate portion usually described as the upper and outer part of the adductor magnus, but as it is almost invariably separate it is entitled to be regarded as a special muscle; it was recognised by Diemerbröck as the adductor quartus (lib. v. p. 462), and a corresponding separate part I have been able to recognise in nearly every subject; most commonly there is some connexion between this and the foregoing muscle at their origin, but they are frequently separated by a branch of the superior perforating branch of the profunda artery.

Gracilis is liable to few variations; I have seen it vary considerably in width or in the extent of its origin. It has been met with split at its insertion, or with some of its muscular fibres lost in the fascia lata two-thirds down the thigh.

Gluteus maximus has been seen—1. Bilaminar; this was found to co-exist with doubled pectorals and trapezius by Tiedemann in an athlete ("Journal Complementaire du Dictionnaire des Sciences Médicales," vol. vi., p. 272. Paris 1820); I have found it partly bilaminar especially along its lower border, and Otto also notices it; 2. With a separate agitator caudæ attached to the coccyx and lowest part of the sacrum; 3. Some of the component coarse fasciculi of this muscle I have observed to present tendinous intersections; 4. The number and extent of these fasciculi are liable to great variety; 5. I have seen it with a high fascial slip of origin from the lumbar aponeurosis; 6. I have seen it attached only to the last two sacral vertebræ; 7. I have seen its insertion rising

as high as the upper part of the great trochanter; 8. The site of its insertion I have seen in muscular males prominent and ridged; 9. In one very remarkable instance which was shown to me in the Dissecting Room of Steevens' Hospital by Drs. Macdonnell and Bookey, an osseous "rib," or arch, extended in the lower border of this muscle, from the upper third of the linea aspera to near the tuber ischii; this seemed an ossification of the fibrous tissue or gluteal fascia which tucks under the margin of the gluteus in this locality; 10. Luschka describes a special slip of the gluteus as having been found by him inserted into the external lip of the linea aspera ("Anat., p. 139); 11. I have found it with an origin from the tuberosity of the ischium, and greater sciatic ligament.

Gluteus medius varies mainly—1. By being united anteriorly to the gluteus minimus; 2. Or posteriorly to the pyriformis; a slip of tendon may pass from this to the pyriformis tendon, or the two tendons may be separated by a bursa (the bursa of Loder), but this latter condition is only found in one in every fifteen subjects. A separate muscular slip runs, but very rarely, beneath, and parallel to the ligamentum suspensorium trochanteris ("Günther," p. 143); a separate bundle of fibres may be found from the front, or more rarely from the posterior border of the muscle going to the upper part of the trochanter. It has been found bilaminar by Henle (p. 246).

Gluteus minimus is not uncommonly inseparable from the medius along its anterior border; 2. It may have detached from it a slip to the hip-joint-capsule; 3. I have seen it split into an anterior and a posterior part, and the latter may be (α) free, (β) joined to the pyriformis, or (γ) to the gemellus superior; 4. This splitting I have seen to be traversed by the superior glutæal nerve; 5. The anterior fibres occasionally are separate, forming a marginal fourth gluteus or scansorius; 6. I have seen this muscle sending a slip into the outer part of the origin of the vastus externus, and when the anterior, or fourth gluteus has existed, I have seen its tendon either thus continuous with the vastus, or overlapped by it, the vastus forming a tendinous arch over the muscle.

Pyriformis—1. I found once deficient. Budge also describes its absence in a deformed limb (Henle u. Pfeufer's "Zeitschrift," vol. x., p. 128); 2. In one instance of its absence Otto found the superior gemellus unusually large, which was not the case in my instance; 3. Fused with the gluteus medius inseparately,

except at the point of the exit of superior gluteal vessels and nerves ; 4. This nerve has been found by Winslow to perforate the pyriformis ("Exposition Anat.," vol. ii., p. 125) ; 5. It may have but one or two sacral origins (Sömmerring) ; 6. I have seen its insertion inseparately tied to that of the gluteus minimus ; 7. When its sacral origins are small, as in some cases of narrow, straight sacra, then its main origin is from the margin of the great sciatic notch. I have seen it with a distinct slip from the upper border of the notch ; 8. It is not uncommonly cleft by the great sciatic nerve ; 9. Or, by its peroneal portion ; in these cases I have seen both the muscle and tendon split, or only the muscle ; 10. I have seen its tendon joined to the obturator internus tendon, the superior gemellus being attached at the point of union of the two, or absent ; 11. I have seen the tendon inserted into the hip-capsule ; 12. The lowest fibres of this muscle are sometimes united to the upper border of the coccygeus at their origins, when the latter muscle is large, the sciatic nerves and vessels separating them in the first part of their course ; 13. The irregularity in development of the bursa described by Loder and Cloquet I have before noticed ; 14. The gemellus superior has been inserted into this tendon instead of that of the obturator ; 15. A special slip from the gluteus medius may supplement the muscle, or run into its tendon ; 16. It may have an origin from the first piece of the sacrum (Sömmerring) ; 17. Bahnsen found it fused with a lamina of the gluteus medius (Henle u. Pfeufer's "Zeitschrift," vol. xxxiii., p. 49).

Gemellus superior—I. May be absent ; this has been seen and described by many anatomists, and is by no means rare ; 2. I have frequently seen it inseparable from the obturator internus ; 3. It may be joined to the pyriformis, and not to the obturator ; 4. Or to the united tendons of pyriformis and obturator internus ; 5. Or even to the lower border of gluteus minimus ; 6. It has been seen double ; 7. Or inserted into the capsular ligament.

Gemellus inferior—1. May be absent, but not nearly so often as the upper muscle—this anomaly was noticed by Otto, Gantzer, Meckel, and myself ; Meckel noticed absence of both in one subject ; 2. Double—this is more frequent than the corresponding variety in the superior ; 3. The inferior was found very large in a case in which the quadratus femoris was absent (Meckel, French trans., tom. ii., p. 195) ; 4. Inseparably connected to (the ob-

t urator internus for its whole extent; 5. United by connecting fibres to the quadratus femoris.

Obturator internus often exhibits detached slips within the pelvis, separate in origin, and joined on to the rest of the muscle immediately before its emergence through the lesser sacro-sciatic foramen—these may arise—1. From the lower border of the ilio-pectineal line; 2. Or from the tendon of the psoas parvus; 3. Or from the ridge above the inner border of the tuber ischii; 4. Or from the pelvic fascia, or its obturator lamina; 5. Or from the sacro-sciatic ligament; 6. The upper slip I have found separated from the rest by the obturator nerve; 7. A separate retro-ischiatic slip not unfrequently arises from the inner side of the ischium above the spine; 8. Schwegl found a slip arising from the third sacral vertebra, and continuous with this muscle.

Quadratus femoris may be—1. Absent (Albinus, p. 530); Sömmerring, tom. iii., p. 286—a case of its absence occurred during the present Session in the dissecting-room of the Royal College of Surgeons; Cruveilhier, p. 223; (Schwegl); 2. Absent and replaced by a hypertrophied gemellus inferior (Meckel); 3. Closely joined to the gemellus inferior; 4. Or to the adductor magnus; 5. Sending a slip directly into the origin of the semi-membranosus (Kelly); 6. Attached to the capsular ligament; 7. Split into two; 8. An extraordinary case of segmentation is recorded by Jancke ("De Capsulis, Tendinibus et Articulationibus," Lipsiæ, 1753, p. 16), in which the muscle was divided into thirty fasciculi.

Obturator externus may—1. Receive a large slip at its tendon from the adductor brevis (Wood); 2. I have seen it split, one portion arising from the horizontal ramus of the pubis below the pectineus, separated from the rest by the obturator nerve; 3. I have seen the tendon attached to the capsule of the hip joint.

Biceps flexor cruris—1. May have its short head absent (Otto, Meckel, Budge, Henle u. Pfeufer, vol. x., p. 128); 2. With a second long head from the tuber ischii (Sömmerring, p. 276); 3. With a third head from the upper part of the linea aspera (Meckel's "Archiv," Bd. v., p. 117); 4. Or from the inner condylar ridge of the femur (Grüber, Müller's "Archiv.," 1848, p. 430); 5. Or from the fascia under the gluteus maximus, as a rounded fusiform slip, which joins the long head (Wood); 6. From the fascia lata,

near the upper end of the linea aspera (Henle); 7. A slip continued over the great sciatic ligament to the side of the sacrum I saw and described some years ago; 8. The short head has received an accessory fascicle from the vastus externus; 9. Or from the adductor magnus; 10. I have seen the short head split into several fasciculi; 11. A third head arising in common with a middle head of the gastrocnemius, something like Professor Grüber's case (No. 4); in this case it lay behind the vessels and nerves of the popliteal space; of all these supernumerary heads that from the tuber ischii is the commonest—cases of it have been described by Sömmerring (as above), by Gantzer (op. cit., p. 15), and Blandin (op. cit., 524); 12. The two heads have been seen separate until near their insertion; 13. The insertion may be by two tendons separated by the external lateral ligament; 14. One slip is not very rarely attached to the external tuberosity of the tibia; 15. One slip into the fascia of the leg (Kelly); 16. A portion into the posterior aspect of the external condyle of the tibia (ibid.); 17. A slip from this muscle, or from its long head to the tendo Achillis, has been described twice—the first form by Otto ("Path. Anat. South's Trans., p. 244")—the second by Kelch ("Beiträge," p. 42). This reminds us of the insertion of the rectus in many of the lower animals, as in the lion.

Semi-tendinosus has been seen separate from the biceps at its origin—this I have never met with; 2. Cloquet describes a bursa between this muscle and the semi-membranosus at origin; but I think it must be one rarely present, for, though looked for specially, I have not seen it; 3. The tendinous inscription was seen doubled by Sömmerring; in its ordinary condition it is very constant; I never saw it absent, though sometimes it has appeared as if interrupted.

Semi-membranosus has been found—1. Absent (Loschge, "Abhandlungen der phys. med. Soc., Erlangen," Bd. i., p. 25; De Souza, "Gazette Medicale," de Paris, 1855, No. 12, p. 184); in one instance a tendon existed in its place, with a small central muscle; 3. Mr. Kelly communicated to me a description of an interesting case, in which this muscle had a slip continuous with the quadratus femoris; 4. This muscle has been seen fleshy much higher than usual; 5. A slip of insertion may be continuous with the retinaculum ligamenti arcuati; 6. A separate portion, lost in the popliteal fat, is described by

Sandifort ("Thesaurus Dissert.," Roterodi, 1769, p. 250); 7. Having its origin from the great sacro-sciatic ligament for the most part; 8. Doubling of the semi-membranosus is described by Calori (loc. cit., p. 144); 9. Its insertion was found by Retzius fourfold, one into the ligamentum popliteum obliquum, second into the margo infraglenoidalis of the tibia, third into the popliteal fascia, fourth its normal insertion.

Gastrocnemius—1. May have a sesamoid bone in its inner head; I have found this; 2. Or in its outer—this is more common; 3. Or the two heads are separable down farther than usual; 4. Or their tendon is perfectly separate from that of the soleus for a few inches; 5. The outer head may have a few lower fibres attached to the external lateral ligament; 2. Or to the posterior ligament of Winslow; 7. A third head may be found from the middle of the popliteal triangle of the femur, and with a second slip from the posterior ligament of Winslow (Quain, pl. 50, figs. 4 and 5); 8. Or it may spring from the line leading from the inner condyle to the linea aspera, an inch above the inner head—this ended in a short tendon, which crossed the popliteal nerve, and was inserted into the convergence of the two heads; 9. Or it may spring from a tendinous arch from the same ridge, higher up, which extends over the femoral vessels; 10. Or from the inner border of the long head of the biceps, forming a bicipiti accessorius (Haughton), similar to that of the lion, and many mammals (Kelch, p. 42); 11. Or from the fascia of the leg (Meckel); 12. An intermediately arising third head, like No. 7, may be inserted with the outer head; 13. Hyrtl describes an instance of this third head separate for a considerable extent ("Oesterreiche Zeitschrift," vol. viii., 1862, No. 22); 14. The entire muscle has been found bilaminar; 15. The external head with a purely tendinous origin.

Soleus—1. May have a bilaminar fibular head; 2. A second separate tibial portion inside the tendo Achillis inserted into the os calcis by a special tendon (Quain, p. 540); 3. A double soleus (Cruveilhier); 4, A special tendon from soleus to the heel (Hellema, "Eenige Ontleedkundige Afwig-Waargenom in de Seechezvan de Marin Hospital in Willemswoord, Geneesk. Tijdsch, v. de Zeemagt," 5th Jahrgang, 1st Afd.)

Plantaris—1. Not seldom absent (Cheselden, p. 116); absence is described by many authors; but there is considerable difference of opinion as to

its relative frequency. Meckel says it is less constant than palmaris, and in 1861–6 I found it so; Gantzer and Wood say not so; in 1867 I stated that my experience during that Session was, that its absence was to that of palmaris in the proportion of one to three; 2. Blended with the external head of the gastrocnemius (Wood and myself); 3. With a double muscular belly (Wood); 4. With a slip nearly its own size from the inner side of its origin inserted into the ligamentum posticum of Winslow, close to the insertion of the semi-membranosus; 5. Having its tendon enclosed in a sheath inside that of the soleus; 6. Ending in a widely expanded aponeurosis, half way down the leg, between the gastrocnemius and the soleus; 7. Arising from the fascia over the popliteus muscle (Hyrtl); 8. Or from fibula between the peroneus longus and the flexor hallucis longus; 9. Receives a second head from the politeal plane (Hall, "Literaturzeitung," 1808, Bd. ii., p. 211); 10. Or from the knee capsule, near the inter-condyloid notch; 11. Attached by fibres to the inner margin of the tibia, and inserted midway between the inner malleolus and the os calcis (Sandifort., "Thesaurus Dissert., Roterdami, 1769, p. 267); 12. Lost partly in fat, and partly in the tendo Achillis; 13. With a loose coiled tendon (Harrison); 14. Origin sometimes lower down (Winslow, p. 120); 15. Inserted into the plantar fascia (Blandin, p. 537); 16. Or into the internal annular ligament (Brown, "Myographia Nova," London, 1684, Tab. xxxiii., Quain, p. 540).

Popliteus muscle has been described as double ("Fabricius ab Aquapendente, De Motu Locali Animalium; in Opera Anat. et Physiol.," Lipsiæ, 1687, p. 359). The same has been seen by Professor Bevan, of the Royal College of Surgeons, Ireland. Its absence has been found by Ringhoffer in a deformed foot (Virchow's "Archiv," vol. xix., p. 28). A small superior popliteus, arising from the outer condyle, and above the popliteus muscle, to which it was united, and inserted into the upper part of the tibia, was seen by Calori (loc. cit.). A sesamoid bone has been found in its tendon. Slips from the semi-membranosus have been found inserted into the fascia over this muscle.

Tibialis posticus may have—1. A trifurcated tendon, one slip going to the normal site of insertion, and one to the peroneus longus tendon, and a third to the heads of the third and fourth metatarsal bones; 2. The slip which frequently goes to the ento-cuneiform may give attachment to the flexor hallucis

by running into it; 3. A slip to the second and third metatarsal (Harrison); 4. A slip to the cuboid (Winslow); 5. Its absence is described by Budge (Henle u. Pfeufer's "Zeitschrift," vol. x., p. 128).

Tibialis secundus is a muscle described by Bahnsen, which passed from the back of the tibia to the ligamentum cruciatum; this muscle is similar to the tensor capsuli tibio-tarsalis of Henle to be afterwards referred to (Henle u. Pfeufer's "Zeitschrift," vol. xxxiii., p. 52).

The unions of the flexor tendons in the foot have been carefully studied by several anatomists. Professor Turner, of Edinburgh, has given an analysis of fifty specimens examined by him, and has found—1. That no two were alike; 2. That in all, the flexor hallucis gave off a slip to the flexor longus digitorum, which was flat, membranous, or rounded. In eleven this slip went to the second toe alone, forming about one half its tendon in four, as described by Arnold, Henle, and Theile. It forms more than half in six, and forms nearly the whole, as described by Diemerbröck and Meckel, in one. In twenty it bifurcated, and ended in the long tendons of the second and third toes (Church, Rolleston, Huxley). Of these the second toe slip was larger than the third in twelve; in one of which it formed, with the accessorium, the whole of the second toe tendon. In six it was equally distributed to the second and third toes; and in two formed nearly the entire tendons of both toes. In eighteen it went to the second, third, and fourth toes—a larger share going to the second in seven; to the second and third in five; to the second and fourth in one; and equally to all three in five. In one case it went to all four outer toes; and in this case the slips for the second and third were larger than those for the fourth and fifth. In all cases the slip from the flexor hallucis never joined the tendon of the long flexor before the division of the latter into its digital slips; in nine specimens a band passed from the tendon of the long flexor before its division to join the flexor hallucis beyond the origin of the connecting link for the common flexor ("Trans. R. Soc. Ed.," vol. xxiv.) Professor F. Eilhart Schultze, of Rostock, has given the analysis of one hundred tendon arrangements in Siebold u. Kölliker's "Zeitschrift f. wiss. Zool.," vol. xvii., p. 1); in thirty-two of these the slip from the flexor hallucis went to the second toe flexor alone; in fifty-eight to the two toes, second and third; in ten to the second, third, and fourth; in twenty-nine cases the slip from flexor longus to the hallucis was present. (Wood found this six times in thirty-six cases, 1838.)

Mr. Wood found in one case the first tendon of the flexor longus digitorum absent, and its place taken by a slip from the flexor hallucis. Perfect separateness of the tendons is also rare, but has been seen by Wood and myself. The flexor digitorum sent slips to the hallucis, and the latter gave off tendons to the second and third toes ; that to the second was the only perforating tendon for it, as there was none from the common flexor. A slip from the flexor communis joined the third toe tendon. In a paper on the flexor digitorum, and on the unions of the flexor tendons in Reichert and Dubois Reymond's "Archiv," 1868, p. 236, Gies describes in one instance, a tendon from the flexor digitorum joining that of the hallucis opposite the metatarso-phalangeal joint, the flexor digitorum giving off before it joined the accessorius, a slip from which arose two bellies, each of which ends in a tendon which joined the perforating tendon of the fourth and fifth toes respectively.

Of other modes of union, the following have been described or seen. I saw the great toe supplied by the hallucis, the second by a tendon composed of : 1. A slip from the extensor hallucis ; 2. One tendinous from the accessorius ; 3 and 4. Two thin threads from the communis; the third toe supplied by a slip from the communis, and one from the accessorius ; the fourth and fifth by the long toe flexor alone. In one subject, Brugnone found the flexor longus with a fifth tendon, which acted as the perforated flexor of the little toe. Messrs. Flower and Murie saw in a bushwoman the following arrangement. The flexor digitorum supplied the fifth, and partly the fourth, third, and second, the flexor hallucis assisting it in supplying these, and exclusively acting on the hallux.

The flexor tendon for the little toe is often complex, being made up of : 1. A muscular band from below the sustentaculum tali; 2. A slip from the cuboid ; and 3. A slip from the long flexor tendon. I have seen the perforating tendon for this toe from the accessorius alone.

The flexor digitorum longus itself has been found with a second long head from the fibula (Gies). This has been seen united with the ordinary head, and either forming a complete or partial substitute for the caro quadrata, or else it may unite with that portion (Henle, Luschka). An accessory long slip from the tibialis anticus was seen by Gies, joining the rest of the muscle at the ankle. A second flexor longus, either parallel and attached to the ordinary muscle,

or to the accessorius, is described by Otto ("Neue seltene Beobachtungen," p. 40), Meckel, ("Archiv," Bd. iv., p. 480), Hall ("Literaturzeitung," 1808, Band ii., p. 204), Reinhardt (Müller's "Archiv," 1846, p. 298. Union has been described between the muscular parts of the flexor digitorum, and flexor hallucis. A second or superficial head has been seen going from the fascia of the leg to the long flexor. Another from the accessorius ad flexorem arose from the outer condyle of the tibia, ended in a long tendon like that of plantaris, and was inserted into the flexor behind the os calcis.

A flexor digiti secundi proprius was found by Bahnsen as a semipenniform muscle from the back of the tibia (Henle u. Pfeufer's "Zeitschrift," vol. xxxiii., p. 32.) An accessorial head of the flexor digitorum longus over the posterior tibial artery has been described by Glaser ("Berliner klin. Wochenschrift," 1866, p. 29).

The flexor hallucis itself, besides the tendon-unions, is liable to very few anomalies. Bergmann found a single tendon from a bicipital muscle which arose from the outer layer of the tibial fascia going to the flexor hallucis ("Handschriftliche Notiz," quoted by Henle.)

Peroneus longus.—1. Inserted by three tendons in a case of talipes into the fifth, third, and first metatarsal bone at their tarsal end; 2. Walther describes three slips of insertion, two into the metatarsal, and one into the ento-cuneiform bone ("De Artic. Lig. et Musc. Hominis," Lipsiæ, 1728, pl. ii.); 3. Fused with the brevis; 4. A slip to the fifth metatarsal behind the brevis; 5. A true sesamoid bone in its tendon; 6. A slip rising to the external lateral ligament; 7. Tendon sending forward a slip to the head of the first metatarsal (Portal, p. 356); 8. A slip to the external malleolus, was found by Budge ("Henle u. Pfeufer's Zeitschrift," vol. x., p. 128).

Peroneus brevis.—1. Gives off a slip of origin to the abductor minimi digiti; 2. Has a slip joining it to the peroneus longus; 3. The two peronei were found united in a deformed forearm arising from the outer condyle of the femur, and inserted into the outside of the calcaneum, and the outer border of the plantar fascia (Ringhoffer in Virchow's "Archiv," vol. xix., p. 28).

Peroneus quinti digiti is a small muscle which I have found—1. As a tendinous slip from the tendon of the brevis to the extensor aponeurosis of the little toe, arising before the tendon winds round the malleolus, or after, or most

commonly at the ankle; 2. Or inserted into the head of the fifth metatarsus; 3. Or into its base when the peroneus tertius was absent, 4. A completely separate muscle arising from the lower fourth of the fibula under the peroneus brevis in the groove, with which its tendon runs, to be inserted into the extensor aponeurosis on the upper surface of the little toe; 5. Or a special muscle may arise from the peroneus brevis tendon, and be inserted as above. Separate muscular peronei quinti have been seen by Hallett, Wood, and myself.

Tibialis anticus—1. Had a split tendon in a case of talipes, one half going to the scaphoid and ento-cuneiform, and one to the outside of the astragalus and os calcis; 2. In another case of splitting, one half went to the ento-cuneiform, and half to the first metatarsal; 3. Ringhoffer found that this muscle, in a case of congenitally deformed limb, arising from the outer condyle, was inserted into the inner side of the palmar fascia (loco citato supra).

Tibio-fascialis anticus is a small muscle, described by Wood, arising from the lower third of the anterior edge or outer side of the tibia over the tibialis anticus, and inserted into the annular ligament over the extensor digitorum communis, and into the lower border of the deep tibial fascia. It may be a tendinous slip from the tibialis anticus, or a separate muscle. Mr. Wood found it twice in thirty-six. A description of a very large specimen of it was communicated to me by Dr. MacMullen.

Extensor proprius hallucis has been found—1. Double (Meckel, Henle); 2. With a common muscular slip between it and the extensor digitorum; 3. With a slip to the extensor hallucis brevis at the junction of its muscle and tendon; 4. Joined to the extensor ossis metatarsi hallucis.

Extensor ossis metatarsi hallucis may exist as a slip of the last, or with a distinct belly arising beside the ordinary extensor, or with a tendon from the annular ligament, and no belly; or, as an offshoot from the tibialis anticus, or extensor digitorum longus. When separate, it traverses the same groove in the annular ligament as the extensor hallucis.

Extensor primi internodii hallucis is nearly always an offshoot from the common extensor of the hallux. It was described first by Walther ("Haller's Disputationes," vol. vi., p. 559). Mr. Wood found it present in nearly one-half the subjects examined by him; it has been found separate, arising above and external to the extensor hallucis, and with its tendon joining the first slip of the

brevis; rarely it arises below the ordinary extensor, or it may arise from the tendon of the extensor at the ankle, or three inches above it. Mr. Wood saw it as a tendinous offshoot from the extensor longus digitorum; its insertion may be separate from the extensor brevis. Its origin may be from the tibialis anticus (Wood); or a slip from the tibialis may co-exist with an ordinary extensor primi internodii tendon from the extensor hallucis. I have seen it as a slip from the hallucis co-existing with a slip joining the extensor hallucis and the extensor communis. A tendon coming from the extensor hallucis may end by bifurcating, one extremity joining the great toe tendon of the extensor brevis, and the other coming under the tendon of the extensor proprius was inserted into the base of the first phalanx. Tendons with a course and arrangement similar to this have been found with a special muscle either internal or external to the extensor hallucis (Wood).

Tensor capsuli tibio-tarsalis—The "spanner" of the ankle-capsule is a muscle described by Henle ("Muskellehre," p. 293), arising from the lower half of the lateral surface of the tibia under the flexor longus digitorum, the belly is three inches in length, and ends in a tendon which is inserted into the ankle-capsule; it has been once seen. I have never met with it; it seems to be similar to the tibialis secundus mentioned above, and described by Bahnsen.

Tensor membrani synoviale tarsi is a somewhat similar muscle, described by Linhart ("Œsterreichische med. Wochenschrift," 1846, p. 14), which arises from the fascia over the deep flexors, and is inserted into the os calcis.

Peroneus tertius: 1. May be absent, an occurence which happens about once in ten subjects; 2. It may be doubled; 3. Or have a double tendon; 4. Or send a tendinous slip into the fourth metatarsal; 5. Or may go entirely into this bone; 6. As a mere tendinous band from the extensor communis; 7. Sending a slip to the extensor aponeurosis of the little toe; 8. Or a slip to the outer tendon of the long extensor; 9. Or to the corresponding tendon of the short extensor, Bahnsen ("Henle u. Pfeufer's Zeitschrift," vol. xxxiii. p. 33); 10. With a double tendon, one part of which went to the fascia over the fourth interosseous muscle; 11. Arising from the lower half of the fibula; 12. Sending off the extensor tendon to the little toe in a case in which no such tendon arose from the long extensor; 13. Double, the posterior muscle having

two tendons to the fifth metatarsal, the anterior sending slips to the little toe and fourth metatarsal.

Peroneus quartus (Otto), sextus (mihi) is a small muscle arising from the back of the fibula between the peroneus brevis and the flexor pollicis, it is inserted into a tubercle on the os calcis behind the tendons of the peronei longus and brevis. A specimen inserted into the cuboid bone was communicated to me by Dr. MacMullen, now of Her Majesty's Indian army; I have seen it attached to the flexor hallucis, but most commonly it is separate from it, but passes through a separate opening in the annular ligament, behind the other peronei tendons.

Peroneus accessorius is a muscle described by Henle, arising between the peronei longus and brevis, and joining the tendon of the former in the sole.

Peroneo-calcaneus internus is a small muscle which I described some time ago, and seems to resemble the tensor of the synovial membrane of the ankle of Henle and Linhart, above described; it arises below the flexor hallucis longus from an oblique line on the back of the fibula behind the external malleolus, passes over the back of the sustentaculum tali, in the groove with the flexor hallucis to be inserted into a tubercle of the os calcis. This muscle I have elsewhere referred to as the probable homotype of the pronator quadratus.

A remarkable long flexor of the little toe is described by Bartholin (quoted by Diemerbroeck "Anat.," p. 465), arising from the head of the tibia and inserted by two tendons into the little toe. No other author has, as far as I know, ever seen a muscle like this; it seems closely allied to the following.

Tensor fasciæ plantaris is a muscle which has been found arising from the linea poplitea under the soleus, and inserted into the anterior annular ligament near the tuberosity of the os calcis. A muscle not dissimilar is described by Reinhardt (Müller's "Archiv," 1846, p. 299), as the musculus accessorius flexori hallucis longi superior, whose origin is said to be above the flexor hallucis and flexor digitorum, its insertion is into the inner border of the caro quadrata, and thus indicates the relation of this and the preceding two to the accessorius ad calcaneum of Gantzer.

Extensor digitorum longus—1. May send a slip to the base of the first phalanx of the second toe (Wood); 2. Or a connecting slip from its innermost

tendon to the extensor hallucis; 3. The tendon for the little toe may arise by a special belly with the peroneus tertius (Wood); 4. A doubled tendon for the little toe; 5. Or for the second toe; 6. To the base of first phalanx of second toe; 7. A slip to the first interosseous muscle; 8. Slips from this muscle and the peroneus tertius to the anterior end of the fifth metatarsal bone; 9. A slip from the second tendon of the long extensor to the short muscle; 10. A special belly for each tendon (Wood); 11. The same with two slips from peroneus brevis to the outer division; 12. Slips from the tendon for the fourth and fifth toes inserted respectively into the fourth and fifth metatarsal bones, (Wood); 13. The outermost tendon of the longus has been found joined to that of the brevis by a cross band (Wood); 14. A slip to the fifth metatarsal coexisting with the peroneus tertius; 15. This muscle was found by Ringhoffer (Virchow's "Archiv," vol. xix., p. 28); in a case of a deformed limb, with four tendons, one to the second toe, one to the fifth, and two to the fourth toe.

Extensor brevis digitorum pedis—1. Reduced to the first and fourth toe-muscles, in a Bushwoman (Flower and Murie "Journal of Anat.," vol. i., p.189); 2. Or to the second and third alone; 3. A slip from the second tendon joined the first dorsal interosseous, and from the third tendon to the second dorsal interosseous; 4. A doubled tendon for the second toe; 5. A slip from the second tendon of the longus to this muscle (McWhinnie); 6. Two separate slips to the second toe; 7. A separate slip to the little toe; 8. A slip to the fourth toe muscle from the extensor longus; 9. A separate extensor hallucis brevis; this is a commoner arrangement than the union of this with the other parts of the extensor brevis, and is regarded by Prof. Henle as the normal condition (Muskellehre, p. 299); 10. A bicipital second toe extensor, the second head springing from the ectocuneiform and base of the third metatarsal (Lotze, in Henle u. Pfeufer's Zeitschrift, 3. Reihe vol. xxviii., p. 99); 11. Double heads for the second and third toe extensors (ibid); 12. In the case above mentioned, described by Ringhoffer, a special slip has been found passing to the fifth toe, from the tarsus beneath the peroneus brevis; 13. Reduced to a little and third toe muscle.

Interossei pedis—1. The fourth plantar arose from a slip of the peroneus longus; 2. The first plantar has been found bipenniform and perforated by the artery; 3. The first and fourth dorsal with only single heads; 4. An

interosseus plantaris hallucis, homologous with that of Henle in the hand, has been found.

Of the foot lumbricales the following varieties have been seen; 1. All absent but the third; 2. Absence of the second ("Petsche, Syllog. Musc. Obs." p. 771). I have seen the same variety; 3. Absence of the two middle muscles (Behrends, Sömmerring, "Rhodii Mantissa Anat.," p. 31, Hafniæ, 1661); 4. Absence of the third is described by Rudolphi, quoted by Gantzer; 5. Absence of the fourth is described by Heschl, Rudolphi, and Wood; 6. The third has been found arising from the perforated, instead of the perforating tendon (Wood); 7. The fourth has been found double; 8. Or bifurcated at its insertion; 9. And the third has been found double by Mr. Wood; 10. The first may arise from the tibial of the deep tendon for the second toe after the junction of the communicating slip from the flexor hallucis, and sometimes from the tibial side of this slip itself; 11. It may have two heads, one from each tendon; 12. There may be no first and two second lumbricales; 13. The second, third, and fourth may arise from special slips of the tendons; 14. The fourth may get a slip from the accessorius.

A tensor of the fascia of the leg, arising from the os calcis and tendo Achillis, is described by Meyer ("Heymann," p. 15, quoted by Meckel).

Flexor accessorius pedis may have—1. A high origin from the lower third of the fibula, ending in a distinct tendon which passes under the annular ligament, joins with the rest of the accessorius (Gantzer, p. 15); 2. A high origin from the fascia over the flexor hallucis longus, with a course like the last. This has been seen in two bundles, one beside the tibia, and one over the flexor hallucis, the tendon of this bifurcated, and one was inserted into the hallucis brevis communis; 3. With an origin from the insertion of the plantaris tendon and the upper part of the os calcis; 4. With an origin from the calcaneo-cuboid ligament; 5. Mr. Wood describes an interesting anomaly in which this muscle possessed a fibular head and heads like those described in Nos. 3 and 4, together with a normal attachment; these all uniting, formed with the flexor hallucis a distinct, deep, or third set of flexors for the four outer toes, each of these joined the perforans inside the sheath, opposite the first joint; 6. Mr. Wood found the accessorius to arise by a tendon from the inner side of the os calcis; 7. A tendon from the outer head of the accessorius joined the perforated ten-

don of the middle toe; 8. The accessorius may have no connexion to the fourth or fifth toe tendons; 9. Sending a few fibres to flexor hallucis; 10. With a slip from the inner side, or, from the deep surface of the soleus (Turner, l. c.); 11. A slip from the fascia of the back of the leg to the inside of the os calcis, close to the accessorius, is described by Gantzer as the accessorius ad calcaneum; 12. An accessory muscle in a deformed limb was found by Ringhoffer (Virchow's "Archiv," vol. xix., p. 28), it arose from the middle third of the limb, along with the flexor digitorum communis, and was inserted into the caro quadrata; 13. The plantar accessorius receiving a slip from the peroneus brevis; 14. Its insertion may be to the outer border of the tendons; 15. Or to the under surface; 16. With no connexion to the third and fourth tendons; 17. Sending an accessory slip to the deep tendon of the little toe; 18. It may be inserted into the communis, partly to the connecting slip from the hallucis, and to tendons of the second, third, and fourth toes; 19. Sometimes a few fibres go to the hallucis; 20. Inserted into the connecting link alone; 21. A slip from the muscle to the tendon of the flexor brevis digitorum for the third toe; 22. A slip from the inner side of the os calcis sending a tendon to the last phalanx of little toe, and one to the point of insertion of the accessorius.

Flexor brevis digitorum has been seen with; 1. Absence of the little toe tendon (Wood), with no substitute; 2. With its place supplied by a small fusiform muscle from the outermost border of the flexor perforans (Wood); 3. The substitute arising by two slender fusiform bellies, one from the long flexor tendon, and the other from the inner tubercle of the os calcis, deeper than the fibres of the flexor brevis digitorum; 4. A slip of perforans for the middle toe joined the perforatus at the metacarpo-phalangeal joint; 5. A long slip of the perforans tendon, joined the flexor brevis to the middle toe slip of flexor brevis; 6. The same from accessorius; 7. This muscle has been seen with only three tendons; 8. Or with five tendons, two to the second toe (Kelly); 9. Or with the little toe tendon not perforated; 10. Or lost in the anterior fascia of the toe; 11. The little toe short flexor has been seen by Brugnone, Henle, and others, arising from the long flexor tendon; 12. Or it has been found separate, and arising by one head from the external intermuscular septum, and from the flexor accessorius.

Abductor hallucis : 1. Sending a muscular slip to the base of the first phalanx of the second toe (Wood), inserted into the long flexor tendon of the great toe. Lepine finds a slip often passing to the skin of the ball of the great toe ("Dictionnaire Annuel des progrés des Sciénces Médicales," 1864, p. 35), but this, and the corresponding slips in the hand, I have not found as frequently as M. Lepine describes.

Flexor brevis hallucis : 1. Sending a slip to the second toe ; 2. It may arise by an additional slip from the deep tendon ; 3. Or from the tendon of the tibialis posticus ; 4. Its inner head is always united more or less to the abductor, but I have seen them absolutely inseparable ; 5. Similarly the external head may be attached to the adductor hallucis.

Opponens hallucis, though not usually described in works of human anatomy, yet is a frequent constituent element in the human foot ; it is most commonly inseparable from the following muscle, and appears as an offshoot from the adductor inserted into the metatarsal bone of the great toe ; more rarely it is free. Several authors have referred to it.

Adductor hallucis may send a slip to base of the first phalanx of the second toe, or may be attached to the opponens, as above noticed, or to the outer head of the flexor brevis hallucis. It may arise entirely from the sheath of the long peroneal tendon.

Transversalis pedis Casserii : 1. Without a slip from the fifth metatarsus ; 2. Skipping unattached over the fourth or third ; 3. Absent (Böhmer, loc. cit., p. 8).

Abductor ossis metatarsi minimi digiti—A muscle from the external tubercle of the os calcis to the spur-like process of the fifth metatarsal, beneath the outer margin of the external slip of the plantar fascia, first described by Wood as a human muscle, is present in seven out of thirty-four (1866) ; seventeen out of thirty-six (1867) ; forty out of sixty (M. 1867) ; nineteen out of thirty-six (W. 1868) ; that is, nearly once in two subjects. It generally has a distinctly fusiform shape ; may be adherent to the abductor minimi digiti, or completely separate, and ordinarily has a distinct tendinous insertion. In one subject Mr. Wood found it arising from the inter-tubercle of the calcaneum, and inserted by a long tendon into the anterior part of the shaft of the metatarsal. I have twice found this form present, but it was in both cases firmly joined to the

abductor minimi digiti. I have also met with, at least, three instances of the presence of this muscle arising from the outer tubercle of the os calcis, and inserted into the neck of the metatarsal bone.

Abductor minimi digiti, I have found inseparably from the last, or from the flexor brevis minimi digiti, or with a strong head from the spur of the fifth metatarsal. Its insertion, which is usually connected with that of the next muscle, may be perfectly separate and unconnected from it.

Flexor brevis minimi digiti may be attached inseparably to the last, or to the tendon of the abductor ossis metatarsi minimi digiti before referred to. It may arise altogether from the peroneal sheath.

Opponens minimi digiti is a small muscle, more often present than the corresponding slip for the hallux: it usually seems like an internal slip of the last, inserted into the inner side of the metatarsal bone of the little toe; it is not so often detached as the opponens hallucis.

In the foregoing Paper I have given the references to such anomalies as have been described by various authors, in whose works more detailed descriptions are given. Especially would I refer to the valuable Papers of Mr. Wood in the "Proceedings of the Royal Society," for 1854–55–56–57, and to those of Hallett in "The Edinburgh Medical Journal" for 1848, and M'Whinnie in "The London Medical Gazette" for 1830, of Luschka, Gruber, Calori, Budge, and Dursy; all of which are good. In those cases in which I have given no reference, the special forms of anomaly are tabulated on my own authority, and they are all such as have occurred to me, or as have been described to me orally by competent observers. I have endeavoured to make this series as complete as possible, and trust that it may be found as useful to other anatomists as it has been to me.

Since this Paper was presented to the Academy, a few additional forms of anomaly have been described, and several undescribed varieties have been met with by myself; but these I must reserve for a subsequent appendix.

EXPLANATIONS OF THE PLATES.

Pl. I.—Fig. 1. *F.h.l.* flexor hallucis longus. *F.d.l.* flexor digitum longus. *T.p.* tibialis posticus. P.*c.i.* peroneo-calcaneus internus. *F.b.d.* = Flexor brevis digitorum.

Fig. 2. *P.t.*[1] peroneus tertius. *P.t.*[2] normal insertion. *P.t.*[3] insertion into the fourth metatarsal. P.*t.*[4] slip to the little toe.

Fig. 3. *P.p.* pubio peritonealis. *E.a.* = Epigastric artery. *E.o.* = External oblique. *R* = rectus.

Fig. 4. *R.R.*[2]*R*[3]. Triple head of the rectus femoris. *T.v.f.* = tensor vaginæ femoris.

Fig. 5. Gemellus superior inserted into the combined tendons of the pyriformis and obturator internus. *T.* trochanter. *G.c.* gemellus inferior. *O.i.* = obturator internus. *Py.* = pyriformis. *G.s.* gemellus superior.

Fig. 6. Thyreo-triticeus muscle. *H* = hyoid bone. *Th.* = thyroid cartilage. *T.tr.* = thyreo triticeus. *Tr.* cartilago triticea.

Fig. 7. *Pl. Pl*[2] Plantaris. Plantaris expanding into a fascia over soleus. *Po.* = popliteus. *S.* = soleus.

Fig. 8. *C. tr.* crico-trachealis. *C. th.* crico-thyroid.

Pl. II.—Fig. 1. Pronator quadratus, with a slip to the trapezium.

Fig. 2. A trifid pronator quadratus.

Fig. 3. A bilaminar pronator.

Fig. 4. A complexly divided pronator quadratus.

Fig. 5. *S. cl.* = sterno-cleido-mastoid reflected ; *cl.*[2] = a second deeper cleido-mastoid; *o. h.* omohyoid monogastric and clavicular in origin. *Tr. C.* Transversalis cervicis. *Sh* = Sterno-hyoid. *S.Th.* Sterno-thyroid.

Fig. 6. *Pl*[1]. palmaris longus. *P.l.*[2] second palmaris, with an intermediate belly. *Ac* = accessorius ad flexorem minimi digiti.

Fig. 7. Costo-coraco-clavicular membrane split by pectoralis minor tendon.

Fig 8. *P* = Petrous bone. *T.p*[1] = tensor palati. *T.p*[2]. tensor palati secundus. *K.* hamular process.

Fig. 9. *P.l.* palmaris longus. *P.l.*[2] palmaris secundus. *Pl.*[3] second head of palmaris secundus. *P.l.*[4] tendon of palmaris secundus.

Fig. 10. Accessorius ad abductorem minimi digiti manûs.

Fig. 11. *P.q.* Pectoralis quartus. *L.d.* = Latissimus dorsi. *P.m.* = Pectoralis major.

Fig. 12. Peculiar orbital ligament.

Pl. III. Various forms of unions of the long flexor tendons of the toes:—

h = flexor hallucis longus.
d = flexor digitorum longus.
a = accessorius.
a[1] and *a*[2] = parts of the accessorius when split.
l[1] and *l*[2] = lumbricales. 1, 2, 3, 4, and 5 = tendons for the supply of the respective toes.

N. B. These sketches are copied from my note-book, and unfortunately they are not all in the same position; in some the under surface is shown, and in others the upper.

INDEX OF MUSCLES.

Abductor minimi digiti manûs, 98.
Abductor minimi digiti pedis, 129.
Adductor pollicis, 99.
Abductor hallucis, 129.
Acromio-clavicularis, 49.
Accessorii (dorsal), 64.
Achselbogen, 60.
Adductor brevis, 112.
Adductor longus, 112.
Adductor magnus, 113.
Adductor minimus, 113.
Adductor minimi digiti,
Adductor pollicis, 99.
Adductor hallucis, 129.
Anconæus, 104.
Antitragicus, 6.
Aryteno-corniculatus, 32.
Aryteno-epiglottici, 31.
Arytenoideus, 30.
Attollens aurem, 5.
Attrahens aurem, 5.
Auriculo-glossus, 7.
Azygos uvulæ, 41.
Azygos pharyngis, 39.

Biventer cervicis, 65.
Biceps, 80.
Biceps flexor cruris, 116.
Brachialis anticus, 76.
Buccinator, 18.
Bucco-pharyngeus, 39.

Capsularis sub-brachialis, 77.
Cephalo-pharyngeus, 38.
Cervicalis ascendens, 64.
Cervico-costo-humeralis, 49.
Chondro-coracoid, 60.
Chondro epitrochlearis, 46.
Cleido-hyoid, 25.
Cleido-occipital, 21.
Coccygeus, 67.
Co-angustator labii inferioris, 12.
Complexus, 65.
Compressores nasi, 12.
Compressores narium, 12.
Comes obliqui superioris, 7.
Constrictor labii inferioris, 12.
Constrictor labii superioris, 12.
Constrictor pharyngis inferior, 40.
Constrictor pharyngis medius, 40.
Constrictor pharyngis superior, 39.
Coraco-clavicularis, 49.
Coraco-brachialis, 74.
Corrugator supercilii, 11.
Costo-fascialis cervicalis, 31.
Cremaster, 70.
Crico-arytenoideus lateralis, 30.
Crico-arytenoideus posticus, 30.
Crico-corniculatus, 28.
Crico-epiglotticus, 31.
Crico-hyoid, 28.
Crico-thyreoideus, 27.
Crico-thyroideus posticus, 28.
Crico-thyroideus superior, 27.
Crico-trachealis, 29,
Cruræus, 110.
Curvator coccygis, 66.
Cubito-carpeus. 97.

Deltoides, 70.
Depressor anguli oris, 15.
Depressor labii inferioris, 15.
Depressor auriculæ, 7.
Depressor cartilaginis Santorini, 32.
Depressor septi mobilis nasi, 12.
Depressor supercilii, 10.
Depressor palpebræ inferioris, 9.
Diaphragm, 70.
Digastricus, 32.
Dilator naris, 12.

Epitrochleo-anconeus, 78.
Erector spinæ, 64.
Extensor carpi radialis brevis, 102.
Extensor carpi radialis accessorius, 101.
Extensor carpi radialis longus, 102.
Extensor radialis intermedius, 102.
Extensor carpi ulnaris, 104.
Extensor coccygis, 67.
Extensor digitorum brevis manus, 104.
Extensor digitorum brevis pedis, 126.
Extensor digitorum communis, 103.
Extensor digitorum longus pedis, 125.
Extensor indicis, 105.
Extensor medii digiti, 105.
Extensor minimi digiti, 103.
Extensor ossis metacarpi pollicis, 107.
Extensor ossis metatarsi hallucis, 123.
Extensor hallucis proprius pedis, 123.
Extensor pollicis et indicis, 106.
Extensor primi internodii pollicis, 108.
Extensor primi internodii hallucis, 123.
Extensor secundi internodii pollicis, 106.

Flexor brevis minimi digiti accessorius, 98.
Flexor brevis minimi digiti pedis, 130.
Flexor carpi radialis, 85.
Flexor carpi radialis profundus, 90.
Flexor carpi ulnaris, 85.
Flexor digitorum accessorius, 127.
Flexor digiti secundi proprius, 122.
Flexor digitorum brevis pedis, 128.
Flexor digitorum longus pedis, 120.
Flexor digitorum profundus, 90.
Flexor digitorum sublimis, 89.
Flexor pollicis brevis, 99.
Flexor hallucis brevis, 129.
Flexor pollicis longus, 95.
Flexor hallucis longus, 120.
Frontalis, 2.

Gastrocnemius, 118.
Gemellus inferior, 115.
Gemellus superior, 115.
Genio-glossus accessorius, 38.
Genio-hyo-glossi, 35.
Genio-hyoideus, 34.
Gleno-brachialis, 74.
Glosso-epiglotticus, 31.
Glosso-pharyngeus, 39.
Glutæus maximus, 113.
Glutæus medius, 114.
Glutæus minimus, 115.
Gracilis, 113.
Gracillimus orbitis, 7.

Hepatico-diaphragmaticus, 71.
Helicis major, 6.

Helicis minor, 6.
Hyo-glossi, 38.
Hyo-epiglotticus, 31.
Hyo-trachealis, 29.

Iliacus major, 111.
Iliacus minor, 111.
Ilio-costalis, 64.
Infracostales, 56.
Infraspinatus, 72.
Intercostales, 55.
Interossei manus posteriores, 108.
Interossei palmar, 108.
Interossei pedis inferiores, 126.
Interossei pedis superiores, 120.
Interpollicaris transversus, 99.
Intertransversales, 66.

Kerato-hyoid, 29.
Kerato-cricoid, 28.
Kerato-arytenoid, 30.

Latissimus dorsi, 59.
Lacrymalis, 10.
Laxator tympani major, 7.
Laxator tympani minor, 7.
Levator anguli oris, 15.
Levator anguli scapulæ, 57.
Levator ani, 67.
Levator glandulæ thyroideæ, 27.
Levator labii inferioris, 15.
Levator labii superioris alæque nasi, 12.
Levator labii superioris proprius, 13.
Levator menti, 15.
Levator claviculæ, 21.
Levator palati, 40.
Levator palpebræ superioris, 7.
Levator prolabii superioris, 12.
Levator latissimi dorsi, 52.
Levatores costarum, 56.
Linguales, 38.
Longissimus dorsi, 64.
Longissimus capitis, 64.
Longus colli, 44.
Longus atlantis, 44.
Lumbricales manus, 90.
Lumbricales pedis, 127.
Masseter, 18.
Mento-hyoid, 33.
Motor uvulæ, 41.
Multifidus spinæ, 65.
Musculus anomalus, 13.
Musculus sacci lacrymalis, 10.
Mylo-hyoideus, 34.
Mylo-pharyngeus, 40.
Mylo-glossus, 35.

Naso labialis, 12.

Obliquus abdominis externus, 67.
Obliquus abdominis internus, 68.
Obliquus capitis inferior, 44–66.
Obliquus capitis superior, 44–66.
Obturator externus, 116.
Obturator internus, 116.
Occipito-staphylinus, 39.
Occipito-frontalis, 3.
Omo-hyoideus, 22.
Opponens minimi digiti, 98.
Opponens minimi digiti pedis, 130.
Opponens hallucis, 129.
Orbicularis oris, 12.
Orbicularis palpebrarum, 9.

Palato-glossus, 41.
Palato-pharyngeus, 41.
Palmaris brevis, 97.
Palmaris-longus, 86.
Pectinæus, 112.
Pectoralis quartus, 47.
Pectoralis minimus, 51.
Pectoralis major, 44.
Pectoralis minor, 47.
Peristaphylinus externus, 42.
Peristaphylo-pharyngeus, 41.
Peroneo-calcaneus, 124.
Peroneus longus, 122.
Peroneus quinti, 122.
Peroneus tertius, 124.
Petro-salpingo-staphylinus, 42.
Petro-pharyngeus, 40.
Pharyngo-staphylinus, 41.
Plantaris, 118.
Platysma myoides, 15.
Poplitæus, 117.
Procerus nasi, 11.
Præclavicularis lateralis, 49.
Præclavicularis medialis, 49.
Pronator quadratus, 96.
Pronator radii teres, 84.
Protractor labii superioris, dexter et sinister, 12.
Psoas magnus, 112.
Psoas parvus, 111.
Pterygoideus externus, 18.
Pterygoideus internus, 19.
Pterygoideus proprius, 19.
Pterygo-pharyngeus, 39.
Pterygo-palatinus, 42.
Pubio-peritonealis, 70.
Pyramidalis abdominis, 30.
Pyramidalis nasi, 11.
Pyramidalis socius, 13.
Pyriformis, 114.

Quadratus femoris, 116.
Quadratus lumborum, 70.

Radio-carpeus, 90.
Rectus abdominis, 68.
Rectus capitis anticus major, 43.
Rectus capitis anticus minor, 44.
Rectus capitis lateralis, 44.
Rectus capitis posticus major, 66.
Rectus capitis posticus minor, 66.
Rectus capitis lateralis longus, 66.
Rectus femoris, 109.
Rectus oculi externus, 8.
Rectus oculi inferior, 8.
Rectus oculi internus, 8.
Rectus oculi superior, 7.
Rectus sternalis, 53.
Retrahens aurem, 4.
Retro-clavicularis, 51.
Rhombo-atloideus, 63.
Rhomboideus faciei, 13.
Rhomboideus occipitalis, 62.
Rhomboideus major, 62.
Rhomboideus minor, 62.
Risorius Santorini, 14.
Rotatores spinæ, 60.

Salpingo-staphylinus, 40.
Salpingo-pharyngeus, 41.
Sacro-spinalis, 64.
Sacro-coccygeus, 66.
Sacro-lumbalis,
Sartorius, 109.
Scalenus anticus, 42.
Scalenus medius, 42.
Scalenus posticus, 43.
Scalenus minimus, 43.
Scalenus accessorius, 43.
Scalenus lateralis, 43.
Scapulo-clavicularis, 49.
Scapulo-costalis minor, 52.
Scapulo-humeralis digastricus, 70.
Semimembranosus, 117.
Semispinalis colli, 65.
Semispinalis dorsi, 65.
Semitendinosus, 117.
Serratus magnus, 56.
Serratus posticus inferior, 63.
Serratus internus, 56.
Serratus posticus superior, 63.
Solæus, 118.
Sphincter ani externus, 70.
Sphincter ani internus, 70.
Spincter ani tertius, 70.
Spinalis colli, 65.
Spinalis dorsi, 65.
Splenius capitis, 63.
Splenius colli, 63.
Stapedius, 7.
Sterno-cleido-mastoideus, 19.
Sternalis brutorum, 53.
Sterno-omoid, 51.
Sterno-clavicularis, 50.
Sterno-hyoideus, 25.
Sterno-thyroideus, 26.
Stylo-glossus, 35.
Stylo-hyoideus, 36.
Stylo-maxillaris, 37.
Stylo-pharyngeus, 39.
Stylo-pharyngeus alter, 39.
Stylo-auricularis, 7.
Subanconeus, 30.
Subclavius, 51.
Subcruræus, 111.

Subscapularis, 73.
Subscapulo-capsularis, 73.
Supinator radii brevis, 100.
Supinator brevis accessorius, 101.
Supinator radii longus, 99.
Supra-clavicularis proprius, 48.
Supra-clavicularis, 50.
Supraspinatus, 72.
Supracostalis, 52.
Suspensorius testis, 70.

Temporalis, 18.
Tensor palati, 40.
Tensor tarsi, 10.
Tensor capsuli tibiotarsalis, 124.
Tensor ligamenti annularis anterior, 100.
Tensor ligamenti annularis posterior, 10.
Tensor semivaginæ articulationis humero-scapularis, 49.
Tensor membrani synoviale tarsi, 124.
Tensor trochlea, 8.
Tensor fascia plantaris, 125.
Tensor tympani, 17.
Tensor vaginæ femoris, 109.
Teres major, 73.
Teres minor, 73.
Thyro-arytenoideus, 30.
Thyro-epiglottic, 31.
Thyroideus transversus, 30.
Thyro-hyoideus, 27.
Thyro-hyoideus superior, 27.
Thyro-mediastinal, 30.
Thyro-staphylinus, 41.
Thyro-trachealis, 29.
Tibialis anticus, 123.
Tibialis posticus, 119.
Tibialis secundus, 120.
Tibio fascialis anticus, 123.
Trachelo-mastoideus, 64.
Trachelo-acromialis, 22.
Tragicus, 6.
Tranversalis abdominis, 68.
Tranversalis colli, 64.
Tranversalis pedis, 129.
Transversus auriculæ, 7.
Transversus menti, 15.
Transversus nuchæ, 3.
Transversus torbitis, 9.
Transversus colli, 31.
Transversus cervicis anterior, 44.
Transversus thoracis posterior, 56.
Trapezius, 58.
Triangularis sterni, 56.
Triceps extensor cubiti, 79.
Triticéo-glossus, 37.

Vastus externus, 110.
Vastus internus, 110.

Zygomaticus minor, 14.
Zygomaticus major, 14.

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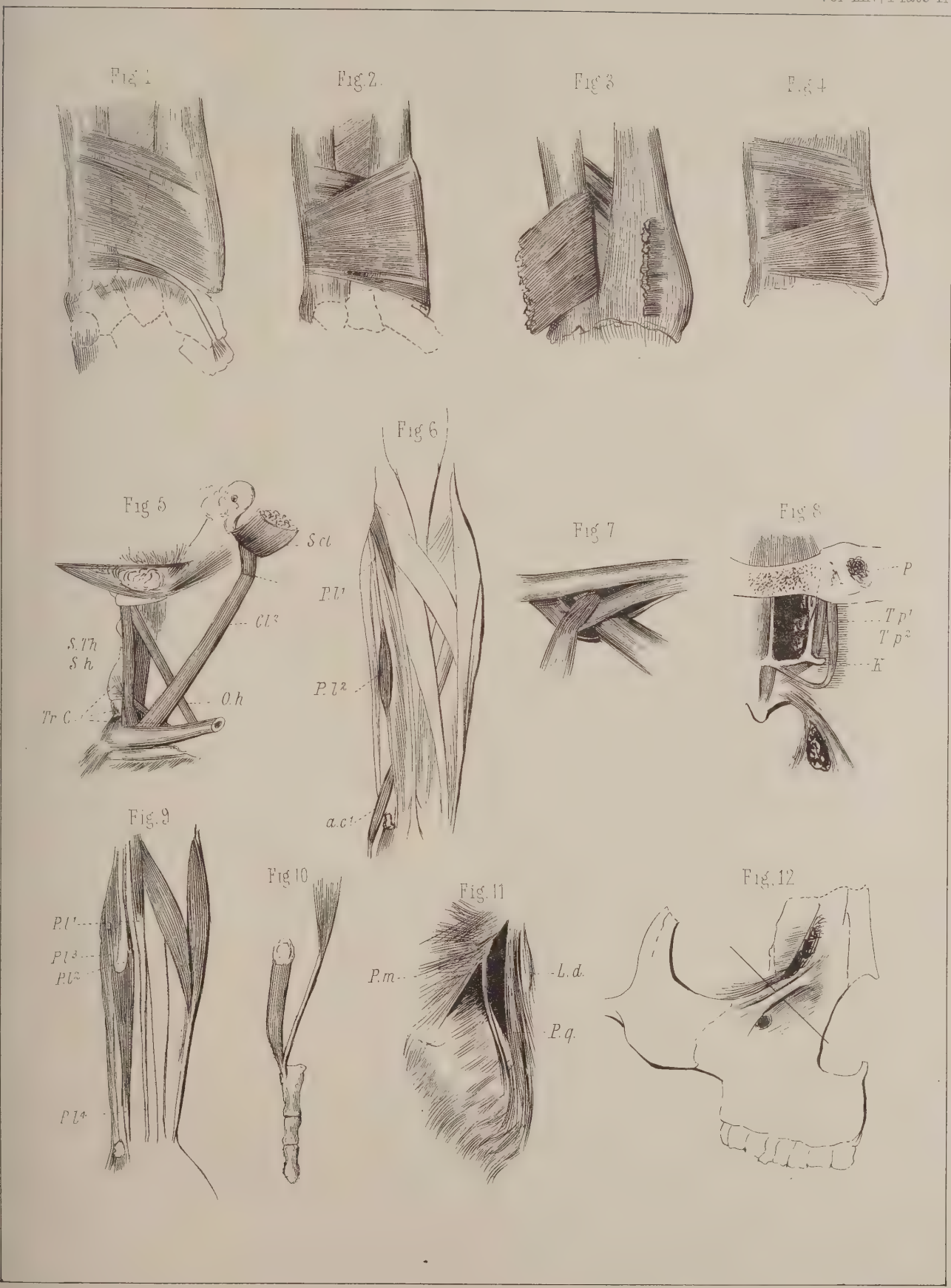

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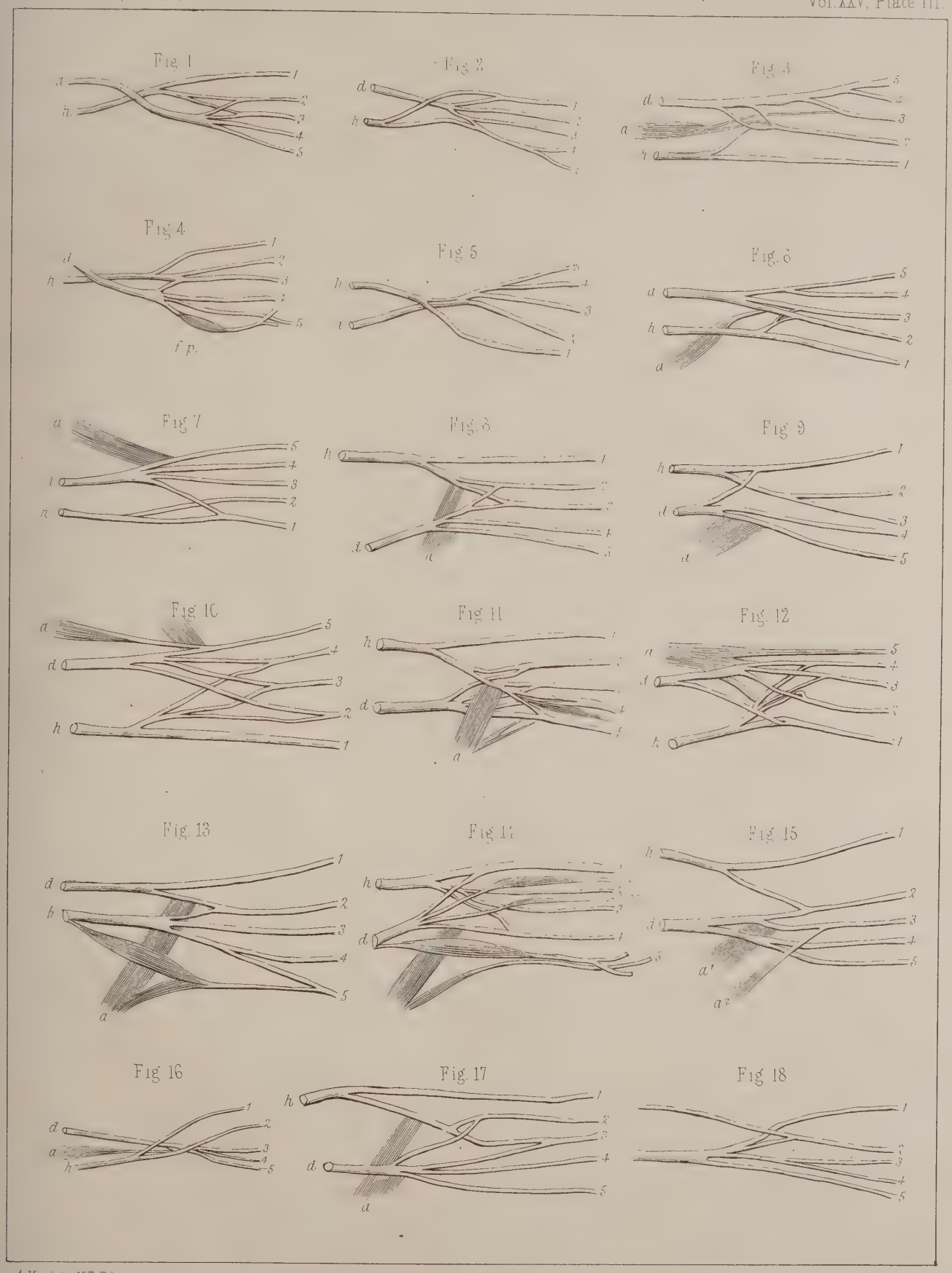

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II.—*Account of Experiments upon the Retardation experienced by Vortex Rings of Air when moving through Air.* By Robert Stawell Ball, LL. D., M. R. I. A., Professor of applied Mathematics and Mechanism, Royal College of Science for Ireland.

Read January 23, 1871.

Introduction—General Description of the Method and Results—The Chronoscope—The Box and Striker—The Target—The Mode of Experimenting—The Discussion of the Results—The Second Series of Experiments.

INTRODUCTION.

1. Since air is not a perfect fluid, a vortex ring is retarded when moving through air. The present inquiry is for the purpose of ascertaining the nature of the retarding force by experiment.

The interest which attaches to vortex motion, in consequence of the beautiful discoveries of Helmholtz and Sir W. Thomson, would suffice to explain the reason why this investigation has been undertaken; it may, however, be added, that any accession, however small, to our knowledge of the properties of air cannot be without value, and also that the question is intimately connected with the mechanical problem of the motion in a resisting medium.

The writer has to acknowledge the liberality with which the Royal Irish Academy placed a grant at his disposal in aid of the research, and also the zeal and intelligence of his assistant in the work, Mr. William Hunt (Associate of the Royal College of Science).

The present paper must be regarded merely as preliminary. The author hopes, in a future series of researches, to carry the experiments to a greater degree of accuracy.

GENERAL DESCRIPTION OF THE METHOD AND RESULTS.

2. Before giving a detailed description of the apparatus, and of the mode of making and discussing the experiments, a brief general account may be convenient.

Air rings (of course invisible, as no smoke was employed) were projected from a large box in the usual manner (Sir William Thomson, "Phil. Mag.," July, 1867). The blows were delivered by a pendulum (called the striker), which, always falling from the same height, ensured that the ring was projected with the same velocity. In one series the ring was 9″ diameter, and was started with an initial velocity of about 10′ per second. After travelling a certain distance varying from 2″ to 20′, the ring impinged upon a target placed to receive the blow. The interval of time between the release of the striker and the impact upon the target was measured by a chronoscope. The time thus found consisted of two parts—namely, the time occupied in the descent of the striker, and the time occupied in the passage of the ring from the orifice of the box to the target. The range being successively increased by intervals of 2′, the time occupied by the ring in moving through definite distances was determined.

3. The observations are next represented in a curve of which the abscissæ are the ranges, and the ordinates the corresponding times. By drawing tangents to this curve, the velocity of the ring at its different positions is approximately found.

A second projection is made, in which the abscissæ are the ranges, and the ordinates the velocities. The points thus determined are nearly in a straight line; from this it follows that the retarding influence may be represented by a force directly proportional to the velocity of which the numerical co-efficient becomes known.

THE CHRONOSCOPE.

4. The chronoscope which was employed was constructed many years ago by Messrs. Elliott (Strand), and it is believed was designed by Sir C. Wheat-

stone. This instrument proved very suitable for the purpose of this inquiry. The chronoscope is shown in Fig. 1, Plate iv. Both front and back views are given. *A* is a weight which animates a train of clock-work, by which the hands shown at *B* and *C* are made to revolve. The weight is raised by turning round the hand on the dial *C*. The dial *C* is numbered from 1 to 10; the dial *B* is numbered from 1 to 30. The hand on *B* makes 10 revolutions for each revolution on the hand on *C*. The rate at which the hands move depends upon the weight *A*; with the weight employed (0·109lb.), the hand on *C* revolved in 1·288 seconds, and one division of the dial on *B* corresponded to 0·0043 seconds. At the back of the chronoscope an escapement wheel is shown, *K* (Fig. 2). The motion of the chronoscope is arrested when a piece, *L*, is pressed against the escapement. *L* is acted upon by *H*, which is on the same axle as *N*, the keeper of the magnet *S*. When the current passes, the motion is stopped, and when the circuit is broken the motion recommences. A screw at *T* (Fig. 1) enables *H* to be pressed against the escapement, so that the chronoscope shall not run down when the battery is out of action. The silken cord supporting the weight is long enough to allow the motion to continue for 5 or 6 seconds.

Before the chronoscope can be employed for the purpose of measuring time, it is necessary to compare its indications with those of a pendulum. It is essential to ascertain whether the reading of the chronoscope is really proportional to the time which has elapsed, and if so to determine the equivalent of the chronoscope unit in seconds.

5. When the circuit is broken, the weight *A* commences to descend. Its velocity must then be accelerated from zero up to a certain maximum, at which the resistance opposed by friction and the escapement becomes equal to the effect of the weight. The motion of the hands is then uniform. It has been ascertained that uniform motion has commenced when the clock has been running for one-third of a second, and that the loss of time arising from the retardation does not exceed one-tenth of a second. In order to avoid the errors inseparable from the initial motion, the observations were so arranged that the chronoscope was only required to give differential results from which the irregularities became eliminated.

6. To compare the chronoscope with the pendulum, it is necessary to arrange a pendulum which shall open a circuit at one point of its path, and close it at another, and afford us the means of calculating the time between the two. The arrangement for this purpose is shown in Plate v.;* and a diagram of the electric connections are shown in Fig. 1, Plate VI.; *OE*, Plate v., is a pendulum consisting of a fine brass wire, to which a sphere of iron (*E*) 1″·5 in diameter is attached. The sphere is between a pair of jaws, *D*, *D*. These jaws are so arranged that when a stud *S* is pressed the jaws fly open and the sphere is released. When *E* is between the jaws, it forms a bridge across which the current passes, and completes the circuit. The current is of course interrupted when *E* is released. At *H* is a pin which projects from the divided bar *MN*. When the wire of the pendulum comes in contact with *H*, the circuit is complete by a route manifest from Fig. 1, Plate VI. *A* is the battery; at *S* the circuit divides into two branches, which again unite at *T*. The jaws *D*, *F* are in one of the branches, and the pendulum, when in the position *OHK*, completes the other branch. The chronoscope (*B*) commences to move when the ball *E* is released from *D*, *F*, but is stopped when the pendulum reaches *OH*. For convenience in the manipulation of the wires, the commutator of Fig. 1, Plate VII., is employed. *A* receives the pair of wires from the battery; *B* the wires from the chronoscope; *E* the wire from the jaws, and *P*, *Q* the wires from the pin and pendulum.

7. The method by which the time is deduced from the initial and final positions of the pendulum is thus described:—

Let α β be the angles made with the vertical by the initial and final positions of the brass wire of the pendulum. These angles are easily calculated, from the observed positions of the wire upon the graduated bar.

Let T be the time of vibration of the pendulum, then the time of descent from α to β is

$$\frac{T}{2} - T\frac{\phi}{\pi}$$

* The framework used in this apparatus has been constructed from Professor Willis' "System of Apparatus for the use of Lecturers and Experimenters in Mechanical Philosophy."

where

$$\sin \phi = \frac{\sin \frac{\beta}{2}}{\sin \frac{\alpha}{2}}$$

500 vibrations of the pendulum taken with a stop-watch, occupied 693·5 seconds, therefore,

$$T = 1{\cdot}387,$$

and the expression for the time of descent from α to β becomes

$$0{\cdot}693 - 0{\cdot}4415\ \phi.$$

8. In the experiments now described, the pin was always placed in one of the two positions which gave $\beta = 10° \, 28'$ or $\beta = 0$: α has the constant value of 14°.

Denoting by T' the time of descent from $\alpha = 14°$ to $\beta = 10° \, 28'$, the following portions of time can be compared with the indications of the chronoscope.

$$T' \ (1)$$

$$\frac{T}{2} \ (2)$$

$$T + \frac{T}{2} \ (3)$$

$$2\,T - T' \ (4)$$

$$2\,T + T' \ (5)$$

$$2\,T + \frac{T}{2} \ (6)$$

The numbers appended correspond with the reference numbers of column I. of Table I. In column II. the equivalents in time are given; these are deduced by making $T = 1{\cdot}387$, and $T' = 0{\cdot}320$ in the expressions just written.

9. The following are the details of the experiment by which the motion of the chronoscope is compared with that of the pendulum when moving, for

$T + \frac{T}{2}$ seconds (3). This is given as an illustration of the mode of experimenting. The pin is attached to a spring, so that it is always withdrawn unless when forcibly held outwards by pulling a string. The ball is placed between the jaws, the hands of the chronoscope are placed at zero, the battery, consisting of a pair of bichromate cells, is turned on by X, Fig. 1, Plate VII., and the serew T, Fig. 1, Plate IV., is released. When all is ready, the stud S, Plate V., is pressed, the jaws fly open, and the chronoscope hands begin to spin round. Directly the pendulum has passed the centre, the pin is drawn out, so that when the pendulum on its return meets the pin, the circuit is renewed and the chronoscope is stopped. The wire of the pendulum bends round the pin, and remains in contact quite long enough to enable the circuit to be permanently closed by bringing the Key Y from Q to P, Fig. 1, Plate VII. The position of the hands of the chronoscope is then to be noted, after which the weight is wound up, and the hands again placed at zero for another experiment.

The pendulum must then have been in motion for a period of $T + \frac{T}{2}$ $= 2{\cdot}080$ seconds.

The experiment being repeated ten times, the readings of the chronoscope for each are recorded. The units are the complete revolutions made by the more rapidly moving hand B, Fig. 1, Plate IV. The decimals are calculated from the observed position of the hand B, recorded in terms of the thirty equal parts into which the dial B is divided.

The following column gives the readings of the chronoscope:—

	15·17
	15·20
	15·08
	15·37
	15·23
	15·37
	15·43
	15·47
	15·43
	15·37
Mean, . . .	15·31

The probable error of this result is—

$$0{\cdot}03,$$

which is $\frac{1}{310}$ part of the whole amount, and corresponds to a period of time equal

$$0{\cdot}004 \text{ seconds.}$$

In Table I. will be found the mean results (column III.) and the corresponding probable errors (column IV.) for the whole six experiments. Each of these has been deduced from ten or more observations. Column II. contains the corresponding known periods of time.

10. The experiments, of which the results are embodied in Table I., contain all that is necessary for a complete examination of the chronoscope. We commence the discussion of them by a graphic projection shown in Fig. 3, Plate VIII.

From the point A, abscissæ are measured equal to the times of column II., Table I., on the scale of $\frac{2}{3}$ inch to a second; at the points thus determined, ordinates are erected equal to the mean readings of column III., Table I., on the Scale of $\frac{1}{6}$ inch to one chronoscope unit. Thus the points C^1, C_2, C_3, &c., have been determined.

It is found that a line BY can be drawn which passes extremely close to the six points C_1, C_2, &c. This line cuts the axis at a distance AB from the origin $AB = 0''{\cdot}14$, corresponding to a period of 0.08 seconds. This is the initial retardation. The figure proves that after a time, at all events not greater than 0·320 seconds, the motion of the chronoscope is uniform. Draw C_2X, YX, parallel to the axes, it is found by measurement that—

$$C_2X = 4''{\cdot}18 \quad YX = 3''{\cdot}23.$$

but $4''{\cdot}18$ along the abscissa corresponds to 2·508 seconds, and $3''{\cdot}23$ along the ordinate, corresponds to 19·38 chronoscope units, and hence the value of one chronoscope unit after the motion has become uniform is

$$0{\cdot}129 \text{ seconds.}$$

11. The same value of the unit is also deduced from Table II.

Let x be the value of the chronoscope unit in time.
a the initial retardation of chronoscope in time.
n the mean reading of the chronoscope.
t the corresponding time.

then $$t = a + xn.$$

If two observations be made of t and n,

$$t_1 = a + xn_1$$

$$t_2 = a + xn_2$$

$$x = \frac{t_2 - t_1}{n_2 - n_1}$$

In Table II., column II., is found the differences between the times of the experiments 3, 4, 5, 6 of Table I., and 0·693 the time of experiment 2. In column III. is found the differences between the mean readings for the experiments 3, 4, 5, 6 of Table I., and 4·63, the mean reading of experiment 2. By dividing the members of column II. by the corresponding members of column III., we have the quotients shown in column IV. The difference between these quotients and their mean value, 0·1288, is shown in column V.

The probable error of this value for the unit is

0·0002 seconds.

We shall adopt as the final value of the chronoscope unit

0·129 seconds,

which can hardly be in error by one three-hundredth part of its amount.

The circumstances under which the chronoscope may be most suitably employed is, when differential observations alone are necessary. If a period of time be defined as the interval between the breaking and making of a current, the chronoscope will measure the difference between two such periods.

For this it is necessary to have the readings of the chronoscope for the two periods, and multiply their difference by 0·129 ; the product is the number of seconds difference between the periods. Neither of these periods of time should be less than the third of a second. The motion of the chronoscope is found, after it has once become uniform, to continue so during the entire descent of the weight.

THE BOX AND STRIKER.

13. The box (Plate IX.) used in these experiments is a 2′ cube of pine 0″·5 thick. At *M* is a hole of 6″ diameter, while the opposite side is formed of stout sacking ; *D* is a disk of wood about 9″ diameter, to which the sacking is nailed. The box is firmly bolted to a stand. This stand and the rest of the framework is also part of Professor Willis' system of apparatus. *AB* is a wire secured at one end *A*, while a cast-iron ball *B* is attached to the other end. On the surface of the ball a flat face is filed, which can be brought home against the poles of the electro-magnet *K*, the ball can therefore be released by interrupting the current. A pair of india-rubber springs is shown at *S*. The object of these springs is to to keep the disk *D* strained outwards. When the ball falls, *D* receives a blow, and an invisible ring of air is projected from *M*. Since the magnet is fixed in position, it is evident that the box will always receive the same blow. The ball which was used with the electro-magnet weighed 4·5lbs., and, with the apparatus in this form, the second series of experiments recorded in Table VI. were deduced.

In the experiments of the series which we shall first describe, a ball weighing 7·3lbs. was employed. In this series the magnet *K* was replaced by a pair of jaws, substantially the same as those shown at *DD*, Plate V. The electrical arrangement is nearly identical in the two cases. It matters little whether the ball be released by the interruption of the current, or the current be interrupted by the release of the ball. The orifice at *C* was 9″ in diameter.

THE TARGET.

15. The ring, after travelling a certain distance, impinges upon the target. This very important part of the apparatus is shown in Plate X. *O* is the back of

the target; it consists of a very slight wooden frame, 16 inches square, having paper strained upon it like a drum. The frame is suspended from the brass springs *C*, *C*.

When the target receives the ring, even at a distance of 22′, the effect of the collision is to complete an electric circuit, by means of an arrangement shown at the back of the target. One wire *H*, which runs along the lower margin of the target, is connected by *Z* with *X*. At *B* is a very slender wire spring. When the target is hanging at rest, *B* and *H* are in close proximity, though not in contact. The effect of the collision of the ring is to press *B* and *H* into contact, and thus complete the circuit. In order to complete the contact permanently, it is necessary that the target be held close to the wire *B*. For this purpose the electro-magnet *A* is fastened to the frame; one wire of *A* is attached to *B*, and the other to the battery. When the circuit has been completed by the contact of *H* and *B*, *A* becomes a magnet, and holds the target firmly in its deflected position by the keeper *E*, which is attached to the target. When the impact has once closed the circuit, it follows that the circuit remains closed.

THE MODE OF EXPERIMENTING.

16. The electrical arrangement is shown in a diagrammatic manner in Fig. 2, Plate VII. *B* is the battery (two or three Grove's cells) the circuit divides at *P*, one branch passing through the rheostat *C*, the other through the chronoscope *D*. The circuit reunites at *Q*; it divides again at *R*, one branch passing through the dropping magnet *E* (or the pair of jaws when this arrangement is used); the other branch passes through the target *H*; these branches unite *S*, and return to the battery.

The object of the rheostat is to relieve the chronoscope (which has considerable resistance) from having to transmit the entire current which is necessary for the magnet *E*. When the jaws are used, the rheostat, though convenient, is not indispensable.

17. For convenience in the manipulation of the wires, the commutator of Fig. 1, Plate VII. is used. *A* receives the pair of wires from the battery, *B* from the chronoscope, *D* from the rheostat, *E* from the target, *P* and *Q* from

the dropping magnet. X being closed, and Y being in contact with Q, the circuit is complete through the dropping magnet. Y being opened, the striker is released, and the circuit is interrupted, until by the blow of the ring on the target the circuit is closed at E. The chronoscope will, therefore, commence to move when the striker is released, and be stopped when the ring meets the target. Where the jaws replace K in Plate IX., as referred to in Article 13, the key Y is placed in contact with Q. The system is then set into action by springing open the jaws, and thus interrupting the current.

18. The experiments recorded in Table III., which will be first described, were conducted in a long passage. The box and striker remained in one position throughout the series, while the target was attached to a moveable stand.

19. The details of one set of experiments is described in order to give an average specimen of the observations, and of the degree of precision attainable. The box has an orifice of 9″ diameter, and the striker has a ball weighing 7·3lbs. These quantities remained the same throughout the series. In the particular set of experiments now described, the target was placed at a distance of 12′ from the orifice.

20. In the adjustment of the target, certain precautions must be observed. It must of course be so placed that the ring shall strike the target each time. This adjustment was made at first by filling the box with smoke, so that the course of the ring became visible. After a little practice, however, this was not found to be necessary. Currents of air arising from doors or windows must be carefully excluded, and the observers should not have crossed the path which the rings are to traverse for at least three minutes before the experiments commence. Up to a range of 20′ no difficulty was found in making the ring strike the target on almost every occasion. At the extreme range of 22′ the ring frequently missed the target, or the blow was so enfeebled that the circuit was not closed. The target should be adjusted so that the distance from the spring B to H, when the target hangs at rest, remains constant. This distance was about 0″·1, and the adjustment was made by means of a screw M, which moves the points of suspension horizontally. A string is attached to the target, the other end of which is carried behind the box.

21. The two observers are stationed behind the box; one takes charge of the chronoscope, to wind it up before each experiment, and record the result after ; the other observer manages the striking apparatus. The current is turned on by closing *X* and *Y*, Plate VII., Figs. 1 and 2. The striker is attached at *E*, Fig. 2, Plate IX. and Plate VII. The hands of the chronoscope are placed at zero, and the stud *T*, Fig. 1, Plate IV., is released; the jaws are then opened, the current is interrupted, the striker falls, the chronoscope runs, the ring darts from the box after the blow, and when it meets the target, the circuit is closed at *H*, Fig. 2, or at *E*, Fig, 1, Plate VII. The chronoscope is stopped, and its reading is recorded. The target is held by the magnet *A*, Fig. 6. This must be released before a new experiment can be made. The current is turned off at *X*, Figs. 2 and 1, Plate VII., and the target is gently detached by the string which is attached to it. The chronoscope being returned to zero, all is ready for a new experiment.

The following are the ten readings of the chronoscope when the target is at a distance of 12′.

	13·63
	13·17
	13·20
	13·20
	13·50
	13·70
	14·13
	14·50
	14·57
	14·03
Mean, ...	13·76

The probable error of the mean result is

0·11,

and corresponds to a period in time of

0·014 seconds.

22. The time indicated by 13·76 chronoscope units is composed of—

(1.) The time of descent of the striker.

(2.) The interval between the blow of the striker and the projection of the ring.

(3.) The time occupied by the ring in moving from the orifice of the box to a distance of 12′.

(4.) The time occupied by the target in closing the circuit.

Of these numbers, 1 and 2 are completely independent of the distance of the target from the orifice, and the variations of No. 4 are, doubtless, extremely minute. We shall consider these quantities to be constant in order to determine the time occupied by the ring in its journey.

DISCUSSION OF THE RESULTS.

23. The results of the experiments when the target was placed at different distances are embodied in Table III.

The probable errors of the several mean results are given in the fourth column. The magnitude of the probable error of experiment 12 amounts to nearly 2 per cent. of the whole, and is proportionally much larger than the probable error of any of the results at shorter ranges. We shall therefore omit this result from the subsequent investigation.

24. In order to compare these observations *inter se*, and for their discussion we shall employ the method of graphical construction. This method is especially suitable for observations which are not capable of extreme precision.

In Fig. 2, Plate IV., is shown a graphic projection of the observations recorded in Table III. The abscissæ of the figure are the ranges of column II. on the scale of 0″·25 to 1′. The ordinates are the corresponding numbers of column III. on the scale of 0″·25 to one chronoscope unit. A curved line is drawn freely, which passes as nearly as possible through the extremities of all the

ordinates. This curve passes very close to all the points, with the exception of No. 8 at a range of 14′. It would appear that in this case, since the probable error 0·12 is not unusual, there was some maladjustment of the apparatus. The time appears to be about

0·08 seconds

too large.

25. It is a remarkable circumstance that the curve appears at first to be slightly convex, and that the point of inflexion has an abscissa of 0″·75. This convexity is sufficiently large to show that it probably does not arise from errors of observation. If we admit that this is the case, it proves that the ring, immediately upon leaving the box, has its motion accelerated until the ring reaches a distance of 3′ from the origin, after which the motion is retarded. It does not seem easy to understand how this should be the case. It is proper to mention that the second series of experiments (Table V.), where the ring was only 6″ diameter, and the velocity of projection less than in the present case, has also been projected into a curve. This curve is concave throughout its entire length, though almost rectilinear at the commencement.

26. When the target was placed within an inch or less from the orifice, the blow of the ring had the effect of drawing the target towards the orifice. This seeming paradox is apparently analogous to the effect on a flat plate when exposed to a stream of air issuing from an orifice in a plane (Professor Willis, Cambridge "Phil. Trans.," vol. iii.).

27. Independently of other considerations, the 2′ cube must be admitted to interfere to some extent with the natural motion of the ring in air so long as the ring remains in the vicinity of the cube. We shall, therefore, neglect the motion of the ring until it has reached a point 4′ from the orifice of the box. This point we shall take as the zero of range, the epoch when the ring is at this point we shall consider the zero of time, and the velocity with which the ring passes this point we shall consider the initial velocity. Other advantages follow from this method. We eliminate the initial irregularities of the chronoscope, and also the constant portions of the observations noticed in Article 22.

28. When the range is 4′ (Experiment 3, Table III.), the mean reading of the chronoscope is 6·50. 4′ is subtracted from all the ranges, and 6·50 from

the corresponding mean readings, and thus columns II. and III., Table IV., are formed. For example, in No. 5, Table IV., we see that while the ring is moving from the chosen position to a distance of 10′, 10·2 chronoscopic units of time have elapsed.

29. These results are represented graphically in Plate VIII., Fig. 2. The abscissæ are the ranges of column II., Table IV., on the scale of 0″·25 to 1′, and the ordinates are the corresponding numbers of column III. on the scale of 0″·25 to one chronoscope unit. The curve being drawn freely through these points, the corrections are deduced which are recorded in column IV., and the corrected values are given in column V. The only correction of any significance is that of No. 5, which has been already referred to in Article 24.

The corrected values of column V. are evaluated in seconds by multiplication with the chronoscope unit (Article 11) of

0·129 seconds.

The values in time are recorded to the hundredth part of a second in column VI.

30. We shall next determine from the construction the velocity of the ring at the different points of its path.

ST is a line drawn parallel to OA at a distance of 1″·9. If the tangent PX be drawn at any point P of the curve, it can be shown that the projection XY of the intercept XZ is the velocity of the ring at the point of its path corresponding to P on the scale of 0″.25 to 1′. By this means the approximate velocities given in Table IV., column IX., have been deduced.

31. The velocity of the ring at different points of its path being approximately known, the nature of the retarding force is determined. In Fig. 1, Plate VIII., the abscissæ are the ranges of Table IV., column II., on the scale of 0″·25 to 1′, and the ordinates are the velocities of column IX. on the same scale.

It is remarkable that a straight line can be drawn passing almost exactly through the extremities of the ordinates, and by measurement is found that

$$\frac{dv}{ds} = -0{\cdot}345;$$

whence

$$v.\frac{dv}{ds} = -\ 0{\cdot}345\ .\ v.$$

It follows that the ring is retarded, as if it were acted upon by a force directly proportional to the velocity.

The value for the co-efficient which has been deduced from the geometrical construction is only to be regarded as an approximate result. A more correct value has been obtained by the method of least squares. From this we derive the following formulæ:—

$$t = 9{\cdot}016 - 6{\cdot}25 \log (27{\cdot}7 - s).$$

$$\frac{ds}{dt} = 0{\cdot}368\ (27{\cdot}7 - s).$$

$$\frac{d^2s}{dt^2} = 0{\cdot}136\ (27{\cdot}7 - s)$$

The first of these formulæ is compared with the observed results in Table IV., columns VII., VIII. It will be observed that in no case does the difference exceed 0·01 seconds. It will, however, be remembered that the observed times have already been corrected by graphical construction.

32. That the resistance may be considered to vary as the velocity can only be affirmed throughout the ranges at which the actual experiments have been made. If we assume it to hold good for higher ranges, we see that the ring could never attain the distance of 27·7 feet.

33. The series of experiments which are recorded in Tables V. and VI. have been conducted with certain variations from the series we have been considering. The diameter of the orifice from which the ring issues is in this case six inches; the initial velocity is 8·8 feet per second; the striker weighs 4·5lbs., and was released from an electro-magnet. The experiments were conducted in a room instead of the long passage already referred to.

34. The graphical construction suggested a law for the resisting force of the air which is proportional to the velocity, but it must not be inferred that the experiments can be reconciled to no other hypothesis. Let us seek whether

the experiments of Table IV. cannot be explained on the supposition that the resistance of the air is a force which varies as the square of the velocity.

We then have an equation of the form

$$\log v + As + B = 0$$

where v is the velocity, s the range, A, B, two constants to be determined.

Substituting the approximate velocities of Column IX., Table IV., and reducing, we find ultimately

$$t = 1{\cdot}855\ (\,(1{\cdot}05212)^s - 1).$$

It will be seen that this formula represents the experiments almost, if not quite, as well as the formula of Column VII.

It is believed that by some modifications of the apparatus, particularly with reference to the target, increased accuracy may be obtained in a future series of experiments. The influence of temperature and barometric pressure is at present undetected. It should, however, be remarked that each of the series of experiments which have been discussed were completely performed on the same day.

TABLE I.

OBSERVATIONS *connecting the Indications of the Chronoscope with known Periods of Time.*

I.	II.	III.	IV.
Reference Number.	Known Period of Time.	Mean of Chronoscope Readings.	Probable Error of Mean.
1	0 320 secs.	1·57	0·04
2	0·693 ,,	4·63	0·03
3	2·080 ,,	15·31	0·03
4	2·454 ,,	18·43	0·03
5	3·094 ,,	23·30	0·06
6	3·467 ,,	26·10	0·06

TABLE II.

THE *Value in Time of the Chronoscope Unit deduced from the Observations of Table I., and Comparison of the several Results with their mean Value.*

I.	II.	III.	IV.	V.
Reference Number.	Time $t_2 - t_1$.	Chronoscope Reading $n_2 - n_1$.	Value of Unit.	Difference from Mean 0·1288.
1	1·387 secs.	10·67	0·1289 secs.	0·0001
2	1·761 ,,	13·80	0·1276 ,,	0·0012
3	2·401 ,,	18·67	0·1286 ,,	0·0002
4	2·744 ,,	21·47	0·1292 ,,	0·0004

TABLE III.

OBSERVATIONS *connecting the Space travelled by a Vortex Ring 9″ diameter in Air, with the corresponding Mean Values of the Readings of the Chronoscope.*

I.	II.	III.	IV.
Reference Number.	"Range" or Distance of Target from Orifice.	Mean of Chronoscope Readings.	Probable Error of Mean.
1	2 inches.	3·80	0·01
2	2 feet.	5·29	0·03
3	4 ,,	6·50	0·04
4	6 ,,	8·12	0·03
5	8 ,,	9·88	0·04
6	10 ,,	11·90	0·08
7	12 ,,	13·76	0·11
8	14 ,,	16·68	0·12
9	16 ,,	18·49	0·12
10	18 ,,	21·01	0·12
11	20 ,,	24·74	0·22
12	22 ,,	27·71	0·52

TABLE IV.

A VORTEX *Ring of* 9″ *Diameter moving in Air with an Initial Velocity of* 10·2 *per second.*

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	XI.
Reference Number.	Distance of Target from Orifice, minus 4 feet S.	Mean Corresponding Reading of Chronoscope minus 6·5.	Correction deduced by Graphical Construction.	Corrected Value of Mean Chronoscopic Reading.	Equivalent Time in Seconds.	Time calculated by Formula $t = 9{\cdot}016 - 6{\cdot}25 \log (27{\cdot}7 - s)$.	Difference between Calculated and Observed Time.	Approximate Velocity deduced from Graphical Construction.	True Velocity calculated by Formula $\frac{ds}{dt} = 0{\cdot}368\,(27{\cdot}7 - s)$.	Retarding Force calculated by Formula $-\frac{d^2s}{dt^2} = 0{\cdot}136\,(27{\cdot}7 - s)$.
1	2 feet.	1·6	0·0	1·6	0·21 secs.	0·20 secs.	− 0·01	9·3	9·5	3·5
2	4 ,,	3·4	− 0·1	3·3	0·43 ,,	0·42 ,,	− 0·01	8·4	8·7	3·2
3	6 ,,	5·4	− 0·2	5·2	0·67 ,,	0·66 ,,	− 0·01	7·7	8·0	3·0
4	8 ,,	7·3	+ 0·0	7·3	0·93 ,,	0·92 ,,	− 0·01	7·3	7·2	2·7
5	10 ,,	10·2	− 0·7 !	9·5	1·23 ,,	1·22 ,,	− 0·01	6·4	6·5	2·4
6	12 ,,	12·0	− 0·1	11·9	1·54 ,,	1·54 ,,	0·00	5·9	5·8	2·1
7	14 ,,	14·5	+ 0·3	14·8	1·91 ,,	1·91 ,,	0·00	5·2	5·0	1·9
8	16 ,,	18·2	− 0·1	18·1	2·33 ,,	2·34 ,,	+ 0·01	4·4	4·3	1·6

TABLE V.

OBSERVATIONS *connecting the Space travelled by a Vortex Ring 6″ Diameter with the corresponding Mean Readings of the Chronoscope.*

Reference Number.	Distance of Target from Orifice.	Mean of Chronoscope Readings.	Probable Error of Mean.
1	0.25 feet	5·73	0·03
2	0.5 ,,	5·88	0·03
3	1·0 ,,	6·33	0·03
4	2 ,,	7·14	0·05
5	4 ,,	9·04	0·06
6	6 ,,	10·85	0·06
7	8 ,,	12·89	0·06
8	10 ,,	15·52	0·06
9	12 ,,	18·50	0·08
10	14 ,,	21·59	0·11

TABLE VI.

A VORTEX *Ring 6″ Diameter moving in Air with an Initial Velocity of* 8·8 *feet per second.*

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	XI.
Reference Number.	Distance of Target from Orifice, minus 4 feet S.	Mean Corresponding Reading of Chronoscope minus 9·0.	Correction deduced by Graphical Construction.	Corrected Value of Mean Chronoscopic Reading.	Equivalent Time in Seconds.	Time calculated by Formula. $t = 5{\cdot}443 - 4{\cdot}425 \log (17 - s)$.	Difference between Calculated and Observed Time.	Approximate Velocity deduced from Graphical Construction.	True Velocity calculated by Formula $\frac{ds}{dt} = 0{\cdot}518\ (17 - s)$.	Retarding Force calculated by Formula $-\frac{d^2s}{dt^2} = 0{\cdot}271\ (17 - s)$
1	2 feet.	1·8	0·0	1·8	0·23 sec.	0·23 secs.	0·00 secs.	7·7	7·8	4·1
2	4 ,,	3·9	0·0	3·9	0·50 ,,	0·51 ,,	+0·01 ,,	6·4	6·7	3·5
3	6 ,,	6·5	0·0	6·5	0·84 ,,	0·83 ,,	−0·01 ,,	5·5	5·7	3·0
4	8 ,,	9·5	0·0	9·5	1·23 ,,	1·22 ,,	−0·01 ,,	4·6	4·7	2·4
5	10 ,,	12·6	0·5	13·1	1·69 ,,	1·70 ,,	+0·01 ,,	3·5	3·6	1·9

FIG. 1.

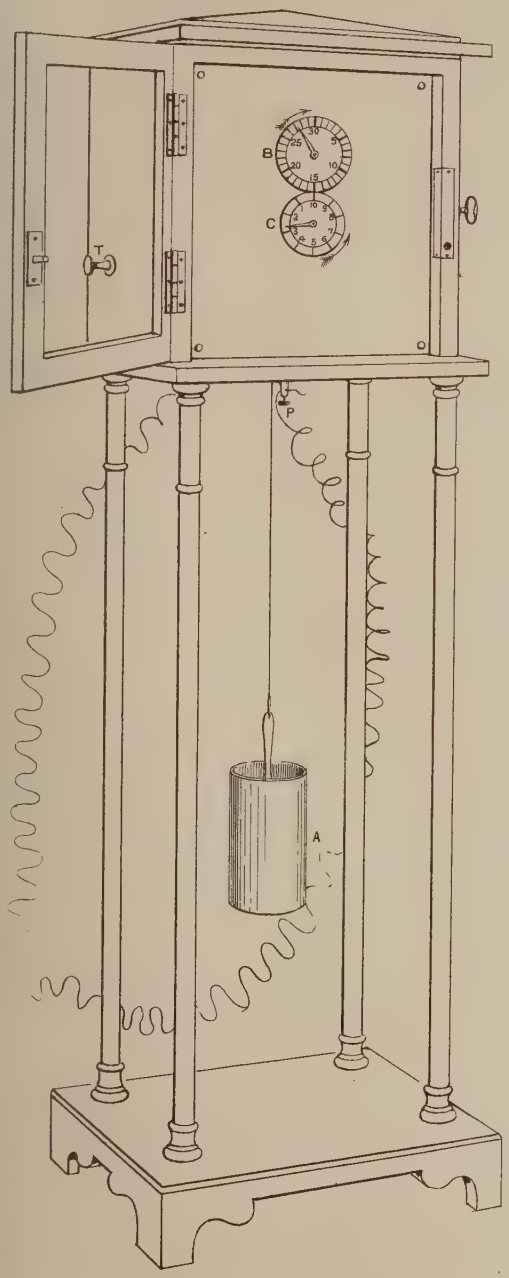
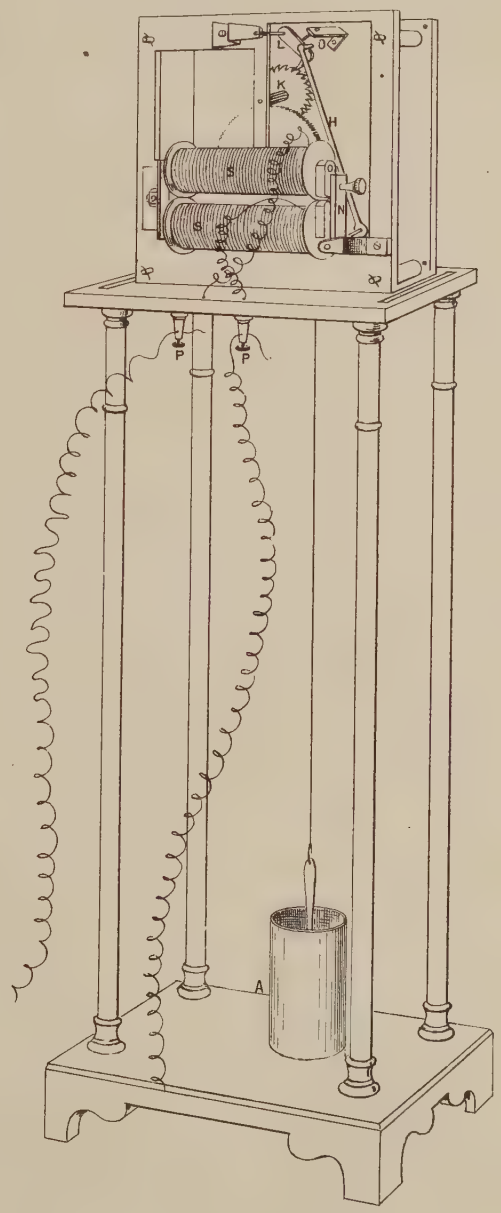


FIG. 2.



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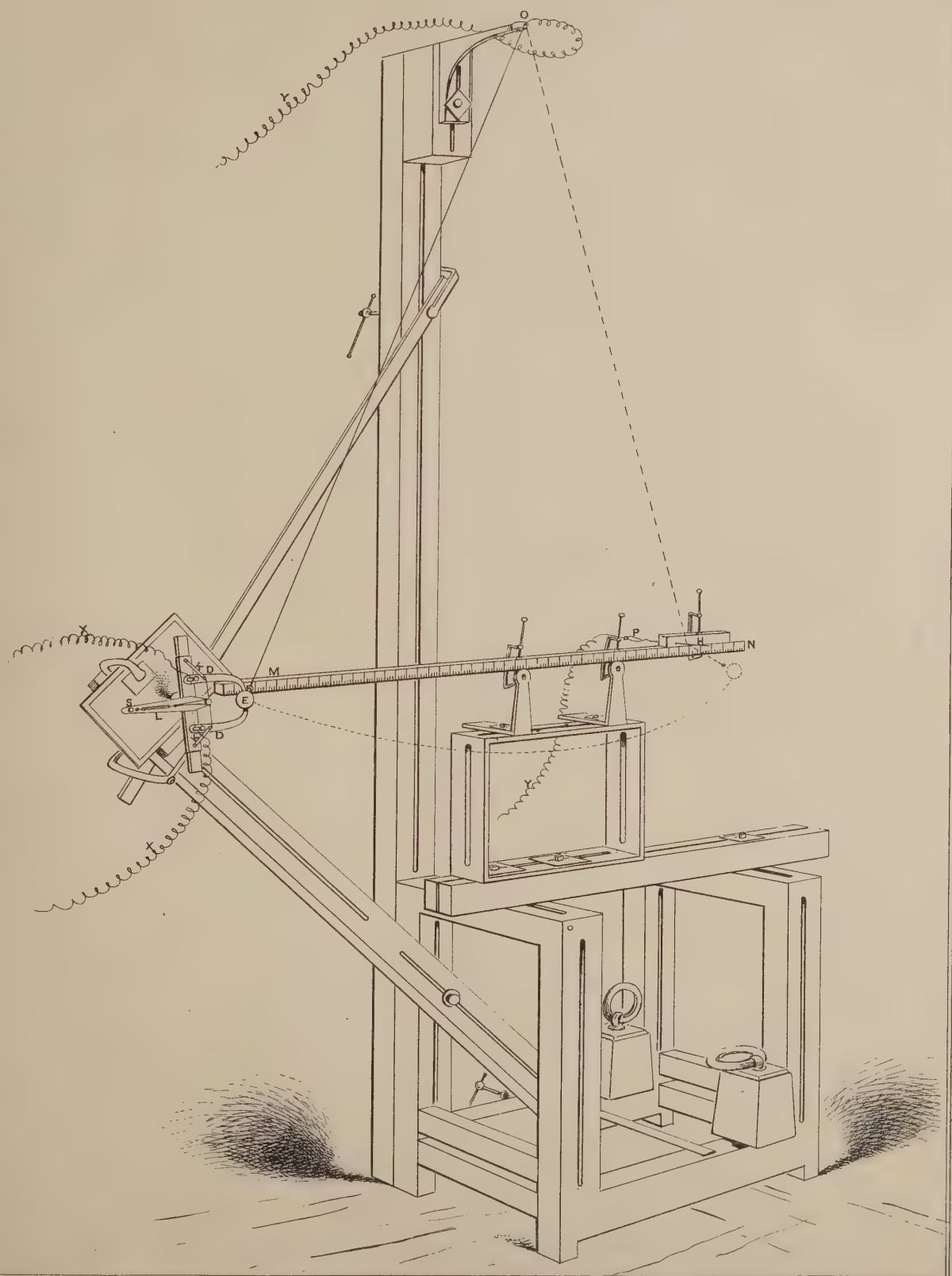

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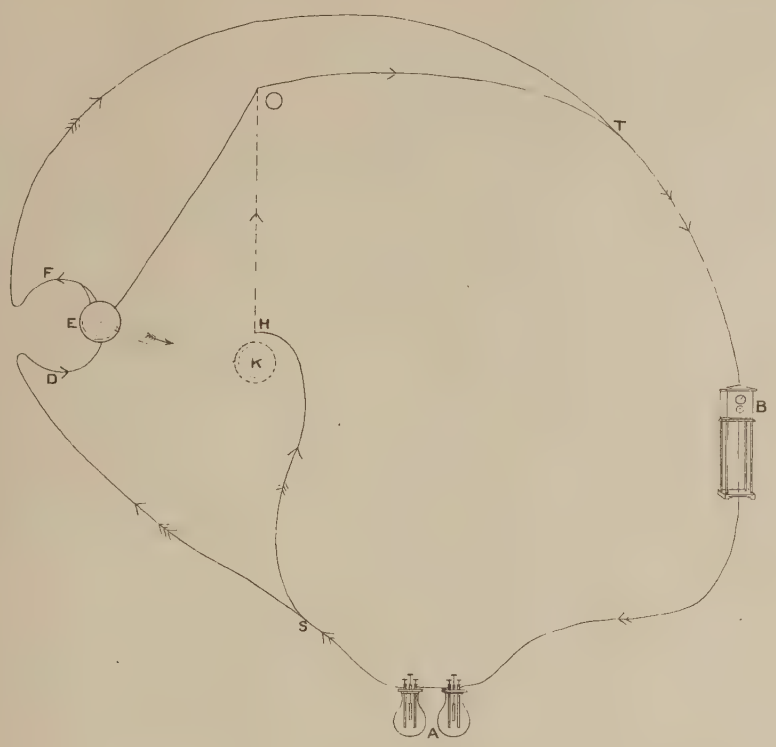

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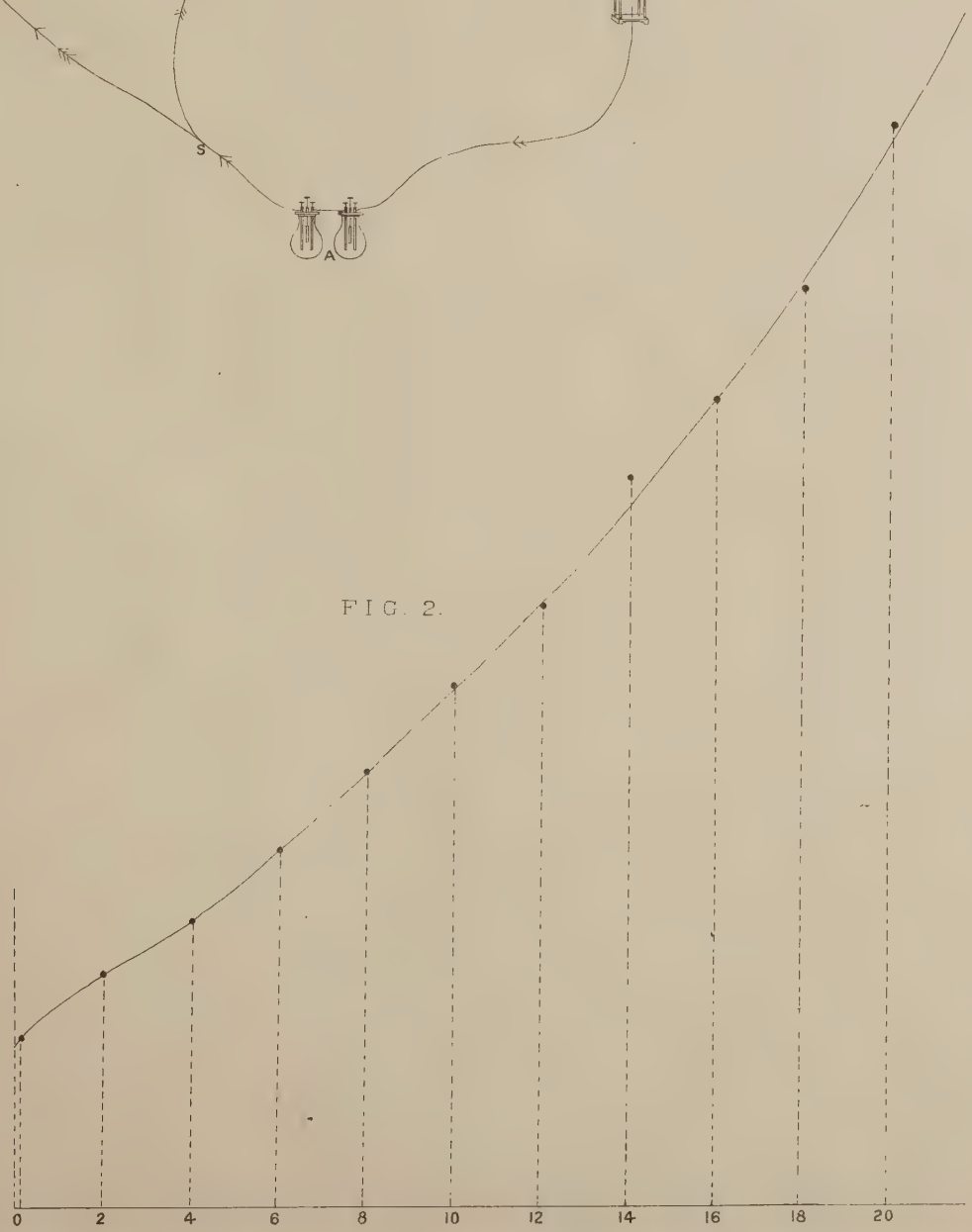

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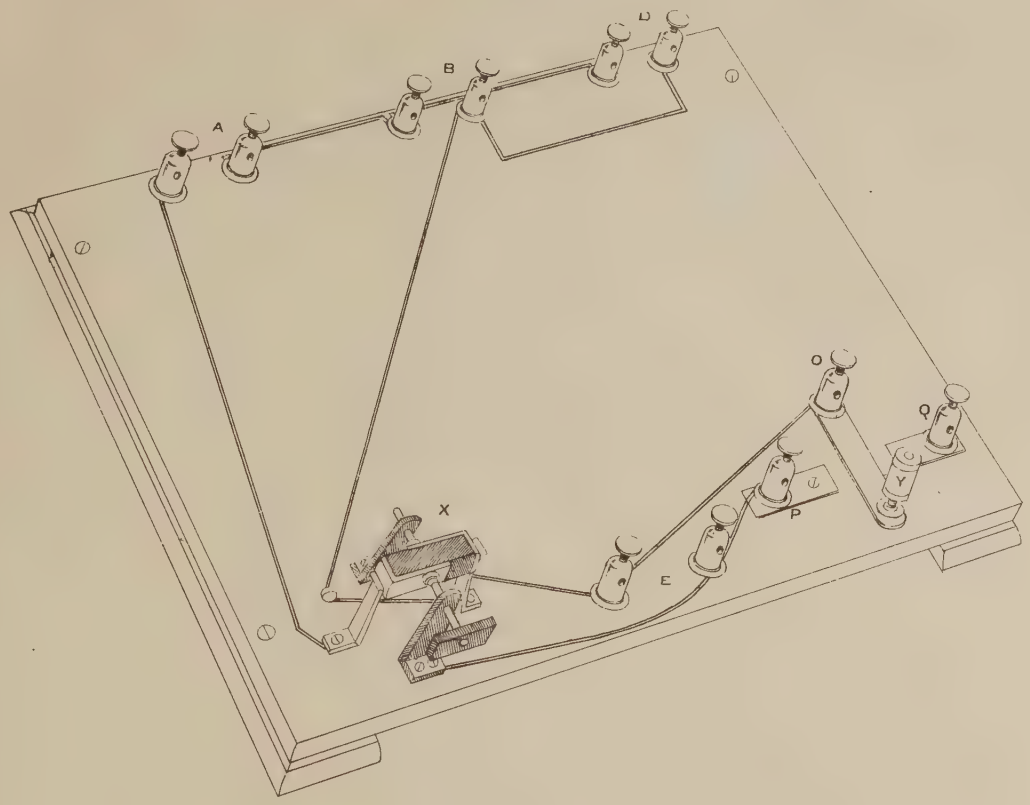
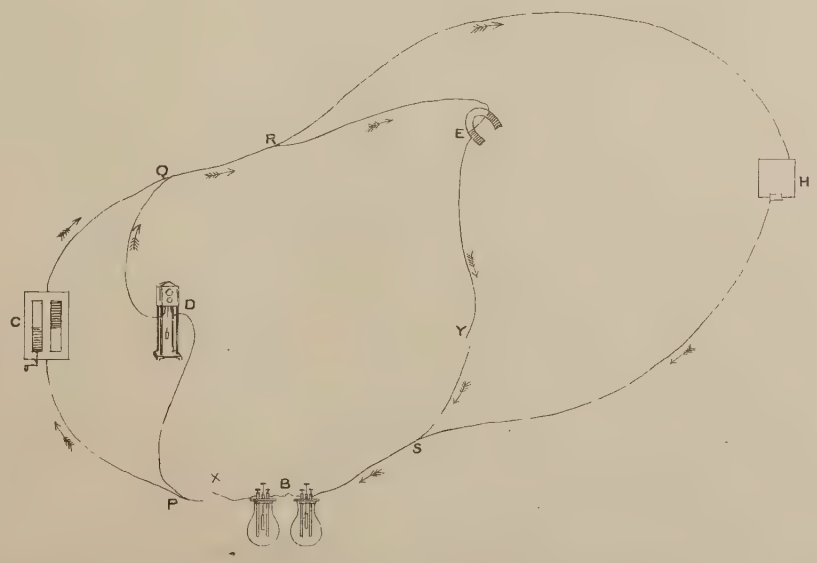


FIG. 2.



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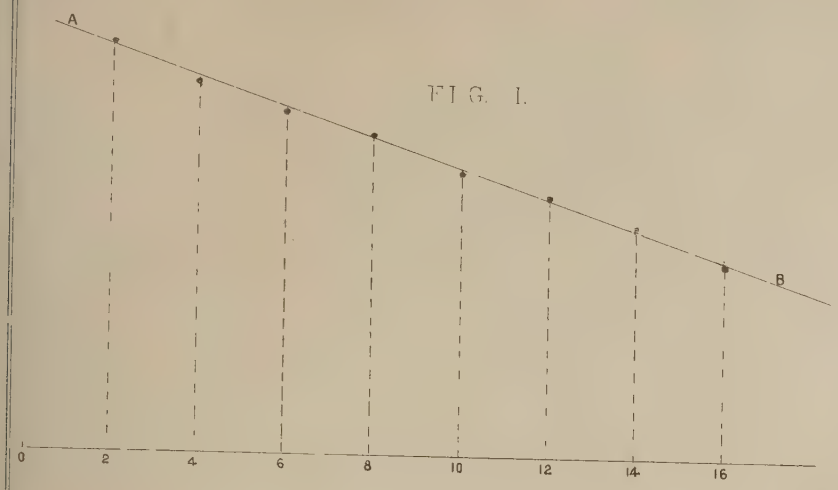

FIG. 1.

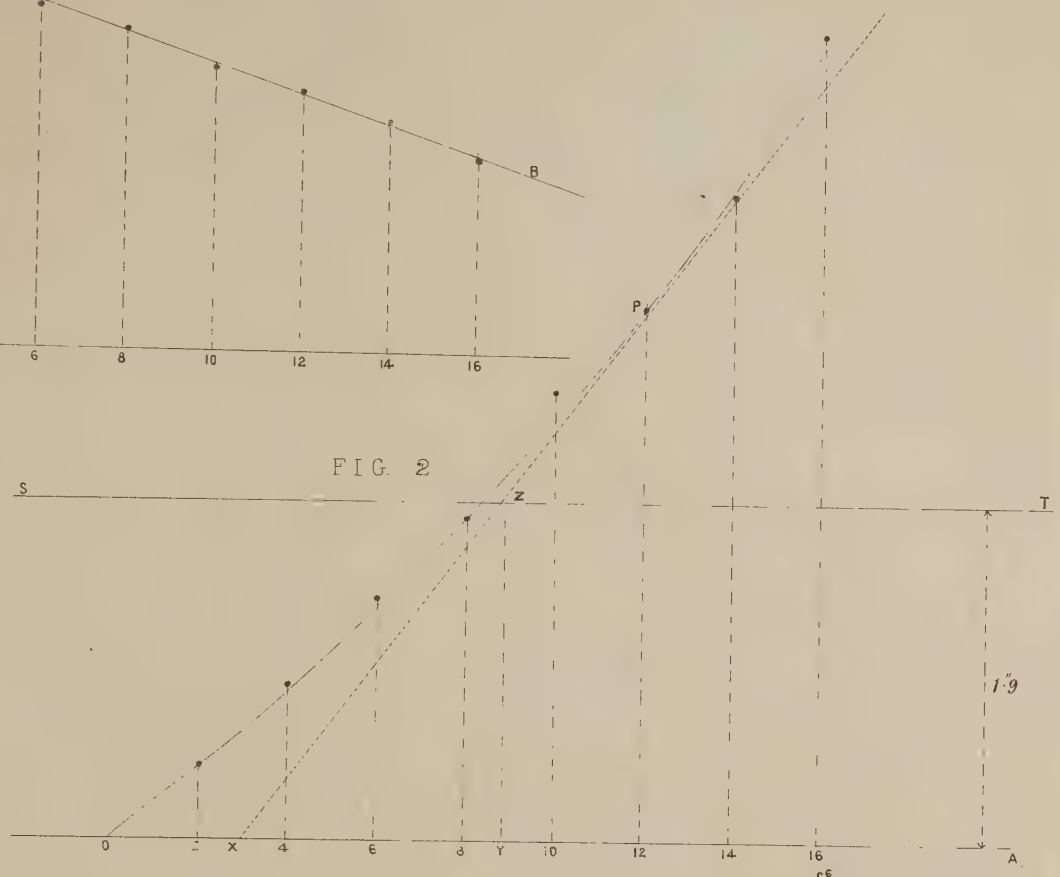

FIG. 2

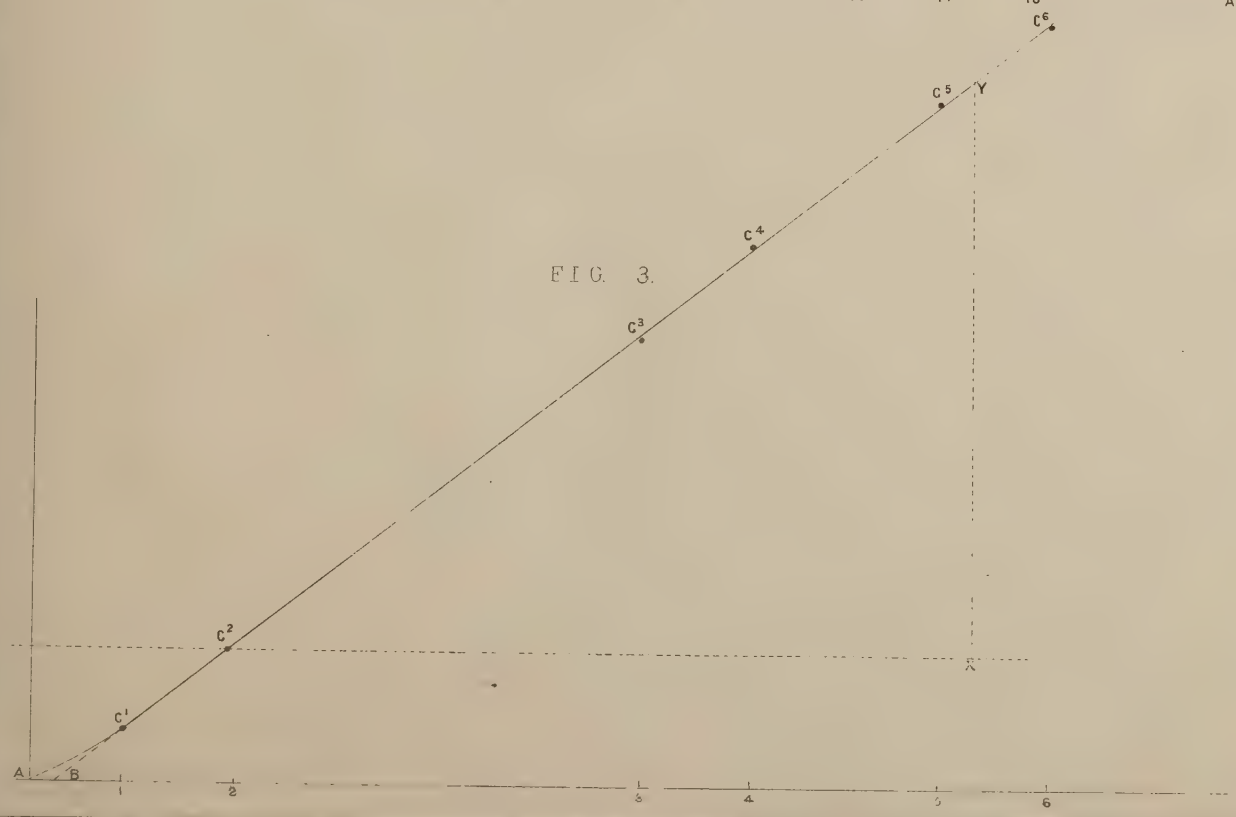

FIG. 3.

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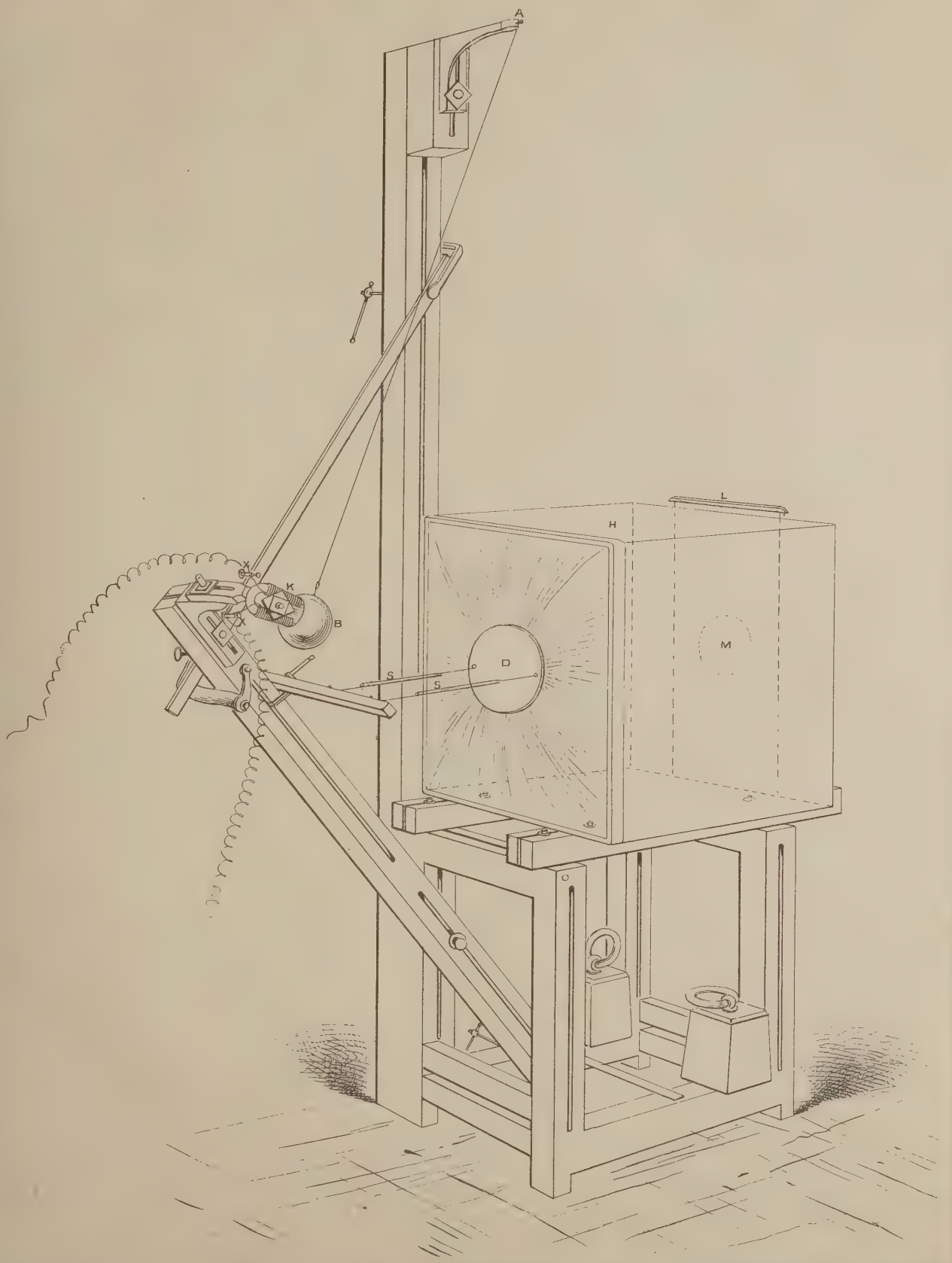

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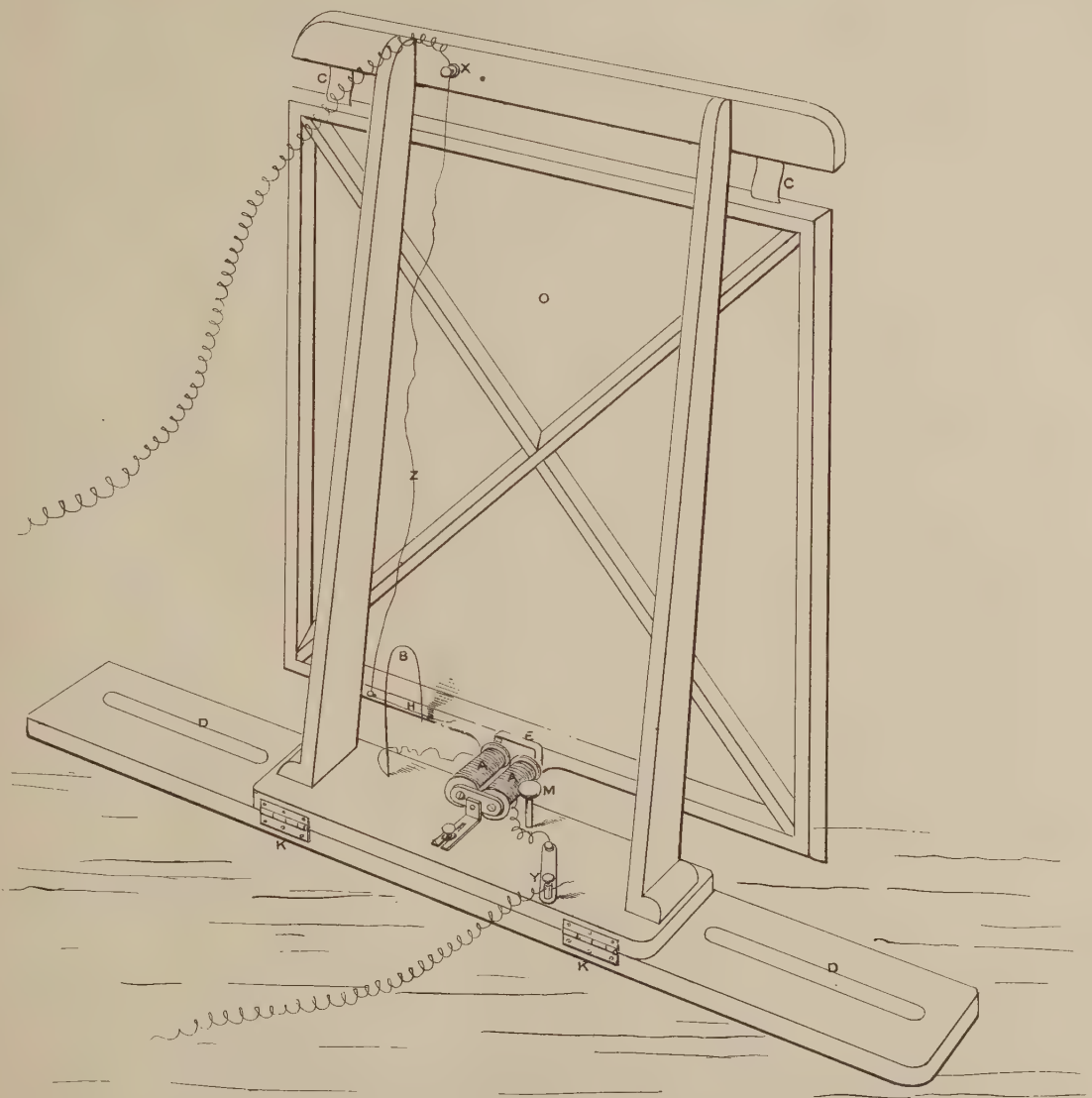

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III.—*The Theory of Screws : a Geometrical Study of the Kinematics, Equilibrium, and small Oscillations of a Rigid Body.* By Robert Stawell Ball, LL.D., M.R.I.A.

Read November 13th, 1871.

TABLE OF CONTENTS.

Preface.
I. Introduction, Articles 1 to 5.
II. The Cylindroid, Articles 6 to 17.
III. Reciprocal Screws, Articles 18 to 22.
IV. The Pectenoid, Articles 23 to 27.
V. Restraint and Freedom, Articles 28 to 49.
VI. Conjugate Screws of Energy, Articles 50 to 57.
VII. The six Degrees of Freedom :—
a. One Degree of Freedom, Articles 58 to 60.
b. Two Degrees of Freedom, Articles 61 to 73.
c. Three Degrees of Freedom, Articles 74 to 88.
d. Four Degrees of Freedom, Articles 89 to 93.
e. Five Degrees of Freedom, Articles 94 to 96.
f. Perfect Freedom, Article 97.
Glossary.
Description of Plates :—
Plate XI.—A Model of the Cylindroid.
Plate XII.—A Model of the Pectenoid.

PREFACE.

Though the second title of this Paper has a familiar sound, yet it is believed that the manner in which the problems therein mentioned are discussed is chiefly, if not entirely new. The object of the Paper may be thus stated :—To present certain problems in the mechanics of a rigid body under the most general conditions, not by means of equations, but by distinct physical or geometrical ideas. The writer has constantly had before him the beautiful Memoir of

Poinsot,* the acknowledgments to which will be made at the proper place; and it has been the ambition of the writer to accomplish for the problems which he has undertaken to solve what Poinsot has done so successfully for the free rotation of a body about a fixed point. Two theorems—one kinematical, the other mechanical—discovered by Poinsot, form the elements from which, as fundamental axioms, the subject has been developed. With the important exception of these theorems, it is believed that all the investigations now presented are novel, at least in form.

It will be well, therefore, at once to call attention to the special features of the method of treatment which has been adopted, and to enunciate a few of the results.

The co-ordinates by which the position of a rigid body in space is usually determined consist partly of angular, and partly of linear magnitudes. In the present method all the co-ordinates employed are homogeneous quantities, denominated "twists" (Art. 1). Thus the position of a rigid body in space is defined by six twists about six given screws.

The importance of such a system of co-ordinates becomes manifest, when compared with the method of compounding and resolving forces adopted in the Paper. Any system of forces may be compounded into a "wrench" (Art. 2), and a wrench about any screw may be resolved into six wrenches about six given screws.

The analogy between twists and wrenches leads to a remarkable property, called the Theorem of Reciprocal Screws (Art. 18). The method of reciprocal screws may perhaps be found as fertile in the mechanics of a rigid body as the method of reciprocal polars is fertile in geometry.

The theory of twists gives a perfectly definite (though completely general) geometrical idea of the latitude enjoyed by a body in any given case of freedom.

The theory of reciprocal screws gives a definite physical idea of the wrenches produced by the reactions of a given system of restraints, and of the screws about which external wrenches are permissible, without disturbing the equilibrium of the body.

* Théorie nouvelle de la Rotation des Corps, Poinsot's Éléments de Statique, Dixième édition, p. 303.

The cylindroid is a surface which generalizes the parallelogram of velocity and of force into a remarkable theorem for compounding any two twists communicated to a rigid body, or any two wrenches applied to the body. The theorem that twists and wrenches are compounded by the same laws is fundamental in the present investigation.

The pectenoid is a surface which solves the kinematical problem of communicating seven twists to a body about seven given screws, so that after the last twist the body shall occupy the same position as it did before the first. The same surface determines seven wrenches about seven given screws, which equilibrate when applied to a free rigid body.

A remarkable property of a conservative system of forces is concisely expressed by the aid of the theory of reciprocal screws.

The small oscillations of a rigid body for each degree of freedom are shown to be compounded of simple harmonic screw vibrations about certain harmonic screws, and all the circumstances of the motion are determined.

I.

INTRODUCTION.

1. It will be necessary to define the special senses which are attached to certain words used in this Paper. The words will be explained as occasion arises; and in the glossary at the end references to the Articles which contain the explanations will be found.

A *screw* is a straight line in space with which a definite linear magnitude, termed the *pitch*, is associated; the *pitch* may have any value from $+\infty$ to $-\infty$.

A body is said to receive a *twist* about a screw when it is rotated around the screw through the *angle of twist*, and is simultaneously translated parallel to the screw, through a distance equal to the product of the pitch, and the *angle of twist* expressed in circular measure.

If the screw be conceived placed along the axis round which the hands of a watch turn, and if the twist be in the direction in which the hands of the watch move, then, for positive pitch, the translation will be from the front of the watch to the back; for negative pitch, the translation will be from the back

of the watch to the front. If in this statement we interchange the words positive and negative, we have the case where the direction of the twist is opposite to the motion of the hands.

If the pitch be zero, the twist is a pure rotation; if the pitch be infinite, the twist is a pure translation.

The motion of a body implied by the word twist is the motion of a nut upon an ordinary screw.

2. A *wrench* about a screw denotes a force (the *wrenching force*), and a couple (the *wrenching couple*). The force is directed along the screw, the axis of the couple is parallel to the screw, and the moment of the couple is the product of the *wrenching* force and the pitch of the screw.

If the pitch be positive, and the force be directed from the front of a watch to the back, then the couple tends to turn the hands round in the proper direction.

If the pitch be zero, the wrench is a force only; if the pitch be infinite, the wrench is a couple only.

3. Two general theorems, due to Poinsot,* may now be stated in the following words:—

Any system of forces acting upon a rigid body can be compounded into a wrench about a certain screw.

A body can be transferred from one position in space to any other position in space by a twist about a certain screw.

4. In the present Paper, small angles of twist only are considered, as the restriction is admissible in questions relating to equilibrium or small oscillations. One great advantage thus gained is, that the *order* in which a series of twists about different screws are communicated to a rigid body has no effect upon the final result; this is an immediate consequence of the principle of the superposition of small movements.

5. Whatever be the motion of a body in space, the body is at any instant twisting about a certain screw, which we shall always call the *instantaneous screw*. The angular velocity with which the angle of twist changes is the *twist velocity*.

If a body have twist velocity (not necessarily infinitely small) about each of

* Éléments de Statique, Dixième Edition, pp. 48 and 310.

several screws, the resulting instantaneous screw and twist velocity is determined by compounding the several small twists performed in an infinitely small time.

II.

THE CYLINDROID.

6. A body receives twists about three screws, A, B, C, situated anywhere in space, the angles of twist being α, β, γ, respectively ; under what conditions will the body, after the last twist, occupy the same position in space as it did before the first ? To solve this problem we require the aid of a conoidal cubic surface, which has been called the cylindroid (at the suggestion of Professor Cayley). The generating lines of the cylindroid are conceived to be screws, the distribution of pitch upon which is regulated by a certain law. When the three screws A, B, C, belong to a cylindroid, and when each of the angles α, β, γ, is proportional to the sine of the angle between the two non-corresponding screws, then the body, after the last displacement, will occupy the same position as it did before the first.

7. The problem may also be stated thus : To compound twists about two given screws, so as to ascertain the single screw, a twist about which would produce the same effect. We shall first examine what appears to be a special case, and then we shall prove the solution to be general.

Take as axes of x and y, two screws intersecting at right angles, whose pitches are $p + m$, and $p - m$; a body is twisted about each of these screws through angles, $\omega \cos \theta$, $\omega \sin \theta$, respectively. The corresponding translations parallel to the axes of x and y, are $(p + m)\, \omega \cos \theta$, and $(p - m)\, \omega \sin \theta$. The resultant of the translations may be resolved into two components, of which $(p + m \cos 2\theta)\, \omega$ is parallel to the resultant of the rotations, and $m\, \omega \sin 2\theta$ is perpendicular to the same line. The latter component has merely the effect of transferring the resultant of the rotations to a distance $m \sin 2\theta$, the resultant moving parallel to itself in a plane normal to the component of the translation $m\omega \sin 2\theta$ (Poinsot, p. 306). The two original twists about the axes of x and y are therefore compounded into a single twist ω, about a screw whose pitch is $p + m \cos 2\theta$.

The position of the screw is defined by the equations

$$y = x \tan \theta,$$
$$z = m \sin 2\theta.$$

Eliminating θ, we have

$$z(x^2 + y^2) - 2mxy = 0.$$

This is the equation of the cylindroid.

8. We shall now show that a cylindroid can be described so as to contain any two screws. When a cylindroid is said to contain a screw, it is not only meant that the screw is one of the generators of the surface, but that the pitch of the screw is identical with the pitch appropriate to the generator with which the screw coincides.

Let the pitches of the screws A, B, be a, b, the length of the common perpendicular be κ, and the angle between the two screws be θ. We shall show that by a proper selection of the origin, the axes, and the constants p and m, a cylindroid can be found which contains A and B. If θ_1, θ_2 be the angles which define the positions of two screws upon the cylindroid, and if z_1, z_2, be the corresponding values of z, we have the equations,

$$a = p + m \cos 2\theta_1, \quad z_1 = m \sin 2\theta_1, \quad \theta = \theta_1 - \theta_2,$$
$$b = p + m \cos 2\theta_2, \quad z_2 = m \sin 2\theta_2 \quad \kappa = z_1 - z_2.$$

From these we deduce

$$m = \frac{\sqrt{\kappa^2 + (a-b)^2}}{2 \sin \theta}, \quad p = \frac{1}{2}(a + b - \kappa \cot \theta);$$

$$\theta_1 = \frac{\theta}{2} + \frac{1}{2} \tan^{-1} \frac{b-a}{\kappa}, \quad z_1 = \frac{1}{2}(b-a) \cot \theta + \frac{\kappa}{2},$$

with similar values for θ_2 and z_2, whence it is obvious that the cylindroid is determined, and that the solution is unique.

It will often be convenient to denote by (A, B) the cylindroid which contains the two screws A and B.

9. It is remarkable that the equation,

$$z\,(x^2 + y^2) - 2\,mxy = 0$$

contains but a single parameter m, and that, consequently, all cylindroids are similar surfaces, differing only in absolute magnitude. The constant p which occurs in the expression for the pitch

$$p + m \cos 2\theta,$$

has no connexion with the form or magnitude of the surface. This amounts to the assertion that the fundamental property of the cylindroid is not affected by adding any positive or negative constant to the pitches of all the screws contained upon the surface. This is a very remarkable property (see Arts. 82, 88).

10. The fundamental property of the cylindroid is thus proved. Take any three screws, A, B, C, upon the surface of which let θ_1, θ_2, θ_3 be the defining angles, and let the body be twisted about these screws, through angles α, β, γ. It is manifest (Art. 7) that each of these twists can be decomposed into two twists about the axes of x and y. Thus the twist α about A is decomposed into the twists, $\alpha \cos \theta_1$ and $\alpha \sin \theta_1$, about the axes of x and y. The entire effect of the three twists is, therefore, reduced to two rotations around the axes of x and y, through angles equal to

$$\alpha \cos \theta_1 + \beta \cos \theta_2 + \gamma \cos \theta_3$$

and

$$\alpha \sin \theta_1 + \beta \sin \theta_2 + \gamma \sin \theta_3,$$

and two translations parallel to the axes of x and y through distances

$$(p + m)\,(\alpha \cos \theta_1 + \beta \cos \theta_2 + \gamma \cos \theta_3),$$

and

$$(p - m)\,(\alpha \sin \theta_1 + \beta \sin \theta_2 + \gamma \sin \theta_3).$$

These four quantities vanish if

$$\frac{\alpha}{\sin\,(\theta_2 - \theta_3)} = \frac{\beta}{\sin\,(\theta_3 - \theta_1)} = \frac{\gamma}{\sin\,(\theta_1 - \theta_2)},$$

which proves the theorem of the cylindroid.

11. The cylindroid affords the means of compounding any two twists

which a rigid body can receive, by a rule as simple as that which the parallelogram of velocities affords for the composition of two displacements of a particle in a plane. The rule may be thus expressed:—Draw the cylindroid which contains the screws A, B, about which the body receives twists α, β. Select the generator, C, which makes angles with A, B, whose sines are in the inverse proportion of α to β; then C is the resultant screw. The pitch is determined by the position of C on the cylindroid, and the resultant twist is found by compounding α and β like two forces in a plane inclined to each other at the angle between A and B. Any number of screw displacements can be compounded by taking two first and then a third, &c. Throughout this Paper we shall postulate that a cylindroid can always be described through a given pair of screws.

12. The theorem of the cylindroid includes, as particular cases, the well-known rules for the composition of two translations parallel to given lines, or of two small rotations about intersecting axes. If the parameter m be zero, the cylindroid reduces to a plane, and the pitches of all the screws become equal to the constant p. When p is infinite, we have the theorem for translations; when p is zero, we have the theorem for rotations. We see, therefore, that, so far as the composition of two displacements is concerned, the plane can only be regarded as a degraded form of the cylindroid, from which the most essential feature (the distribution of pitch) has disappeared.

13. When first the author became acquainted with the cylindroid, he wrote to Professor Cayley on the subject. The object of writing was to ascertain whether this surface had been previously described, or its kinematical properties pointed out. Professor Cayley replied with that kindness which always characterizes him, and an extract from his letter is given here. "I do not know that there is anything *specially* on the surface

$$z(x^2 + y^2) - 2mxy = 0.$$

The equation may be written

$$(z - mw)\,(x + y)^2 + (z + mw)\,(x - y)^2 = 0 \quad (w = 1),$$

which is of the standard form

$$x^2z + y^2w = 0\,;$$

so that we have $x = 0$, $y = 0$, as a nodal line, and $z = 0$, $w = 0$ (line at infinity in the plane, $z = 0$) as a simple line. Any section is a cubic curve, having a nodal point where the plane meets $x = 0$, $y = 0$, and an asymptote parallel to the intersection of the plane by $z = 0$; so that, starting with such cubic, any plane $z = c$ will meet the cubic in two finite points, say A, B; and, if it also meets the line $x = 0$, $y = 0$, in a point C, then the two generating lines in the plane in question are CA, CB. In particular, taking the plane of the section through a generating line, the cubic is replaced by a line and conic, and, taking a particular position, we have the following simple construction. Cut the cylinder $x^2 + y^2 = m^2$ in an ellipse by the plane $z = x$, and consider the line $x = 0$, $y = -m$, of the cylinder: then, if any plane $z = c$ cuts the ellipse in the points A, B, and the line in C, the generating lines in the plane in question are CA, CB.

"There are evidently two positions of the plane, for each of which the points AB coincide, and the whole surface is included between these planes."

14. A model of the cylindroid represented in Plate XI. has been exhibited to the Academy. A duplicate of this model is deposited in the collection of the London Mathematical Society, 22, Albemarle-street, London. In order to to realize from the model the actual form of the surface, the diameter of the central cylinder must be supposed evanescent, and the cylinder be replaced by the nodal line. The radiating wires, which form the generating lines, are to be supposed to extend to infinity. The parameter m (Art. 9) in the model is 66 millimeters. The distribution of pitch upon the generating lines is shown by colouring a length of 66 cos 2θ millimeters upon each wire. Positive pitches have been coloured green, and negative have been coloured red. Any constant p may be added (algebraically) to the pitches of all the screws, without affecting the fundamental property (Art. 6) of the surface.

15. If a body receive two twist velocities about two given screws, the two twist velocities may be compounded into a twist velocity about one screw upon the cylindroid, determined by the two given screws. This will be easily seen by compounding the twists which the twist velocities produce in an infinitely short time.

16. The fundamental property of the cylindroid with reference to forces may be thus stated :—

If three screws be taken upon a cylindroid, and three wrenches act about these screws upon a rigid body, then these wrenches will be in equilibrium, provided each of the wrenching forces is proportional to the sine of the angle between the two remaining screws.

The demonstrations given in Arts. 7 and 10 apply to prove the present theorem also. It is merely necessary, in the former demonstrations, to replace the words twist, rotation, translation, by the words wrench, force, couple, respectively.

17. Poinsot has pointed out* the analogy between the composition of rotations about axes, and the composition of forces, and the corresponding analogy between the composition of translations, and the composition of couples. We have now proved the general analogy between the composition of twists and the composition of wrenches. This includes, as particular cases, the two analogies indicated by Poinsot.

III.

RECIPROCAL SCREWS.

18. If a body only free to twist about a screw A (of pitch a) be in equilibrium, though acted upon by a wrench about the screw B (of pitch b), then conversely a body only free to twist about the screw B will be in equilibrium, though acted upon by a wrench about the screw A.

A and B are called *reciprocal screws* when this relation is fulfilled.

Let κ be the length of the common perpendicular to the two screws.

Let ϕ be the angle between the two screws.

In order to avoid ambiguity with reference to ϕ and its supplement, ϕ is the angle between those portions of A and B that follow the direction of a right-handed screw along the common perpendicular.

Let P be the wrenching force about B (Fig. 1). A is in the plane of the paper ; B is above the paper ; whence ϕ is the angle indicated in the figure. The wrenching force P is resolved into $P \cos \phi$, $P \sin \phi$; the former may

* p. 306, loc. cit.

be transferred to A, with the introduction of a neutralized couple. The component $P \sin \phi$ produces a negative couple about A, whose moment is $P \kappa \sin \phi$. The wrenching couple $b P$ is resolved into $b P \sin \phi$, which is neutralized, and $b P \cos \phi$, which may be directly transferred to A. Thus the total effect of B upon A consists of the force $P \cos \phi$ along A, and the couple whose moment is—

$$b \cos \phi \, P - \kappa \sin \phi \, P.$$

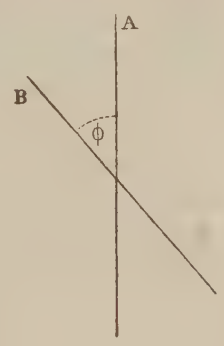


Fig. 1.

This couple and force compound into a wrench about a screw, whose direction is the line A, and of which the pitch is—

$$b - \kappa \tan \phi.$$

Under what conditions will this wrench be unable to disturb the equilibrium of a body only free to twist about A? If the body be twisted through a small angle ω, the virtual moment is—

$$P\,[(a+b) \cos \phi - \kappa \sin \phi]\, \omega\,;$$

and therefore the wrench about B will be inoperative upon a body only free to twist about A, if

$$(a+b) \cos \phi - \kappa \sin \phi = O.$$

The symmetry of this equation shows that the same condition must be fulfilled when a wrench about A is unable to disturb a body only free to twist about B. Whence the theorem of the reciprocal screws.

19. Intersecting screws are reciprocal when the sum of their pitches is zero. Screws at right angles are reciprocal either when they intersect or when the pitches of one or both of the screws is infinite.

20. The importance of the theorem of the reciprocal screws is due to the step which this theorem often enables us to take from the kinematical relations of a rigid body to its mechanical relations, and *vice versâ*. In the mode of studying mechanics adopted in the present Paper, the doctrine of reciprocal screws is of fundamental importance.

21. If a screw X be reciprocal to two given screws A and B, then X is reciprocal to every screw on the cylindroid (A, B).

For a body only free to twist about X would be undisturbed by wrenches

about A and B; but any pair of wrenches about A and B compound into a wrench about a third screw C upon the cylindroid (A, B); therefore a wrench about C cannot disturb a body only free to move about X, therefore C and X are reciprocal: since the wrenches about A and B may have any ratio, C is typical of every screw upon the cylindroid, therefore every screw upon the cylindroid is reciprocal to X. We may say that X is reciprocal to the cylindroid.

X cuts the cylindroid in three points, and one screw of the cylindroid passes through each of these three points; these three screws must, of course, be reciprocal to X. Now two intersecting screws can only be reciprocal when they are at right angles, or when the sum of their pitches is zero. The pitch of the screw θ upon the cylindroid is

$$p + m \cos 2\theta.$$

This is also the pitch of the screw $\pi - \theta$. There are, therefore, two screws of any given pitch; but there cannot be more than two. It follows that X can at most intersect two screws upon the cylindroid of pitch equal and opposite to its own; and, therefore, X must be perpendicular to the third screw. Hence any screw reciprocal to a cylindroid must intersect one of the generators at right angles. We easily gather, also, that a line intersecting one screw of a cylindroid at right angles, must cut the surface again in two points, the screws passing through which have equal pitch. This theorem can be easily proved by direct calculation. The application of this Article will appear in the discussion of 2 and 4 degrees of freedom.

IV.

THE PECTENOID.

22. The surface to which this name has been given, though of much less importance than the cylindroid, is convenient in certain parts of the subject. We shall postulate its construction in the solution of some problems. We proceed to define the pectenoid.

Every pair of screws in space are reciprocal, when the condition (Art. 18) is fulfilled. Given a screw A (of pitch a), and a point O. Through O a

screw X (of pitch x) is to be drawn reciprocal to A. If on the line X a radius vector OP be marked off, so that—

$$OP = a + x.$$

Then the locus of P is an unclosed surface of the fourth order (the pectenoid).

23. We shall show how the equation of the pectenoid may be determined.

LM (Fig. 2) is the common perpendicular to A and X. HK is the projection of LM on the plane of z; the perpendicular OH, let fall from O on A is h. LM being perpendicular both to OZ and OP, is perpendicur to the plane containing OZ and OP, and therefore to the trace OQ of that plane on the plane of z; therefore HK is perpendicular to OQ, and we have

$$LM = h \sin \theta.$$

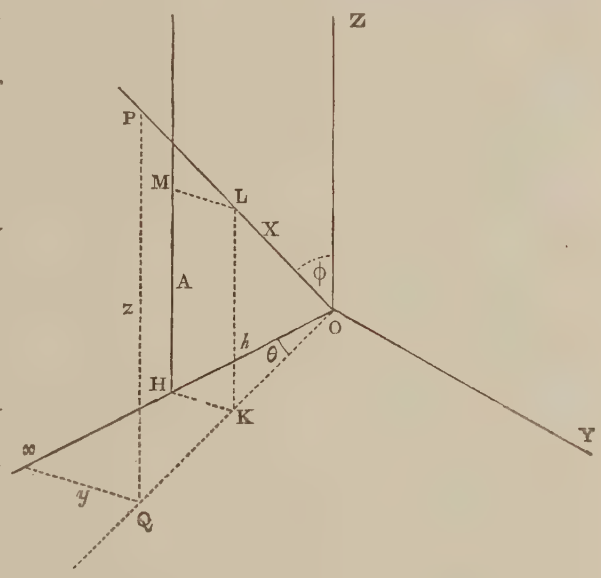


Fig. 2.

Since A and X are reciprocal, and OP is the sum of the pitches—

$$OP \cos \phi - LM \sin \phi = O,$$

making $OP = \rho$, we deduce—

$$\rho z - hy = O,$$

which is the equation of the pectenoid in its most convenient form.

Squaring we have—

$$z^2 (x^2 + y^2 + z^2) - h^2 y^2 = O.$$

24. All pectenoids are similar surfaces, merely depending upon the single parameter h. The screw A, which, in conjunction with the point O, com-

pletely determines the surface, we shall call the *cardinal screw.* The perpendicular from the origin upon A, is a nodal line. This is the only line upon the surface. The pectenoid although unclosed, yet is entirely contained between the pair of parallel planes $z = +h$, $z = -h$. Sections parallel to the plane of z are hyperbolas. Sections by planes, containing the nodal line, are circles.

25. A plaster model of the pectenoid has been exhibited to the Academy (Plate XII.). The model shows enough of the surface to enable its form to be distinctly realised. The resemblance of the surface to the valves of a scallop-shell has suggested the name *pectenoid,* which has been applied to designate it. A wire projecting from the nodal line, marked on the model, indicates the cardinal screw. The distance from the cardinal screw to the centre of the surface is the parameter; in the model this distance is four centimeters. A second wire also projects from the surface; this is introduced to show concisely the theorem which the pectenoid expresses. If this wire be a screw whose pitch, when added to the pitch of the cardinal screw, is equal to the intercept from the centre to the superfices of the surface, then the two screws are reciprocal. The interpretation of the nodal line is the obvious truth that when two screws intersect at right angles, they are reciprocal, whatever be the sum of the pitches.

26. One of the circular sections made by a plane, passing through the nodal line, is also indicated in the model. The physical interpretation is, that all the screws of given pitch, drawn through a point reciprocal to a given screw, lie in a plane.

27. From the symmetry of the surface, it appears that a second cardinal screw can be drawn, whose equation is $y = o\ x = -h$. The interpretation of this second screw is, that it is reciprocal to a screw along a radius vector, when the sum of the pitches is negative, though equal in magnitude to the radius vector. We shall not refer to the second cardinal screw again.

V.

RESTRAINT AND FREEDOM.

28. The geometrical conditions by which we may conce ive a rigid body to be restrained are of infinite variety. Viewed with reference to all positions

which the body may assume, the restraints appear to defy classification. Suppose, for example, a body be free to turn about a certain axis, and at the same time to slide along the axis. We make a tacit assumption that the axis is infinite, and assert that the body has two degrees of freedom; but if the axis be bounded by a plane at one end, and terminate abruptly at the other, the body may move from where it has only one degree of freedom (in contact with the plane) along the axis where it has two degrees, and off into space where it is perfectly free.

If, however, we narrow the extent of the displacement to an infinitely small quantity, we are in general entitled to assume that the body in the new position is subjected to the same conditions of restraint or freedom as in the old. We receive the benefit of this restriction in the present Paper.

29. If a rigid body occupy the position A in space the body can, if not actually fixed, be moved by a motion M, to a new position B. The same change could have been produced by a twist about a screw X, of definite position and pitch (Art. 3). The intermediate series of positions assumed by the body while moving from A to B, in the manner prescribed by M, will in general differ from the intermediate positions assumed when the body moves from A to B, by twisting about X, though the final positions are the same. It is, however, most important to notice, when the displacement is infinitely small, that whatever be the character of the restraints to which the body is subjected, an adequate knowledge of the freedom enjoyed by the body is acquired when all the screws have been ascertained, about which it is possible for the body to receive a twist.

30. The analysis and classification of geometrical freedom (or restraint) is therefore reduced to a determination of all the screws in space about which the body can receive a small twist. We shall conceive the process to be thus conducted :—Let a line in space be taken ; let this be conceived to have successively every pitch from $-\infty$ to $+\infty$: we then ascertain whether the body can receive a twist about one (or more) of the screws thus obtained, and record the result. The same process is to be repeated for every line in space. The results of these experiments will give all the information conceivable with reference to the geometrical freedom.

31. Suppose that k screws A_1, &c., A_k have been found by these trials, about each of which the body can receive a twist. It is evident, without further trial, that twisting about an infinite number of other screws must also be possible ($k > 1$): for suppose the body receive any k twists about A_1, &c., A_k the position attained could have been reached by a twist about some single screw A. It follows that the body must be free to twist about A. Now since the k twists may have any magnitude (each not exceeding an infinitely small quantity), A is merely one of an infinite number of screws, about which twisting must be possible. All these screws, together with A_1, &c., A_k, we call a system of *co-ordinate screws of freedom* k ($k < 6$).

If it be found that the body cannot be twisted about any screw, which is not one of the coordinate system of freedom k, then the body is said to have k degrees of freedom. It may be necessary to remark that A_1, &c., A_k must not be themselves members of a coordinate system of degree lower than k. If this were the case, the entire system would only consist of screws which form a coordinate system of that lower degree of freedom.

32. Whatever be the nature of the restraints to which a body is subjected, these restraints may be changed in an infinite number of ways, without affecting the freedom of the body. It is admissible, in the solution of any mechanical problem, to replace the given system of restraints by any equivalent system. It is obvious that this liberty may often facilitate an inquiry, by permitting the selection of a system of restraints which, while equivalent to the given system, are more favourably circumstanced with reference to the forces than the given system.

We thus replace every system of freedom k by an equivalent system derived from the theory of twists. Suppose a set of k screws to be properly selected in space, and nuts upon each screw (in the ordinary sense of the word screw). Let the body be rigidly attached to one of these nuts, and twist with it through a small angle (while the body is twisting about any screw it is supposed that the other screws are annihilated, or at least rendered incapable of interfering with the movement of the body). Let the body next be rigidly attached to a nut upon the second screw, then receive a twist about this screw. Let this process be repeated for all the k screws. By this means all the positions can be attained in which it is possible for the body

to be placed. Now no system of geometrical restraints could be devised, the freedom consistent with which is not identical with the freedom enjoyed by a body, when capable of twisting about k properly selected screws in the manner described above.

We now see how an infinite number of systems of restraints can be chosen, equivalent to any given system. For the body, having freedom k, any restraints which will permit the body to be twisted about any k screws of the co-ordinate system, must be equivalent to the given restraints. We define the freedom in the present method, not by the character of the restraints, but by a co-ordinate system of screws. The importance of this ultimate analysis of all freedom into twisting about screws is manifest when we call to mind the intimate connexion that exists between a twist and a wrench (Art. 20). We proceed to develope this branch of the subject.

33. A screw is determined when five elements are given; two of them may be the co-ordinates of the intersection of the screw, with a given plane; two more indicate the direction, while the pitch of the screw is the fifth.

34. When a body has k degrees of freedom, a screw of the co-ordinate system has $k - 1$ disposable elements—viz.: the $k - 1$ ratios of the k angles of twist about any k screws of the system.

35. A body free to twist about one screw, and one only, is not disturbed from a state of rest by a wrench about a reciprocal screw. From this we infer that the reaction of a screw about which a body twists is necessarily a wrench about a reciprocal screw.

36. When a body has k degrees of freedom, a screw X which is reciprocal to k screws A_1, &c., A_k of the system is necessary reciprocal to every screw A of the system. For since twists and wrenches are compounded by the same laws (Art. 16), wrenches about A_1, &c., A_k compound into one wrench about A, but each of the wrenches about A_1, &c., A_k are reciprocal to X; therefore, the wrench about A cannot disturb the equilibrium of a body only free to twist about X; therefore, A and X are reciprocal.

37. If a body have k degrees of freedom ($k < 6$), one or more screws reciprocal to the entire system (called A for brevity) can be found; for since one condition only is necessary for a pair of screws to be reciprocal, a screw X will be reciprocal to the system, if it fulfil k conditions. The screw X has

therefore, $5 - k$ elements still disposable, and consequently ($k < 5$) in general are infinite number of screws reciprocal to the system A can be drawn. The theory of reciprocal screws will show us that all the screws X really form a co-ordinate system of freedom $6 - k$. It is manifest, in the first place, that all the screws X must form a co-ordinate system of some degree, because (by the law of reciprocity) a body which can twist about all the screws X is yet not perfectly free, since wrenches about A_1, &c., A_k do not disturb equilibrium. Now any screw X has $5 - k$ elements disposable. It therefore follows, from Art. 34, that X must be one of a co-ordinate system of freedom $6 - k$.

We have thus learned that to any system P of freedom k a reciprocal system Q of freedom $6 - k$ corresponds. Every screw of the system P is reciprocal to all the screws of Q, and *vice versâ*. This theorem provides us with a definite test as to whether a given screw A is a member of the co-ordinate system P of freedom k. Construct any $6 - k$ screws of the reciprocal system ; if then A be reciprocal to these $6 - k$ screws, A must be co-ordinate with P. We thus have $6 - k$ conditions in the form of equations (Art. 18) to be satisfied by any screw, when a member of a co-ordinate system of freedom k. The significance of these theorems will be more fully appreciated after we have studied the special cases of freedom.

38. If a body have freedom about a co-ordinate system P, the body will remain in equilibrium, though acted upon by a wrench, about any screw of the reciprocal system Q. This is perhaps the most general theorem that can be enunciated with reference to the equilibrium of a rigid body. This theorem is thus proved. Suppose a wrench to act about a screw B of the system Q, if the body does not remain in equilibrium it must commence to twist about some screw A. We may then suppose that (for an instant) the restraints are such that the body can only twist about A ; hence a wrench about B would have to produce motion about A, which is impossible, since A and B are reciprocal. In precisely the same manner it may be shown that a body free to move about the co-ordinate system Q cannot be disturbed by a wrench about any one of the screws of the system P. Thus of two reciprocal systems of co-ordinate screws, each expresses the latitude permissible to a wrench which does not disturb the equilibrium of a body free to twist about any screw of the other.

39. An important consequence follows from this theorem. Suppose that

a statical problem of freedom k has been discussed ; that the co-ordinate system of screws of freedom have been determined, and also the reciprocal system of screws, the latter being necessary to the solution of the problem, in order to determine the locus (in a general sense) of the screw—a wrench about which is consistent with equilibrium ; the principle of reciprocity then shows that a different statical problem for a rigid body possessing freedom $6-k$, is stated and solved by merely interchanging the words twist and wrench, whenever they occur in the first problem, and its solution. For example, when we have solved a problem for two degrees of freedom, we have solved a corresponding problem of four degrees of freedom. We can employ this method to deduce the higher cases of freedom IV. and V. from the cases of freedom II. and I. In the case of freedom III. the same theorem also indicates a pair of related problems, of very great physical and geometrical interest.

40. It follows from Art. 38, that the reactions exerted by the restraints appropriate to a co-ordinate system P can only be wrenches about screws of the reciprocal system Q, for otherwise wrenches not belonging to Q could hold the body in equilibrium.

41. We shall now inquire into the number of elements necessary to completely define a co-ordinate system of freedom. If the body be capable of displacement about k screws, it might appear that 5 data for each screw (in all $5\,k$ data) should be given; but it must be remembered that any k screws of the system perfectly define the freedom, and (Art. 33) in the selection of each screw of the system, we have $k-1$ arbitrary elements. It follows that the total number of data absolutely necessary is—

$$5\,k-k\,(k-1)=k\,(6-k).$$

This expression is unaltered when k and $6-k$ are interchanged. Comparing this with Art. 39, we infer that the most general solution of the case of freedom k is determined when the most general solution of the case of freedom $6-k$ is known.

42. It will now be necessary to give a brief sketch of some of the different co-ordinate systems of screws. The details of the subject will be fully

examined when the mechanical problems of equilibrium and small oscillations or each degree of freedom are discussed (Arts. 58 to 97).

For freedom I. the co-ordinate system consists of a single screw A; the reciprocal system has freedom V. (Art. 37); but about any point O a pectenoid (Art. 23) can be described, of which A is the cardinal screw (Art. 24); therefore (Art. 41), in the case of freedom V., about any point O, a pectenoid can be described, such that every line through O is a screw co-ordinate with the system of freedom V., provided the pitch of the screw only differs by a certain constant (the pitch of the cardinal screw) from the radius vector of the pectenoid. Every line in space, when it receives a proper pitch, is therefore a co-ordinate screw of freedom V.

43. In the case of freedom II. all the screws upon the cylindroid form the co-ordinate system. All the screws of freedom IV. must be reciprocal to a cylindroid, and therefore (Art. 21) every screw co-ordinate with freedom IV. must cut a generator of a cylindroid at right angles. It follows that a cone, every generator of which is a screw, co-ordinate with freedom IV., can be drawn through every point of space.

44. Given a screw P, and a cylindroid (A, B), one screw can always be found upon the cylindroid reciprocal to P—for, draw any four screws Q_1, Q_2, Q_3, Q_4, intersecting generators of the cylindroid at right angles, and having the proper pitches (Art. 21) to make the screws reciprocal to the cylindroid: a pectenoid can be drawn about any point O, co-ordinate with the five screws P_1, Q_1, Q_2, Q_3, Q_4 (Art. 42). The cardinal screw C of this pectenoid must lie upon the cylindroid, for otherwise it would not be reciprocal to Q_1, Q_2, Q_3, Q_4 (Art. 37); also C must be reciprocal to P; therefore C is the screw required.

It is further to be noticed that the solution is unique; for, suppose a second screw on the cylindroid were reciprocal to P, then P must be reciprocal to every screw on the cylindroid, which in general is not the case, for it requires P to fulfil special conditions (Art. 21). We shall have many occasions to make use of this lemma in the proof of theorems of importance.

45. Being given five screws anywhere in space (any k screws of which are not members of a co-ordinate system of degree lower than k), it is manifest

from what has been said (Art. 42) that one screw and one only, reciprocal to the five screws, can be determined. It will be shown subsequently how this screw can be found by construction, (Art. 95).

46. We now come to a very important problem. Given seven screws A_1, &c., A_7 anywhere in space (any k screws of which are not members of a co-ordinate system of degree lower than k) ; to determine seven wrenching forces X_1, &c., X_7 such that when the seven corresponding wrenches are applied about the screws A_1, &c., A_7 to any rigid body, the body will be in equilibrium.

The following geometrical construction will enable the ratio of any two of the wrenching forces, for example $X_1 : X_2$ to be determined. Determine a screw $B_{1,2}$ reciprocal to the five screws A_3, &c., A_7, and suppose the wrenches X_1, &c., X_7 applied to a rigid body free to move about $B_{1,2}$ this body must be in equilibrium ; but by the property of reciprocal screws, the wrenches X_3, &c., X_7 are inoperative ; therefore, the wrenches X_1, X_2 must equilibrate when applied to a body only free to twist about $B_{1,2}$. X_1 and X_2 compound into a single wrench about a screw $C_{1,2}$ upon the cylindroid (A_1, A_2) therefore, $C_{1,2}$ must be the screw upon the cylindroid reciprocal to $B_{1,2}$ Art. (44) and is therefore determined. Hence (Art. 16) $X_1 : X_2$ is also determined. The solution is manifestly unique.

This result enables us to resolve a given wrench about a given screw, into six wrenches about six given screws. Thus any system of forces can be revolved into six wrenches. A particular case is the well-known resolution into three forces and three couples.

47. The solution of the last problem involves the solution of the following problem:—Given seven screws in space, to determine the magnitudes of the twists which a body must receive about each of these screws, in order that after the last twist the body shall occupy the same position as it did before the first. By this we are enabled to decompose any twist into six twists about six given screws, or any twist velocity into twist velocities about six given screws. These results are of the utmost utility in the present method of studying the mechanics of a rigid body.

48. On account of the importance of the problem of the last Article we may solve it in a different manner. Let A_1, &c., A_7 be the screws. The twists about A_1, A_2 could have been produced by a twist about some single screw

$C_{1,2}$ upon the cylindroid (A_1, A_2); but this twist about $C_{1,2}$ must be annulled by the five twists about A_3, &c., A_7. Hence $C_{1,2}$ and A_3, &c., A_7 must be members of a co-ordinate system of freedom V. Hence a certain screw $B_{1,2}$ is reciprocal to the six screws $C_{1,2}$, A_3, &c., A, but $B_{1,2}$ can be determined by A_3, &c., A_7, and then $C_{1,2}$ can be found by Art. (44). The line of reasoning here used would also have solved the problem for the seven wrenches, Art. (46).

49. When a body has k degrees of freedom, the effect of any wrench upon the body can be produced by suitably chosen wrenches about k given screws, provided that none of the k screws are members of the co-ordinate system, reciprocal to the freedom. For take the k screws, and any $6 - k$ screws of the reciprocal co-ordinate system, the given wrench may be resolved into six wrenches, about these six screws, but of these $6 - k$ being about reciprocal screws are inoperative (they are in fact counteracted by the reactions of the restraints), consequently wrenches about the k given screws alone remain.

VI.

CONJUGATE SCREWS OF ENERGY.

50. Suppose that a rigid body be in stable equilibrium under the action of a conservative system of forces, the body being either perfectly free or constrained in any way. Let the body be displaced from its position of equilibrium by a twist through a small angle ω, about a screw A. This screw A is called the *displacement screw*. The body is no longer in a position of equilibrium, and consequently a wrench commences to act about a screw X. This we call the *restoration screw*. It is manifest that the wrenching force is the product of tha angle of twist ω, and a certain coefficient due to the acting forces, and specially related to the displacement screw. It is further evident that, in order to give the body a twist ω about A, a definite quantity of energy is required. This quantity is proportional to ω^2, as the coefficient of the term involving ω is zero, since the position from which the body has been displaced is one of equilibrium. The quantity of energy consumed may, therefore, be represented by the positive quantity—

$$P\omega^2.$$

This expression is also a measure of the potential energy due to the position indicated by a twist ω about A.

We can find an expression for the potential energy in a different manner. The wrench about the restoration screw can be resolved into a wrench about A, and five wrenches about five screws reciprocal to A (Art. 49). With regard to a twist about A, the wrenches about the reciprocal screws are inoperative, and therefore the energy required for the displacement is absorbed in doing work against one wrench, acting about the screw A. Let $F\omega$ be the component of the wrenching force, which acts about the screw A, whose pitch is p. For an infinitely small displacement $d\omega$, the energy necessary for the translation $pd\omega$ is—

$$Fp\omega d\omega.$$

The same expression $Fp\omega d\omega$ expresses the energy required for the rotation $d\omega$; thus the entire energy necessary for the twist $d\omega$ is—

$$2\,Fp\omega d\omega.$$

Integrating between the limits ω and zero we have $Fp\omega^2$ for the total potential energy due to the twist ω.

51. The fundamental property of conservative forces, which is useful for our present purpose, is thus stated :—The quantity of energy necessary to move a body from one position to another position, is independent of the route by which the change has been effected. This property will prove a remarkable relation, which the theory of reciprocal screws enables us to enunciate with reference to a conservative system. Suppose A_1, A_2 be a pair of displacement screws, and X_1, X_2 be the corresponding pair of restoration screws ; then if X_1 be reciprocal to A_2, A_1 must be reciprocal to X_2. For, suppose the body receive a twist ω_1, about A_1, a quantity of energy $P\ \omega_1^2$ has been consumed, and the wrench about X_1 has been called into action. Now, since X_1 is reciprocal to A_2, the wrench about X_1 will not prejudice the quantity of energy necessary to produce a twist ω_2 about A_2. This, of course, would not be the case unless X_1 and A_2 were reciprocal. Hence the quantity of energy required for the twist ω_2 is—

$$P_2\omega_2^2\,;$$

and thus the total quantity of energy is—

$$P_1\omega_1^2 + P_2\omega_2^2.$$

Now, since the forces are conservative, the total quantity of energy required must be the same, if the body first receive the twist ω_2 about A_2, and then ω_1 about A_1. The twist ω_2 requires, as before, a quantity of energy $P_2\omega_2^2$, and calls into action a wrench about X_2. It follows (since $P_1\omega_1^2 + P_2\omega_2^2$ is constant) that the twist ω_1 about A_1 must require the same quantity of energy $P_1\omega_1^2$, when the wrench about X_2 is in action, as it did when no wrench acted about X_2; but this can only be the case if A_1 and X_2 are reciprocal.

This theorem expresses a very interesting property of a conservative system. It will be noticed that, though the property has been deduced from the principles of energy, yet energy is not concerned in the enunciation of the result: the relation is a conditional proposition between the pitch and position of a pair of displacement screws, and the pitch and position of the corresponding restoration screws.

52. We proceed to investigate a general expression for the potential energy due to the position of a body in the vicinity of a position of stable equilibrium, under the action of a conservative system of forces. Taking any six screws in space, the position of the body is defined by six twists (Art. 47) about these six screws. The potential energy will, therefore, be a quadratic function of the six angles of twist, containing in general 21 terms. But by proper choice of the six screws it is possible to represent the potential energy by the sum of six square terms. The theorem of Art. 51 will enable this choice to be made in an infinite number of ways.

Select any screw A_1, and let X_1 be the corresponding restoration screw; choose a second screw A_2, reciprocal to X_1, but restricted by no other condition; X_2 is the restoration screw corresponding to A_2; choose A_3 reciprocal to X_1, X_2; A_4 reciprocal to X_1, X_2, X_3; A_5 reciprocal to X_1, X_2, X_3, X_4; A_6 reciprocal to X_1, X_2, X_3, X_4, X_5. From this method of selection it follows (Art. 51) that each displacement screw is reciprocal to the five non-corresponding restoration screws. The quantity of energy necessary to produce

a displacement ω_1 about A_1 is therefore quite unaffected by wrenches about $X_2X_3X_4X_5X_6$, and hence the total quantity of energy required to give the body any position. is—

$$P_1\omega_1^2 + P_2\omega_2^2 + P_3\omega_3^2 + P_4\omega_4^2 + P_5\omega_5^2 + P_6\omega_6^2.$$

In general the selection of six screws involves thirty arbitrary elements. We shall see how many elements remain arbitrary in the selection of a set of six screws, possessing the property we have been considering. A_1 is quite arbitrary, and therefore five elements are arbitrary. A_2 has four elements; A_3 has three; A_4 has two; A_5 has one. In all fifteen elements, or half the entire number, remain arbitrary.

The analogy presented by these screws to a set of conjugate diameters in an ellipsoid has suggested the designation of the system as six *conjugate screws of potential energy.*

53. Our knowledge of the forces which hold a system in equilibrium will be adequate, if we have means of ascertaining the restoration screw corresponding to every displacement screw, and the magnitude of the wrenching force. We proceed to study this subject.

Suppose A_1, A_2 are a given pair of displacement screws, and X_1, X_2 the corresponding restoration screws (also given). If B be another given displacement screw, and Y its restoration screw, Y must be reciprocal to a screw which can be determined. Draw the cylindroid (X_1, X_2), and find a screw Z upon the cylindroid reciprocal to B. Decompose a small wrench about Z into two wrenches about X_1 and X_2. Find the twists about A_1 and A_2, that would have produced these wrenches about X_1 and X_2; compound these twists into a twist about a screw C upon the cylindroid (A_1, A_2), then C being considered a displacement screw, Z is the corresponding restoration screw; but Z is reciprocal to B, therefore (Art. 51) Y is reciprocal to C.

54. Let us suppose that we are given six conjugate screws of potential energy A_1, &c. We are then given the six corresponding restoration screws X_1, &c., for each one is determined by being reciprocal to the five non-corresponding displacement screws. Let us also suppose that we are given a specific constant P_1, &c., appropriate to each of the six conjugate screws. This constant, when multiplted into the angle of twist ω_1, &c., about each displacement screw, gives the wrenching force about the corresponding

restoration screw. Now, a body can receive any position by a suitable selection of twists about A_1, &c., and the six corresponding wrenches about X_1, &c., become determined; by compounding these wrenches the screw of restoration due to any position is determined, and also the wrenching force of restoration.

55. We next describe what may be called a system of *conjugate screws of kinetic energy*. They are analogous to the screws of potential energy which we have been considering. Suppose a wrench (about an *impulsive screw* commence to act upon a rigid body, not acted upon by any other forces, the body commences to twist about a certain *instantaneous screw*. The *impulsive screw*, and the *instantaneous screw*, are related to each other in a similar manner as the restoration screw, and the displacement screw, in the case of potential energy.

The kinetic energy due to a certain twist velocity about a screw is expressed by the product of the square of the twist velocity with a certain constant. The unit of energy most convenient to adopt is the quantity of work done by the absolute unit of force in moving over the unit of distance. The quantity of energy in the mass M, moving with the velocity $p\dot{\omega}$, is $\frac{1}{2}Mp^2\dot{\omega}^2$, and the quantity of energy due to the rotation is $\frac{1}{2}Mk^2\dot{\omega}^2$, where k is the radius of gyration. The kinetic energy is therefore—

$$\tfrac{1}{2}M\,(p^2+k^2)\,\dot{\omega}^2.$$

We shall denote this expression by $T\,\dot{\omega}^2$.

56. We shall now prove the following theorem; X_1, X_2, are a pair of screws about which impulsive wrenches are communicated; and A_1, A_2 are the corresponding instantaneous screws; if X_1 be reciprocal to A_2 then X_2 must be reciprocal to A_1.

For let the body have, in consequence of the impulses, instantaneous velocities $\dot{\omega}_1$, $\dot{\omega}_2$ about A_1, A_2, then the kinetic energy must be of the form,

$$P\dot{\omega}_1^2 + 2Q\dot{\omega}_1\,\dot{\omega}_2 + R\dot{\omega}_2^2.$$

The rate of change of this expression with reference to $\dot{\omega}_1$ is

$$2\,(P\dot{\omega}_1 + Q\dot{\omega}_2)$$

If X_1 be reciprocal to A_2 the rate of change cannot depend upon ω_2, and there-

fore $Q = 0$, but when $Q = 0$ we can show conversely that X_2 is reciprocal to A_1. This might also be shown in a manner similar to that used in Art. 51.

57. It follows that six conjugate screws of kinetic energy can be selected, leaving fifteen elements arbitrary, out of the thirty which determine the six screws: see Art. 52. The impulsive screw corresponding to any one of these screws is reciprocal to the remaining five screws. The twist velocity of a body about any screw can be decomposed into six twist velocities about the six conjugate screws, and the total kinetic energy of the body is the sum of six square terms.

$$T_1\dot{\omega}_1^2 + T_2\dot{\omega}_2^2 + T_3\dot{\omega}_3^2 + T_4\dot{\omega}_4^2 + T_5\dot{\omega}_5^2 + T_6\dot{\omega}_6^2.$$

VII.

THE SIX DEGREES OF FREEDOM.

a. One Degree of Freedom.

58. If a body have one degree of freedom it is capable of twisting about one screw A. The body is in equilibrium when acted upon by a wrench, provided that wrench be about a reciprocal screw. The reaction of the screw A upon the body is a wrench about a screw reciprocal to A.

If two wrenches about any two screws X, Y act upon the body, it may be in equilibrium, provided the wrenching forces have a proper ratio. For, draw the cylindroid (X, Y), select on this surface the screw Z, reciprocal to A, Art. (44). If the wrenches about X and Y compound into a wrench about Z, the body will remain in equilibrium. It follows from this that the effect of a wrench about any screw X may be produced by a wrench (of proper magnitude) about any other screw Y (not reciprocal to A).

59. Suppose the body receive an impulsive wrench about a screw X, we can determine the impulsive wrench produced by the reaction of the screw A. Let Y be the screw, which, if the body were perfectly free, would be the impulsive screw correspondiug to A as the instantaneons screw; the reaction of A, and the wrench about X must compound into a wrench about Y. Draw the cylindroid (X, Y); select on this cylindroid the screw Z, reciprocal to A;

then the impulsive strain produced by the reaction of A must be a wrench about Z, and the wrenching force is found by decomposing the wrench about X into wrenches about Y and Z.

60. We now suppose the body to be displaced from a position of stable equilibrium, under the action of conservative forces. Let the body receive a small twist ω about A; it is no longer in equilibrium. The resultant wrench, which before the displacement was about a reciprocal screw, is no longer about a reciprocal screw, though about a screw in the immediate vicinity of its original position. This wrench can be replaced by a wrench about the screw A (Art. 49). Let F be the wrenching force and p the pitch of A, then (Art. 50) the potential energy of the position is

$$Fp\omega^2$$

The kinetic energy is,

$$\tfrac{1}{2}\, M\,(k^2 + p^2)\, \dot{\omega}^2$$

The forces being conservative the sum is constant: differentiating the sum we have,

$$M(k^2 + p^2)\,\frac{d^2\omega}{dt^2} + 2\,Fp\omega = 0,$$

and the period of a small oscillation is,

$$\pi\sqrt{\frac{M(k^2 + p^2)}{2\,Fp}}.$$

The twist oscillation here discussed may be physically conceived thus:—Support a screw (in the ordinary sense of the word) horizontally; let a nut move upon the screw without friction; attach a heavy pendulum to the nut, and make the pendulum swing in a plane normal to the screw; the nut then executes twist oscillations. All small oscillations of a rigid body must be ultimately composed of the twist oscillations here described.

b. Two Degrees of Freedom.

61. When a body is capable of being twisted about two screws, it is capable of being twisted about every screw on the cylindroid determined by those

screws. In discussing this case, we therefore concentrate our attention upon the cylindroid, and not upon the two original screws, the discovery of which enabled the cylindroid to be drawn. Eight data determine a cylindroid and the pitches of all the screws. Four data are necessary for the axis ($x = 0$, $y = 0$ Art. (7) ; two more indicate the extreme points in which the axis cuts the surface ($x = 0$, $y = 0$, $z = \pm m$) ; the seventh gives the direction of one generator, and the eighth (a) determines the distribution of pitch. The two original screws were defined by ten data, but it must be remembered that any pair of screws upon the surface would have answered equally well. The ten data suffice not only for the perfect construction of the surface, but also for pointing out two special screws upon that surface (compare Art. 41).

62. We shall first consider the conditions under which a body free to twist about every screw upon the cylindroid will remain in equilibrium. It has been shown (Art. 21) that a screw reciprocal to two screws of a cylindroid is reciprocal to every screw, and that a reciprocal screw cuts one generator at right angles, and two other generators, each of which has pitch equal, and opposite to that of the reciprocal screw. A wrench about any screw reciprocal to the cylindroid must hold the body in equilibrium; and we therefore proceed to examine fully into the relations which the reciprocal screws bear to the cylindroid.

63. It is manifest that a cone of reciprocal screws can be drawn from any point, since a perpendicular may be let fall from the point upon every generator. We shall first prove that this cone is of the second degree.

Take any point O; about O describe a pectenoid reciprocal to one screw A of the cylindroid, the equation of the pectenoid is—

$$\rho M - kN = 0$$

where M and N are planes passing through O (Art. 23).

Let p be the pitch of A, and ϕ be the pitch of a screw reciprocal to the cylindroid through O, then

$$\rho = p + \phi$$

and

$$(p + \phi)\, M - kN = 0.$$

The pectenoid reciprocal to a second screw A' upon the cylindroid whose pitch is p' is

$$(p' + \phi)\, M' - k'\, N'.$$

Eliminating ϕ we have

$$(p - p')\, MM' - kNM' + k'\, N'\, M = 0\,;$$

but this is the equation of a cone of the second order, whose origin is at O.

64. It is not a difficult exercise in analytic geometry to find the locus of lines drawn from the point O perpendicular to the generators of

$$z\,(x^2 + y^2) - 2mxy = 0,$$

and to deduce the following geometrical construction for the reciprocal cone. (*N.B.* A pair of screws upon the cylindroid, which have equal pitch, are called *corresponding screws*).

From the point O let fall a perpendicular upon the plane of z; this will cut the cylindroid in one real point P. A plane drawn through P, and through the screw corresponding to the generator through P, cuts the surface in an ellipse; the cone drawn from O to the ellipse thus found is the cone required.

The plane of the ellipse is a tangent plane to the cylindroid, touching the surface at the foot of the perpendicular let fall from P upon the generator corresponding to the generator passing through P.

65. We next examine the distribution of pitch upon the generators of the reciprocal cone drawn to a cylindroid from any point. It has been proved (Art. 21) that the pitch of each reciprocal screw is equal and opposite to the pitches of the corresponding screws in which it meets the cylindroid: now the greatest and least pitches upon *real* generators of the cylindroid are $p + m$ and $p - m$, consequently the generators of the cone which meet the cylindroid in three real points must have pitches intermediate between $-p - m$ and $-p + m$. Since only one line can be drawn from a point intersecting both of two corresponding screws, it follows that only one generator of the cone can be found, having a given pitch. This is otherwise evident; for suppose two generators had equal pitches, every screw in the plane of those screws which has the

same pitch, and passes through the vertex of the cone, would also be a reciprocal screw, which is impossible except in one case (see Art. 68). One screw can be found upon the reciprocal cone of every pitch from $-\infty$ to $+\infty$. The line drawn through P perpendicular to the plane of z is a generator of the cone which has infinite pitch; setting out from this line around the cone, the pitch gradually decreases to zero, then becomes negative, and increases to negative infinity when we reach the line from which we started. It is manifest that a screw whose pitch is infinite is the same whether the infinity be positive or negative. If we conceive distances marked upon each generator of the cone from the vertex, equal to the pitch of that generator, then the perpendicular from the vertex of the cone upon the plane of z, on the cylindroid, forms a sort of asymptote to the curve so traced upon the cone. It is manifest that we must admit the cylindroid to have imaginary screws, whose pitch is nevertheless real.

66. We easily infer that, through any point in space, one screw of given pitch can be drawn reciprocal to a given cylindroid.

67. When a body has two degrees of freedom, the small movements of any point are restricted to a plane. This is evident; for any displacement of the point can be produced by two twists, but each of these twists will move the point in a certain line, and therefore no displacement can cause the point to leave the plane determined by those lines.

68. The reciprocal cone drawn from a point to a cylindroid is decomposed into two planes when the point lies upon the cylindroid. The first plane is normal to the generator passing through the point. Every line in this plane, passing through the point, must, when it receives the proper pitch, be a reciprocal screw. The second plane is that drawn through the point and the corresponding screw. Every line in this plane, passing through the point which has a pitch equal, but of opposite sign to the generator passing through the point, is a reciprocal screw.

69. We have therefore solved in the most general manner the problem of the equilibrium of a body possessing two degrees of freedom. We have shown that the necessary and sufficient condition is, that the resultant wrench be about a screw reciprocal to the cylindroid expressing the freedom; and we have shown the manner in which the reciprocal screws are distributed through

space. We now proceed to study the effect of an impulsive wrench in producing motion of a body possessing two degrees of freedom.

We shall first show that, from a co-ordinate system of screws A, possessing k degrees of freedom, one screw can always be selected reciprocal to any other system of screws B, possessing $k-1$ degrees of freedom. For, form the reciprocal system to A, this system (A') will have $6-k$ degrees of freedom; now take any $6-k$ screws of A' and any $k-1$ screws of B, in all five screws, determine the single screw P (Art. 45) reciprocal to these five screws: then P is the screw required, for P is reciprocal to B, and also to A'; but if P be reciprocal to A' it must be co-ordinate with A. For example, being given a cylindroid and a system of screws possessing three degrees of freedom, a screw can be chosen from the latter reciprocal to the cylindroid.

70. Let A_1 and A_2 be a pair of screws upon a cylindroid, and let X_1, X_2 be the pair of impulsive screws corresponding to A_1, A_2 (Art. 55). X_1 and A_1 are related to each other in a manner which is completely determined by the configuration, &c., of the rigid body. Let us now suppose that the body has its freedom prescribed by the cylindroid (A_1, A_2), and that an impulsive wrench acts upon the body about the screw Z. It is required to determine the instantaneous screw B upon the cylindroid, about which the body will commence to twist, and also the impulsive wrench which is produced by the reaction of the restraints. Let the wrench of reaction act about a screw Y, of course reciprocal to the cylindroid. The instantaneous twist about B is produced by compounding a pair of wrenches about Y and Z. But since B is upon the cylindroid (A_1, A_2) the twist about B could have been produced by an appropriate composition of twists about A_1 A_2, and these twists could have been produced by appropriate impulsive wrenches about X_1, X_2.

This can only have been the case if the cylindroid (X_1, X_2), have a screw in common with the cylindroid (Y, Z) (Art. 84); in other words, if X_1, X_2, Y, Z are co-ordinate screws of a system with three degrees of freedom, hence Y must be co-ordinate with the system determined by (X_1, X_2, Z), and must be reciprocal to $(A_1 A_2)$. Thus Y is known (Art. 69). The wrench about Z is then decomposed into wrenches about X_1, X_2, Y thus giving the value of the reactionary wrench about Y. The twist velocities about A_1 A_2 are known when the components of the wrenches about Z upon X_1, X_2 are known. Com-

pounding the twist velocities about A_1, A_2 the required instantaneous screw B and its twist velocity are known. The decomposition of the wrench about Z along X_1, X_2, Y is thus effected. Let S be the screw common to the cylindroid (Y, Z), and (X_1, X_2) Z is first decomposed about Y, S, and the component about S is then decomposed about X_1, X_2.

71. Suppose a body be in equilibrium under the action of conservative forces. Let the body receive a small twist about some screw upon the cylindroid to which its movements are restrained, and also a small initial twist velocity about another screw on the cylindroid, it is required to determine the small oscillations of the body. The problem will be solved when the screw is determined, a twist about which of a determinate magnitude will bring the body from the initial position to the position it occupies at a given time. We should also know the instantaneous screw about which the body is twisting at any instant, and the twist velocity about the instantaneous screw. We proceed to show how the problem as thus stated is to be solved.

The body receives a twist about the screw A upon the cylindroid: a wrench of restoration about the screw X is immediately evoked: a screw B upon the cylindroid can be found which is reciprocal to X: A and B are thus conjugate screws of potential energy. Suppose that Y is the screw, an impulsive wrench about which would, if the body were free, produce an instantaneous twist about A. Let B' be the screw upon the cylindroid reciprocal to Y, then A and B' are conjugate screws of kinetic energy. In general B and B' will not be identical, but it is manifest by appropriate choice of A, that B and B' can be made to coincide. Let us call A_1, A_2 the two screws thus determined, which are conjugate screws both of potential and kinetic energy. Let X_1, Y_1 be the restoration screw and impulsive screw corresponding to A_1, and X_2, Y_2 the corresponding screws for A_2. Draw four screws P_1, P_2, P_3, P_4, reciprocal to the cylindroid; the six screws X_1, Y_1, P_1, P_2, P_3, P_4, are all reciprocal to A_2; therefore the six screws are co-ordinate members of a system of freedom V. The wrench of restoration about X_1 can, therefore, be decomposed into five wrenches about the screws Y_1, P_1, P_2, P_3, P_4. The four last are about screws reciprocal to the cylindroid, and therefore the components of the wrench along these screws are destroyed by the reactions of the system. The final result is, that the wrench of restoration

about X_1 may be replaced, so far as the body only free about the cylindroid is concerned, by a wrench about Y_1. We now suppose that the body receives a displacement ω about A_1. The wrench of restoration is to be replaced by a wrench about Y_1. Now, this wrench continually acts when the body is removed from its position of equilibrium. Since an impulsive wrench about Y_1 would make the body twist about A_1, it is manifest that the action of the component of the restoration wrench along Y_1 will tend by an infinite series of impulses to make the body twist about A_1. The only other wrench acting upon the body is one which (following Poinsot) may be regarded as produced by "centrifugal force." This wrenching force being proportional to the square of the velocity, is of a higher order of small quantities than the wrench we have been considering, and is therefore negligible. It follows that the body must perform twist oscillations about the screw A_1 for ever, and that the angle of twist is an harmonic function of the time. We have, in fact, reduced the case to that already discussed (Art. 60).

72. Let us now suppose that the body receives an initial twist velocity about A_1, with or without an initial displacement. It is manifest that this merely affects the constants introduced in the integration, but it does not in the least affect the character of the motion, or even the periodic time.

73. Taking the most general case, let us suppose that the body receives a certain initial twist, the wrench of restoration may be decomposed along Y_1, Y_2, P_1, P_2, P_3, P_4, the four last wrenches are destroyed by reactions leaving wrenches about Y_1 and Y_2. Since these are reciprocal to A_2 and A_1, twist oscillations about A_2 and A_1 subsist quite independently. The initial twist velocity can be decomposed about A_1 and A_2, and thus we have the following general solution of the problem of the small oscillations of a rigid body possessing two degrees of freedom.

Draw the cylindroid expressing the freedom; determine upon this surface the pair of screws, which are conjugate screws both of potential energy and kinetic energy. These screws may be conveniently called *harmonic* screws*. If the

* In the abstract of this Paper, published in the "Proceedings of the Royal Irish Academy," and reprinted in the "Quarterly Journal of Mathematics," the expression *normal screw* was used: the more significant designation of *harmonic screw* has been adopted at the suggestion of the Rev. Professor Townsend, F. T. C. D.

body be initially displaced by a small twist about a harmonic screw, and receive a small twist velocity about the same screw, the body will continue for ever to perform harmonic twist oscillations of determined period about that screw. If the body be displaced about any screw upon the cylindroid, and receive an initial twist velocity about any screw, the movement is compounded of two twist oscillations about the two harmonic screws. The oscillations about the harmonic screws being determined, the instantaneous screw and twist velocity are found by compounding upon the cylindroid the twist velocities about the harmonic screws. By compounding the two twists about the screws, the single screw is determined, about which the body must receive a twist of determinate magnitude, in order to be brought from the initial position to the position occupied at a given time.

The physical meaning of the harmonic screws upon the cylindroid may be thus stated:—The quantity of energy necessary to give the body a small displacement against the forces about one harmonic screw is unprejudiced, though the body may have previously received a displacement about the other screw. Also, the quantity of energy necessary to give the body a twist velocity about one harmonic screw is unprejudiced, though the body may have previously received a small velocity about the other screw.

c. Three Degrees of Freedom.

74. The case of three degrees of freedom, including, as it does, the celebrated problem of the rotation of a rigid body about a fixed point, is of the greatest interest. The present mode of viewing the subject presents this case under a peculiar aspect. We have here k and $6-k$ equal, and consequently the reciprocal system is of the same order as the original system. This is very fertile in interesting theorems. We shall strictly confine our inquiries to the most general case, as we have done uniformly throughout the Paper. The application of the general principles to definite problems would, however, be very interesting, and might be made the subject of future communications.

75. Let us take three screws A_1, A_2, A_3 (not members of the same cylindroidal system), whose pitches are a_1, a_2, a_3; we shall show in general that

through any point O in space, three screws can be drawn, each of which is reciprocal to A_1, A_2, A_3.

Draw through O three pectenoids reciprocal to A_1, A_2, A_3. Through O draw a line, whose direction cosines are α, β, γ. This line cuts the three pectenoids in points P_1, P_2, P_3. If now α, β, γ, be properly chosen, we can have—

$$OP_1 = a_1 + \theta, \quad OP_2 = a_2 + \theta. \quad OP_3 = a_3 + \theta.$$

Or from the equations of the pectenoids,

$$(a_1 + \theta)\ L_1 + k_1 M_1 = 0.$$
$$(a_2 + \theta)\ L_2 + k_2 M_2 = 0.$$
$$(a_3 + \theta)\ L_3 + k_3 M_3 = 0.$$

Where L_1, M_1, &c., are linear functions of α, β, γ, and k_1, &c., are constant, eliminating α, β, γ, we have a cubic for θ. The values of θ are the pitches of the three reciprocal screws, which can be drawn through O, and the corresponding values of α, β, γ can be determined from the equations just written.

76. Given three screws passing through a point; three reciprocal screws can be drawn through the same point, for each of the reciprocal screws may be perpendicular to the plane of two of the original screws, and of pitch equal and opposite to the third screw. In fact, if a sphere be described about the point, and the three original screws intersect the sphere in the points A, B, C, the reciprocal screws intersect the sphere in the points A', B', C', such that the spherical triangle A', B', C' is the polar of the triangle A, B, C; the pitch of A being equal to the pitch of A', with the sign changed, &c.

77. We can now show that, given any three screws A_1, A_2, A_3 in space, three screws, co-ordinate with A_1, A_2, A_3, may be drawn through any point O in space; for draw through O three screws R_1, R_2, R_3 (Art. 75), reciprocal to A_1, A_2, A_3, then (Art. 76) draw three screws P_1, P_2, P_3 through O, reciprocal to R_1, R_2, R_3, and P_1, P_2, P_3 must be co-ordinate with A_1, A_2, A_3 These theorems show us that a body free to twist about A_1, A_2, A_3 must be free to twist about an infinite number of screws, three of which can be drawn through every point in space. We also learn that a body free to twist about three screws will remain in equilibrium, provided the wrench

acting upon the body be about one of an infinite number of reciprocal screws, three of which can be drawn through any point.

78. In order to facilitate the discussion of three degrees of freedom, it is manifestly desirable to use, for the purpose of expressing the freedom, three judiciously chosen screws out of the infinite number forming the co-ordinate system. We may take any point in space, and draw three screws through it. In the selection of the point we have three arbitrary elements to dispose of. We are, therefore, entitled to select a point, the three co-ordinate screws through which are at right angles. We thus see that a full discussion of the freedom of a body capable of twisting about three screws intersecting at right angles really contains the solution of the most general problem which could be stated with reference to three degrees of freedom. We shall, therefore, continue the discussion of the subject in this simplified form. The three reciprocal screws, which can be drawn through this point, are in the same lines as the original screws, and with equal and opposite pitches.

79. OA, OB, OC (Fig. 3), are three screws, intersecting at right angles, whose pitches are a, b, c. Through the point $P\ (x', y', z')$ a screw PQ is drawn whose pitch is ρ. We shall determine the conditions under which this screw is reciprocal to OA, OB, OC. Let p be the common perpendicular AQ to the two lines OA, PQ; then HT being the projection of PQ is perpendicular to OH, the projection of AQ; OR being parallel to PQ, the condition of reciprocity is—

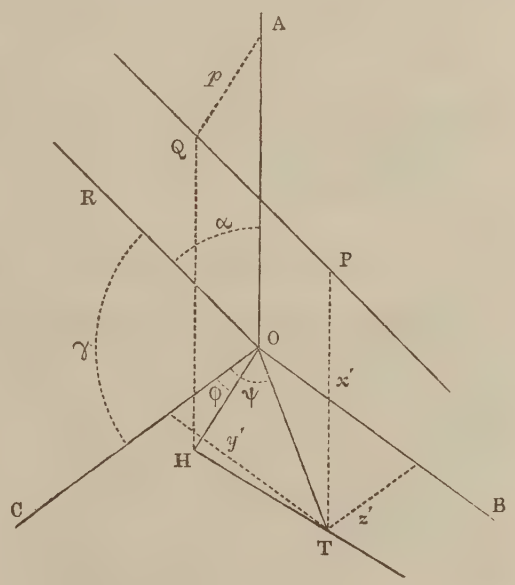


Fig. 3.

$$(a+\rho)\cos a - p\sin a = 0.$$

$$p\sin a = OH\sin a = OT\cos(\psi-\phi)\sin a.$$

$$= z'\cos\phi\sin a + y'\sin\phi\sin a.$$

But from Fig. 4, which represents the angles upon a sphere, with letters corresponding to Fig. 3,

$$\cos\phi \sin\alpha = -\cos\beta \qquad \sin\phi \sin\alpha = \cos\gamma,$$

whence—

$$(a+\rho)\cos\alpha + z'\cos\beta - y'\cos\gamma = 0,$$

is the condition that PQ shall be reciprocal to OA. If PQ be reciprocal to OA, OB, OC, we have, therefore—

$$\begin{aligned}(a+\rho)\cos\alpha + z'\cos\beta - y'\cos\gamma &= 0.\\ -z'\cos\alpha + (b+\rho)\cos\beta + x'\cos\gamma &= 0,\\ +y'\cos\alpha - x'\cos\beta + (c+\rho)\cos\gamma &= 0.\end{aligned}$$

Eliminating $\cos\alpha$, $\cos\beta$, $\cos\gamma$.

$$\frac{x'^2}{(b+\rho)(c+\rho)} + \frac{y'^2}{(a+\rho)(c+\rho)} + \frac{z'^2}{(a+\rho)(b+\rho)} + 1 = 0.$$

This is the cubic by which ρ is determined, ρ being known, the corresponding values of $\cos\alpha$, $\cos\beta$, $\cos\gamma$, are easily found.

80. Supposing ρ be regarded as a known quantity, what is the interpretation of the equation?

$$\frac{x^2}{(b+\rho)(c+\rho)} + \frac{y^2}{(a+\rho)(c+\rho)} + \frac{z^2}{(a+\rho)(b+\rho)} + 1 = 0.$$

This is an hyperboloid of one sheet, and proves the very interesting theorem that the locus of screws of given pitch reciprocal to a system possessing three degrees of freedom is a hyperboloid. By giving ρ successive values we have an infinite series of hyperboloids, showing the distribution of the reciprocal screws in space. Three of these hyperboloids can be drawn through any point.

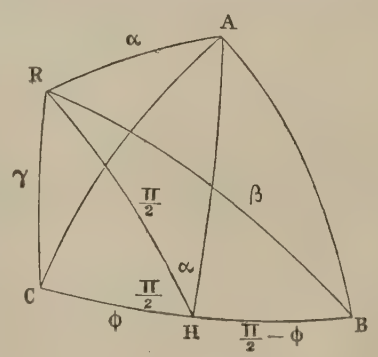


Fig. 4.

81. We can determine the conditions under which a screw, drawn through a given point x', y', z', is *co-ordinate* with the three screws OA, OB, OC. If we take three screws along OA, OB, OC, of pitches $-a$, $-b$, $-c$, a screw reciprocal to these screws

is manifestly co-ordinate with the original screws. A screw, therefore, of pitch ρ, drawn through the point x', y', z', and making angles α, β, γ, with the axes, will be a member of the co-ordinate system, if—

$$(-a+\rho)\cos\alpha + z'\cos\beta - y'\cos\gamma = 0.$$
$$-z'\cos\alpha + (b+\rho)\cos\beta + x'\cos\gamma = 0.$$
$$+y'\cos\alpha - x'\cos\beta + (-c+\rho)\cos\gamma = 0.$$

We deduce, as before, that a screw co-ordinate to the system of pitch ρ lies upon the hyperboloid—

$$\frac{x^2}{(b-\rho)(c-\rho)} + \frac{y^2}{(a-\rho)(c-\rho)} + \frac{z^2}{(b-\rho)(a-\rho)} + 1 = 0.$$

By comparing this result with the equation of the hyperboloid previously obtained, we learn that the co-ordinate screws, whose pitch is ρ, and the reciprocal screws, whose pitch is $-\rho$ lie upon the same hyperboloid; and we deduce the theorem, which may be enunciated as follows:—

When a system has three degrees of freedom an hyperboloid can be constructed for every pitch, such that, if to all the generators of the first system that pitch be assigned, the screws thus produced are co-ordinate screws of the system; while, if to all the generators of the hyperboloid of the second system the same pitch, with an opposite sign, be assigned, all the screws thus produced are reciprocal screws to the co-ordinate system.

82. It follows, from the equation of the hyperboloid just written, that if ρ, a, b, c be each increased by a constant, the screw whose pitch is the new ρ in the same line as the old, will be co-ordinate with the three axes, each provided with the new pitch. This reminds us of the property of the cylindroid (Art. 9)—namely, that if a constant be added to all the pitches, the screws still belong to a co-ordinate system. We can further generalise this property, and prove that if $k+1$ screws be members of a co-ordinate system of freedom k, a constant value c added to all the pitches will make a new set of $k+1$ screws also members of a co-ordinate system of freedom k. For, take $6-k$ screws reciprocal to the first system, now the condition (Art. 18) that a pair of screws be reciprocal depends upon the sum of their pitches. If, therefore, the pitch of one screw be increased by c, a reciprocal screw will still

be reciprocal, if its pitch be diminished by c. If the pitches of the $6-k$ screws be diminished by c, the $k+1$ screws, whose pitch has been augmented by c, will still be reciprocal to the $6-k$ screws; but $k+1$ screws cannot be reciprocal to $6-k$ screws, unless they are members of a co-ordinate system of freedom k.

83. The hyperboloid—

$$\frac{x^2}{(b-\rho)\,(c-\rho)}+\frac{y^2}{(a-\rho)\,(c-\rho)}+\frac{z^2}{(b-\rho)\,(a-\rho)}+1=0$$

is now to be discussed. Suppose—

$$a>b,>c.$$

If $\rho<c$ this equation is the sum of four positive terms, and is, therefore, impossible. No co-ordinate screws of the system can have a pitch $<c$. If $\rho=c$ the equation reduces to the axis of z.

When ρ is intermediate between b and c, the equation is that of an hyperboloid of one sheet, the sections parallel to the plane of z being ellipses. When $\rho=b$ the surface becomes a pair of planes through the axes of y; when $\rho>b<c$ the elliptic sections of the hyperboloid are parallel to x, and the hyperboloid finally vanishes into the axis of x, when $\rho=a$. If $\rho>a$ the equation again becomes the sum of four positive squares, and is, therefore, impossible.

84. Take any four screws of the system A, B, X, Y, then the cylindroid (A, B) must have a screw in common with the cylindroid (X, Y). For twists about the four screws can be of such a magnitude that the body, after the last twist, shall occupy the same position as it did before the first. Therefore the twists about A, B must be counteracted by the twists about X, Y. Now, any pair of twists about A, B must compound into a single twist about a screw C upon the cylindroid (A, B), and the twists about X, Y must compound into a single twist Z upon the cylindroid (X, Y); hence, if the body is to be restored to its original position by the four twists, C and Z must be identical. This theorem provides a ready test as to whether four screws belong to the same co-ordinate system of freedom III. It also enables a twist about any one screw of the system to be decomposed into twists about any three of the co-ordinate screws.

85. Given any four screws in space, it is possible for four properly chosen

wrenches about the these screws to hold a body having freedom III. in equilibrium. For, take the four given screws, and three screws reciprocal to the system, wrenches of proper magnitude about these seven screws will equilibrate, but those about the reciprocal screws are destroyed by the reactions, and, therefore, the four wrenches about the four screws equilibrate. It is manifest that this theorem may be generalised into the following :—If a body have k degrees of freedom, then properly selected wrenches about any $k + 1$ screws (not reciprocal to the co-ordinate system) will hold the body in equilibrium.

86. If an impulsive wrench act upon a body possessing three degrees of freedom, the instantaneous screw, about which the body commences to twist, can be determined in a manner similar to that already employed (Art. 70). It will be just as short to write the mode of proceeding in the case of freedom k. Let A_1, A_2, &c., A_k, be the instantaneous screws, which, if the body were free, would correspond with impulsive wrenches about X_1, X_2, &c., X_k. Let the freedom be defined by A_1, &c., A_k, which we may call the system A, denoting the other system by X. Let Y be an impulsive wrench, and let B be the instantaneous screw, co-ordinate with A, about which the body will commence to twist, after receiving the wrench about Y. Let Z be a screw reciprocal to A, such that a wrench about Z represents the initial reaction of the restraints. We have to determine B and Z, the instantaneous twist velocity about B and the instantaneous wrench about Z. Since B is co-ordinate with A, the initial twist velocity about B can be decomposed into twist velocities about A_1, &c., A_k; but these twist velocities could have been produced by impulsive wrenches about X_1, &c., X_k. Hence X_1, &c., X_k, Y, Z, must be members of a co-ordinate system of order $k+1$: because wrenches about Y, Z are equivalent to wrenches about X_1, &c., X_k. Hence Z is selected from a system of $k+1$ freedom, reciprocal to a system of k freedom, and is, therefore, determinate (Art. 69). The position of Z being known, the components of the wrench about Y, resolved along X_1, &c., X_k, Z, are known. From knowing X_1, &c., X_k, we know the instantaneous velocities about A_1, &c., A_k. Compounding these B is determined, and the problem is solved.

87. By a similar method to that used in Art. 71 we can show that, when

a body has three degrees of freedom, and is under the action of conservative forces, that, if the body be slightly displaced from a position of stable equilibrium, the body will perform movements compounded of twist oscillations about three harmonic screws; and that, if the body be displaced by being merely twisted about one of the harmonic screws, and receive a small twist velocity about that screw, the body will continue for ever to twist to and fro about that screw, the angle of twist being connected with the time, in the same manner as the arc of an ordinary circular pendulum is connected with the time.

We shall prove this theorem for the case of k degrees of freedom. The following lemma is a necessary preliminary:—The screws selected from a co-ordinate system A, possessing freedom a, reciprocal to a system B, possessing freedom b $(a > b)$, form a co-ordinate system of freedom $a-b$. Take $6-a$ screws reciprocal to A, then the screws required must be reciprocal to $b+6-a$ screws. They must, therefore, be the members of a co-ordinate system of freedom, whose order is $6-(b+6-a)=a-b$.

Let us denote by A the co-ordinate system of freedom k. Select any screw A_1 of this system. Let X_1 be the corresponding restoration screw of potential energy. In the selection of A_1 we have $k-1$ arbitrary data. Select now any screw A_2 from A reciprocal to X; in this choice we have only $k-2$ arbitrary elements (from the preceding lemma). Let X_2 be the restoration screw corresponding to A_2, and let A_3 be chosen from A, so as to be reciprocal to X_1 and X_2; in this choice we have $k-3$ arbitrary data. Finally, let A_1, &c., A_k be chosen in this manner, and we have in all—

$$k-1+k-2\,..+1=\frac{k(k-1)}{2}$$

arbitrary data.

Now, in the choice of k screws of the system taken at random, we have $k(k-1)$ arbitrary elements. It follows that, in the selection of a set of k conjugate screws of potential energy from a co-ordinate system of freedom k, one-half the entire number of quantities required to determine the screws remain disposable. We may use precisely the same language with reference to a set of conjugate screws of kinetic energy. Hence it is manifest that k screws of the co-ordinate system can be chosen, which are conjugate screws both of kinetic and potential energy. The screws thus defined are the harmonic

screws of which we are in search. Let A_1, &c., A_k be the k harmonic screws. Let X_1, &c., X_k be the corresponding impulsive screws; and let Y_1, &c., Y_k be the corresponding screws of potential energy. We shall first show that, if a body be displaced by receiving a twist about A_1, the motion will consist of twist oscillations for ever, performed about A_1. Let P_1, &c., P_{6-k} be $6-k$ screws, reciprocal to the system A_1, &c., A_k. Then the $8-k$ screws, X_1, Y_1, P_1, &c., P_{6-k}, must all be reciprocal to A_2, &c., A_k; therefore, X_1, Y_1, P_1, &c., P_{6-k} must be members of a co-ordinate system of freedom $7-k$. Hence the wrench about Y_1, produced by the twist about A_1, can be decomposed into wrenches about X_1, P_1, &c., P_{6-k}. But of these wrenches those about P_1, &c., P_{6-k}, to the number of $6-k$, are counteracted by the reactions of the system. Hence the remaining component of Y_1 is a wrench about X_1. But the effect of a wrench about X_1 will be merely to twist the body about A_1. It is true that X_1 is the screw about which an impulsive wrench produces an instantaneous twist velocity about A_1, whereas the wrench we are now considering cannot be considered an impulsive wrench; but it is manifest that it can be considered resolved into an infinite number of infinitely small impulsive wrenches. The "centrifugal force" produces a wrench of an order proportional to the square of the angle of twist, and is, therefore, negligible in comparison with the wrench we have been considering, which is directly proportional to the twist. For, since the sum of the kinetic and potential energies is constant, the kinetic energy, which is proportional to the square of the twist velocity, must be of the same order as the square of the angle of twist; but the wrench produced by centrifugal force is proportional to the square of the twist velocity.

We shall now show that, in general, the small oscillations must be compounded of k twist oscillations about the k harmonic screws.

The initial twist about any one screw of the co-ordinate system can be decomposed into k initial twists about the k harmonic screws. The initial twist velocity can be decomposed into k twist velocities about the k harmonic screws. The k twists about A_1, &c., A_k produce wrenches about Y_1, &c., Y_k, which may, as we have already shown, be replaced by wrenches about X_1, &c., X_k. Now, each of these wrenches will operate independently of the others; but each of them tends to make the body perform harmonic twist oscillations about a

harmonic screw. Hence the entire motion is compounded of twist oscillations about harmonic screws.

88.* The case of three degrees of freedom possesses such exceptional interest that we shall enter into some further detail. The theorems now to be enunciated are, it is believed, but a small proportion of those which may be discovered by diligent inquiry. We shall, in the present and following articles, approach the subject from a somewhat different point of view than that previously adopted. It is believed that the additional insight into freedom III., gained by a different method of attack, will more than compensate for the circumstance that some theorems already proved will be again demonstrated in a different manner.

A body has three degrees of freedom when its six co-ordinates are subject to such conditions that only three can be arbitrarily chosen. The most well-known case is, where the body has one point fixed; but other cases may be mentioned. If three of the points be constrained to certain surfaces, or, if one point be restrained to a curve, and another to a surface, or if a surface, rigidly connected with the body, be constrained to touch three surfaces, or if a body be free both to revolve around and slide along an axis confined to a given ruled surface. In all such cases, of which an infinite number can be given, the body enjoys three degrees of freedom.

To study the nature of the freedom, so far as small displacements are concerned, it is sufficient to bring the body from its initial position into any three adjacent positions. The three screws are then to be found, twists about which would have accomplished the small displacements. These screws determine every other displacement which it is competent to the body to accept.

Let the three screws be P, Q, R, and let the pitches of these screws have magnitudes p, q, r $(p > q > r)$. Draw the cylindroid (P, R), and select the pair of screws P_1, R_1 upon the cylindroid, which have the pitch q. The three screws P_1, Q, R_1 are members of the co-ordinate system determined by P, Q, R.

* The present Article was read as a Supplement to the original Paper at a Meeting of the Academy, on June 24th, 1872, and has been added by permission.

If the body receive all possible displacements, the system of screws corresponding to these displacements form the co-ordinate system. The theory of reciprocal screws provides three critical equations to ascertain whether any screw belongs to the co-ordinate system. For every co-ordinate screw must be reciprocal to each of three screws, which themselves are reciprocal to the given screws.

We might also proceed as follows :—Draw the two cylindroids (P, R) (R, Q): on each of these cylindroids there are, in general, two screws, whose pitch is zero. Taking two of these screws from one cylindroid, and one from the other cylindroid, we have three screws of zero pitch, P', Q', R', co-ordinate with the three original screws. Should the screws of zero pitch be imaginary, we select from the two cylindroids any three screws of equal pitch Then diminish all the pitches by the magnitude of the pitch of these three screws (Art. 82). We thus have a new co-ordinate system, three of whose screws have zero pitch. We shall, therefore, suppose that, without the slightest sacrifice of generality we have replaced the three given screws P, Q, R by P', Q', R', whose pitches are zero.

Draw three transversals L, M, N to meet the three screws P', Q', R'. If we consider L, M, N as screws, each with zero pitch, then they will be members of the reciprocal system. It follows from Art. 37 that any transversal of zero pitch meeting L, M, N is a member of the co-ordinate system, while any transversal with zero pitch meeting P', Q', R' is a member of the reciprocal system.

Remembering that a constant may be added to all the pitches of a co-ordinate system, we infer that—When a body has three degrees of freedom, all the screws of a given pitch lie upon an hyperboloid, and form one of its system of generators, while the other system of generators, with an equal and opposite pitch, belong to the other system (see Art. 81).

We shall now prove that the axes of the hyperboloid, when proper pitches have been assigned to them, are also members of the co-ordinate system.

Draw tangent planes to the hyperboloid at the extremities of one of the principal axes, and also draw a plane through the centre parallel to the tangent planes. Each tangent plane contains a co-ordinate screw; these

screws are at equal distances from the centre, and they are inclined to the principal axes in the parallel section at equal angles. It follows that, if a cylindroid be described through the two screws, then the principal axes of the hyperboloid will be generators of the cylindroid, and therefore co-ordinate with the system, when they receive the proper pitch.

Let the hyperboloid be—

$$ax^2 + by^2 +, cz^2 + abc = 0\,;$$

and let us determine the pitches Y, Z of the axes of y, z.

The pitch distribution on the cylindroid is represented by—

$$\tfrac{1}{2}(Y+Z) + \tfrac{1}{2}(Y-Z)\cos 2\theta.$$

The screws common to the hyperboloid and cylindroid are parallel to the asymptotes of the section—

$$by^2 + cz^2 + abc = 0\,;$$

whence—

$$\cos\theta = \sqrt{-\frac{c}{b}} \quad \cos 2\theta = \frac{c+b}{c-b}.$$

Also, we have the equation (Art. 7)—

$$z = \tfrac{1}{2}(Y-Z)\sin 2\theta\,;$$

whence we deduce—

$$\tfrac{1}{2}(Y-Z)\frac{\sqrt{-4bc}}{b-c} = \sqrt{-bc}.$$

$$\tfrac{1}{2}(Y+Z) + \tfrac{1}{2}(Y-Z)\frac{c+b}{c-b} = 0.$$

Combining these equations with those found by taking the other pairs of axes, we deduce—

$$X = a.$$
$$Y = b.$$
$$Z = c.$$

Each of the axes of the hyperboloid is a co-ordinate screw, whose pitch is inversely proportional to the square of the corresponding axis.

If we diminish the pitches of all the screws by T, the quadric—

$$(a-T)\,x^2+(b-T)\,y^2+(c-T)\,z^2+(a-T)\,(b-T)\,(c-T)=0,$$

is the locus of the screws of zero pitch in the altered system, and therefore of the pitch T in the original system. The other system of generators, with the pitch $-T$, are reciprocal screws.

The centre O of the system of hyperboloids is the only point through which three co-ordinate (or reciprocal) screws can be drawn at right angles. It is easy to see that O must be unique; for, if there be a second point O' possessing the same property, it would be possible to draw about each as centre an hyperboloid, containing co-ordinate screws of pitch T; but this is impossible, for it is easily shown that all the screws of pitch T lie upon the same hyperboloid.

The point O being unique is manifestly of the greatest importance in the theory. It may be termed the *kinematic centre* of a body having three degrees of freedom.

The quadric—

$$ax^2+by^2+cz^2+abc=0,$$

must, when a real surface, be an hyperboloid of one sheet, and may be termed the *pitch hyperboloid.* The pitch hyperboloid determines the co-ordinate system of three degrees of freedom, and its reciprocal system, as completely as the cylindroid determines the co-ordinate system of two or four degrees of freedom.

We have already seen (Art. 41) that nine data are necessary to determine completely a co-ordinate system with three degrees of freedom. The nine most suitable for this purpose will be those which determine the pitch hyperboloid. We have, therefore, to show how, when the pitch hyperboloid is known, all the screws of the co-ordinate system are determined. The first theorem to be proved is thus enunciated.

The pitch of any screw of the system is proportional to the inverse square of the parallel diameter in the pitch hyperboloid.

This we have already seen to be true in the case of the three co-ordinate screws which lie along the axes of the pitch hyperboloid.

The equation of the cone drawn through the kinematic centre, whose generators are parallel to all the screws of the co-ordinate system which have the pitch T is—

$$(a-T)\,x^2+(b-T)\,y^2+(c-T)\,z^2=0.$$

If ρ be the intercept on a generator of this cone by the pitch hyperboloid, we have—

$$T=-\frac{abc}{\rho^2}$$

which proves the theorem.

This is a theorem of importance, giving, as it does, a concise view of the distribution of pitch throughout the co-ordinate system. The pitch hyperboloid, with two other quadrics (to be presently considered) contain the entire theory of the mechanics of a rigid body, with three degrees of freedom, so far as its equilibrium, small oscillations, and small displacements are concerned.

Up to the present we have been analysing the system with three degrees of freedom, by classifying the co-ordinate system into families of screws, each of which possesses the same pitch. Some interesting features will be presented by adopting a new method of classification. We shall now divide the general system into groups of screws, which are parallel to the same plane.

We shall first prove that any one of these groups must lie upon the same cylindroid. For—

Draw a normal to the plane, and consider it as a screw of infinite pitch, then all the screws of the group parallel to the plane are reciprocal to the screw of infinite pitch, normal to the plane. Therefore, the screws of the group are reciprocal to four screws (viz., any three screws of the original reciprocal system and this normal screw), and must consequently themselves form a system with two degrees of freedom—that is, they must lie upon a cylindroid.

We shall now prove this in another manner.

In a system with three degrees of fredom, the hyperboloid containing a screw must touch every plane passing through the screw. The number of

screws of the system which can lie in a given plane, is, therefore, equal to the number of the hyperboloids of the system which can be drawn to touch that plane.

The quadric—

$$\frac{x^2}{(b-T)(c-T)}+\frac{y^2}{(a-T)(c-T)}+\frac{z^2}{(b-T)(a-T)}+1=0$$

touches the plane

$$Px+Qy+Rz+S=0.$$

If—

$$P^2(b-T)(c-T)+Q^2(c-T)(a-T)+R^2(a-T)(b-T)+S^2=0;$$

whence it follows that two values of T can be found, or that two hyperboloids can be made to touch the plane, and that, therefore, two screws of the system and, of course, two reciprocal screws, lie in the plane.

From this it follows that all the screws of the system parallel to a plane must lie upon a cylindroid. For, take any two screws parallel to the plane, and draw a cylindroid through these screws. Now, this cylindroid will be cut by any plane parallel to the given plane in two screws, which must belong to the system; but this plane cannot contain any other screws; therefore, all the screws parallel to a given plane must lie upon the same cylindroid.

We propose now to solve this problem. Given a plane, determine the cylindroid, which contains all the screws selected from the co-ordinate system of three degrees of freedom parallel to that plane.

Draw through O the kinematic centre a plane A parallel to the given plane. We shall first show that the centre of the cylindroid required lies in A.

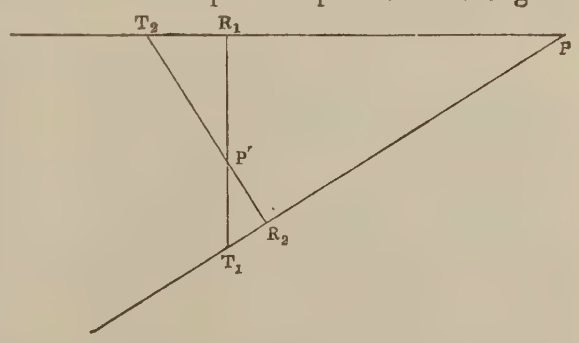


Fig. 5.

Let T_1, T_2, (Fig. 5) be two points in which the hyperboloids touch any plane parallel to A; then P is the intersection of the pair of co-ordinate screws, which lie in that plane, and P' is the intersection of the pair of reciprocal screws. Since $R_1\ T_1$ is to be reciprocal

to T_2 R_1 it is essential that R_1 be a right angle, similarly R_2 is a right angle. The reciprocal cylindroid, whose axis passes through P', will be identically the same as the co-ordinate cylindroid, whose axis passes through P; but the two will be differently posited. If the angle at $P = 90°$, the points T_1 and T_2 are at infinity; therefore, the plane touches the quadric at infinity; it must, therefore, touch the asymptotic cone, and must, therefore, pass through the kinematic centre; but P is the centre of the cylindroid in this case, and, therefore, the centre of the cylindroid must lie in the plane A.

The position of the centre of the cylindroid in the plane A is to be found by the following construction:—Draw through the kinematic centre O a line conjugate in the pitch hyperboloid to the plane A. Let this line intersect the pitch hyperboloid in the points P_1, P_2, and let S, S' (Fig. 6) be the feet of the per pendiculars let fall from P_1, P_2 upon the plane A. Draw the asymptotes OL, OM to the section of the pitch hyperboloid, made by the plane A. Through S and S' draw lines in the plane A, ST, ST', $S'T$, $S'T'$, parallel to the asymptotes, then T' and T are the centres of the co-ordinate and reciprocal cylindroids, all the screws of which are parallel to A.

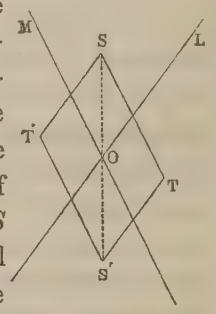


Fig. 6.

This construction is thus demonstrated:—

The tangent planes at P_1, P_2 each intersect the surface in lines parallel to OL, OM. Let us call these lines P_1L_1, P_1M_1 through the point P_1, and P_2L_2, P_2M_2 through the point P_2.

If then $P_1 L_1$, $P_2 M_2$ be co-ordinate screws, P_1M_1, $P_2 L_2$ are reciprocal screws.

For, since OL is a tangent to the pitch hyperboloid, it therefore must be intersected by two rectilinear generators, one of each system. These two generators lie in a plane which contains OL; but since OL touches the hyperboloid at infinity, the lines on the surface must be parallel to OL, and therefore their projections on the plane of A must be $S'T$, ST'. Similarly for ST, $S'T'$; hence ST' and $S'T'$ are the projections of two co-ordinate screws, and therefore the centre of the cylindroid is at T'. In a similar way it is proved that the centre of the reciprocal cylindroid is at T.

Having thus determined the centre of the cylindroid, the remainder of the

construction is easy. The pitches of the screws on the surface must be proportional to the inverse square of the parallel diameters of the section of the pitch hyperboloid made by A. Therefore, the greatest and least pitches will be on screws parallel to the principal axes of the section. Hence, lines drawn through T' paralled to the external and internal bisectors of the angle between the asymptotes are the two principal screws of the cylindroid. Thus the problem of finding the cylindroid is completely solved.

The following theorem is evident from the preceding. If two screws at right angles can be drawn through a point co-ordinate with a system possessing three degrees of freedom, the plane of the two screws passes through the kinematic centre.

It can also be shown that the three tangent planes to the three pitch hyperboloids, which can be drawn through a given point, intersect in a line.

Only one screw of the system can be drawn parallel to a given line; for, draw any two planes through this line, then all the screws parallel to the given line must lie upon the two cylindroids, determined by the two planes. But the two cylindroids cannot have more than one screw in common; for, if they had two screws in common, they would be identical.

This theorem, with many others, would require modification, if one or more of the original screws had infinite pitch. In this exceptional case, the symmetrial features of the system become lost. It is not proposed, in the present Paper, to discuss such exceptions.

The preceding theorems have reference merely to the kinematical relations involved in three degrees of freedom: we proceed now to some mechanical questions of considerable interest.

The first problem to be discussed is the following:—

A certain small quantity of energy being given, it is proposed to determine the magnitude of the twist velocity which this quantity of energy could communicate to the body around any of the screws about which it is free to move. We find that an ellipsoid, which we shall call the *ellipsoid of equal kinetic energy*, presents the solution of this problem in a vivid manner. The solution is as follows:—

Draw the *ellipsoid of equal kinetic energy*; then the quantity of energy necessary to give the body a twist velocity ω around any screw S of the

system is found, by multiplying $\dot{\omega}^2$ by a coefficient proportional to the inverse square of the diameter of the ellipsoid of kinetic energy, which is parallel to S.

This theorem is thus demonstrated :—

The angular velocity $\dot{\omega}$, about any screw S, could be compounded of angular velocities $\dot{\omega}_1$, $\dot{\omega}_2$, $\dot{\omega}_3$, about any three screws of reference A, B, C.

The quantity of energy necessary to impart the angular velocity $\dot{\omega}$ about S would, therefore, have the form—

$$P\dot{\omega}_1^2 + Q\dot{\omega}_2^2 + R\dot{\omega}_3^2 + 2\,L\,\dot{\omega}_1\,\dot{\omega}_2 + 2\,M\dot{\omega}_1\,\dot{\omega}_3 + 2\,N\dot{\omega}_1\,\dot{\omega}_2.$$

Six disposable constants are available in the selection of A, B, C; we may choose these constants, so that A, B, C pass through a point, and so that L, M, N are all zero. We, therefore, have for the quantity of energy to produce a twist velocity $\dot{\omega}$ about any screw of the system, the expression—

$$P\dot{\omega}_1^2 + Q\dot{\omega}_2^2 + R\dot{\omega}_3^2\,;$$

where $\dot{\omega}_1$, $\dot{\omega}_2$, $\dot{\omega}_3$ are twists about three screws of the system which pass through a point.

Taking these three screws as axes of co-ordinates, construct the ellipsoid—

$$Px^2 + Qy^2 + Rz^2 = K.$$

We have then, if ρ be the semi-diameter of this ellipsoid parallel to S—

$$\frac{x}{\rho} = \frac{\dot{\omega}_1}{\dot{\omega}} \quad \frac{y}{\rho} = \frac{\omega_2}{\dot{\omega}} \quad \frac{z}{\rho} = \frac{\dot{\omega}_3}{\dot{\omega}}$$

Whence, for the quantity of energy necessary to give the twist velocity ω about S, we have the expression—

$$\frac{\dot{\omega}^2 K}{\rho^2},$$

which proves the theorem.

In the particular case of a body rotating about a fixed point, the ellipsoid of equal kinetic energy reduces to the well-known momental ellipsoid.

Suppose a body be in stable equilibrium under the action of a conservative system of forces (forces which have a potential). A certain small

quantity of energy being given, it is proposed to determine the magnitude of the twist which this quantity of energy could communicate to the body around any of the screws about which the body is free to move. We find that what will be called the *ellipsoid of equal potential energy* gives the solution, which may thus be enunciated :—

Draw the *ellipsoid of equal potential energy*, then the quantity of energy necessary to give the body a twist around any screw S of the system is found by multiplying ω^2 by a co-efficient proportional to the inverse square of the diameter of the ellipsoid of equal potential energy, which is parallel to S.

This is proved by similar reasoning to the previous theorem.

The parallel sets of conjugate diameters in these two ellipsoids are parallel to the harmonic screws of the system.

The relations of these two ellipsoids to the rigid body, to each other, to the pitch hyperboloid, and to the momental ellipsoid, present a most interesting field of inquiry.

The following theorem is a simple example of the connexion between kinematical and mechanical properties, which is given by the theory of reciprocal screws.

If a rigid body, having three degrees of freedom, be in equilibrium under the action of gravity, the restraints must be such as permit the body to be rotated about an axis passing through the centre of gravity.

This is evident, because the pitch of one of the reciprocal screws through the centre of gravity being zero, the pitch of one of the co-ordinate screws through the same point must also be zero.

d. Four Degrees of Freedom.

89. From the theory of reciprocal screws it follows that the discussion of four degrees of freedom is involved in two degrees. We thus see that a cone of screws of the second order, co-ordinate with the system, can be drawn through a point, and that the cylindroid whose generators are intersected at right angles by the screws upon the cone, is the locus of the reciprocal screws. All the details in the discussion of statical and kinematical questions about two degrees of freedom may be reciprocated into theorems concerning four degrees of freedom.

90. It will, however, be interesting to show independently that a cone of screws of the second order can be drawn through any point in space co-ordinate with any four given screws. Take any point P in space, through P three screws can be drawn co-ordinate with any three of the four screws; therefore, twelve screws at least, co-ordinate with the system, can be drawn through P. Now, we can show that, when even four screws pass through a point, twists about these four screws can compound into a twist about a fifth screw, provided the fifth screw (of proper pitch) lie upon a certain cone of the second degree passing through the four screws. The twelve screws we have mentioned will, of course, lie upon this cone.

Let four screws be P_1, P_2, P_3, P_4, whose pitches are p_1, p_2, p_3, p_4, and direction cosines α_1, β_1, γ_1, &c. If a screw Q, of pitch q, have direction cosines x, y, z, it will belong to the system co-ordinate with P_1, P_2, P_3, P_4, provided that a twist ω about Q can be counteracted by twists about the four screws. For this to be the case we must satisfy the six equations:—

$$\left\{\begin{array}{l}
\omega_1\alpha_1 + \omega_2\alpha_2 + \omega_3\alpha_3 + \omega_4\alpha_4 + \omega x = 0. \\
\omega_1\beta_1 + \omega_2\beta_2 + \omega_3\beta_3 + \omega_4\beta_4 + \omega y = 0. \\
\omega_1\gamma_1 + \omega_2\gamma_2 + \omega_3\gamma_3 + \omega_4\gamma_4 + \omega z = 0. \\
\omega_1 p_1\alpha_1 + \omega_2 p_2\alpha_2 + \omega_3 p_3\alpha_3 + \omega_4 p_4\alpha_4 + \omega q z = 0. \\
\omega_1 p_1\beta_1 + \omega_2 p_2\beta_2 + \omega_3 p_3\beta_3 + \omega_4 p_4\beta_4 + \omega q y = 0. \\
\omega_1 p_1\gamma_1 + \omega_2 p_2\gamma_2 + \omega_3 p_3\gamma_3 + \omega_4 p_4\gamma_4 + \omega q z = 0.
\end{array}\right.$$

From these equations the five quantities, $\frac{\omega_1}{\omega}$, $\frac{\omega_2}{\omega}$, $\frac{\omega_3}{\omega}$, $\frac{\omega_4}{\omega}$, q can be eliminated, leaving a single relation between x, y, z, which is, of course, the equation of the cone. It is easily seen that this cone is of the second order; for, solving for ω_2, ω_3, ω_4 from the three first equations, we have expressions of the form—

$$\omega_2 = A\omega_1 + L\omega.$$
$$\omega_3 = B\omega_1 + M\omega.$$
$$\omega_4 = C\omega_1 + N\omega.$$

Where A, B, C are independent of x, y, z, and L, M, N contain x, y, z in the

first degree. Substituting these values in the three last equations, and eliminating ω_1, q, ω, we see x, y, z rise to the second degree in the result.

91. In forming the equation of this cone, certain simplifications will aid. In general a set of three rectangular generators can be found, for two may be made to slide upon the cone until the third also lies upon it. We, therefore, take three screws, P_1, P_2, P_3, at right angles for the axes, and let α, β, γ be the direction cosines of P_4.

The equation connecting x, y, z is—

$$(p_2-p_1)\,(p_3-p_4)\,\gamma xy+(p_3-p_2)\,(p_1-p_4)\,\alpha yz+(p_1-p_3)\,(p_2-p_4)\,\beta xz=0.$$

It is to be remembered that only the differences of pitches occur in this equation (Art. 82). If we make one pitch zero, we do not diminish the generality of the solution, making $p_4=0$, we have—

$$p_1\,(p_3-p_2)\,\alpha yz+p_2\,(p_1-p_3)\,\beta xz+p_3\,(p_2-p_1)\,\gamma yz=0.$$

It is easy to see that the pitch of a screw upon this cone is one of the three equal quantities.

$$\frac{p_1\,p_2\,(\alpha y-\beta x)}{p_1\,\alpha y-p_2\beta x}=\frac{p_2\,p_3\,(\beta z-\gamma y)}{p_2\,\beta z-p_3\,\gamma y}=\frac{p_1\,p_3\,(\gamma x-\alpha z)}{p_3\,\gamma x-p_1\,\alpha z}.$$

The pitch is zero for the line α, β, γ, and infinite for the line $p_1\alpha$, $p_2\beta$, $p_3\gamma$. Therefore, when a body has four degrees of freedom, one screw can be drawn through every point in space, whose pitch is infinite. In general, one line of given pitch ρ, co-ordinate with four screws, can be drawn through any point in space being the intersection of the cone with the plane—

$$\rho=\frac{p_1\,p_2\,(\alpha y-\beta x)}{p_1\,\alpha y-p_2\beta x},$$

which also passes through the axis of z.

92. The general method already described (Art. 86) shows how the instantaneous screw, corresponding to a given impulsive screw, is to be determined. We also see from Art. 87 the following theorem :—

When a body has four degrees of freedom, and is slightly displaced from

a position of stable equilibrium, under the action of a conservative system of forces, the small oscillations of the body are compounded of four twist oscillations about four harmonic screws; and if the original displacement of the body had been a twist about one of the harmonic screws, and the original twist velocity had been about the same harmonic screw, then the body would have continued to twist for ever about that harmonic screw.

93. In order to have a clear conception of the system of co-ordinate screws, when four screws are given, it is necessary to find the reciprocal cylindroid. The generators of this surface may be thus found :—Take any fifth screw, construct (Art. 95) a screw reciprocal to the five screws, then this screw must lie upon the reciprocal cylindroid, since it is reciprocal to the four given screws. Taking any other screw, we can determine a second screw upon the cylindroid in the same manner. Thus the cylindroid is known. Any screw cutting a generator of the cylindroid at right angles, and having a pitch equal and opposite to the pitches of the two other generators of the cylindroid which the screw intersects, must be a member of the co-ordinate system of freedom IV.

*e. Five Degrees of Freedom.**

94. From some points of view the system now to be discussed is, perhaps, the most interesting in the present mode of considering the mechanics of a rigid body. This is not the less true, because, in the ordinary method of treating the subject, this degree of freedom has received but little attention. Related to the fifth degree of freedom is the extremely important theorem, which expresses that one reciprocal screw, and one only, can be found for every five screws in space. This at once connects the fifth degree, or highest degree, short of absolute freedom, with the first degree, or lowest degree, short

* Professor Sylvester, in an interesting memoir, "Comptes Rendus," tom. lii., p. 741, has considered the involution of six lines in space. In the language of the present Paper, a system of lines of involution would be a selection from the general co-ordinate system having five degrees of freedom, of those screws whose pitch is zero. The theorem due to Moebius, that all the lines in involution with five given lines, which can be drawn through a given point, lie in a plane, is a particular case of the theorem of Art. 26.

of absolute fixedness. We thus learn that a wrench about one screw can be destroyed by the reaction of restraints, which yet permit a body to enjoy five degrees of freedom. We also learn that every line in space can be a screw of the co-ordinate system, provided its pitch be such as to make it reciprocal with a certain screw, and that the pitches of all the co-ordinate screws, which can be drawn through a point, differ by a constant quantity from the radius vector of a pectenoid, of which that point is the origin, and of which the cardinal screw is known.

95. It will, however, be useful to prove, independently of the theory of reciprocal screws, the relation borne by the pectenoid to the case of freedom V. Taking any point O upon one A of the five given screws, a cone of screws co-ordinate with the remaining four can be drawn. We can take any four generators of this cone, B, C, D, E, and thus we have five screws drawn through O, viz., A, B, C, D, E, which are co-ordinate screws of the system. We might see in another manner that five (or more) screws can be drawn through a point co-ordinate with the system. Take any five screws in space, A_1, &c., A_5. If a body receive twists about these screws, the body will attain a position which could have been reached by a twist about a single screw. The five angles of twist have four independent ratios, and thus a co-ordinate screw of the system has four disposable quantities. It can thus be drawn through any point, and in any direction, the pitch however depending upon the direction. Five screws through a point will, therefore, adequately express the freedom. Draw three rectangular axes through the point. Let the direction cosines of the screws be α_1, β_1, γ, &c., the pitches be p_1, &c. Let ρ be the pitch of a co-ordinate screw through the point, whose direction cosines are x, y, z. We have the equation (Art. 90) :—

$$\begin{vmatrix} \alpha_1, & \alpha_2, & \alpha_3 & \alpha_4 & \alpha_5, & x \\ \beta_1, & \beta_2, & \beta_3, & \beta_4, & \beta_5, & y \\ \gamma_1, & \gamma_2, & \gamma_3, & \gamma_4, & \gamma_5, & z \\ p_1\alpha_1, & p_2\alpha_2, & p_3\alpha_3, & p_4\alpha_4, & p_5\alpha_5, & \rho x \\ p_1\beta_1, & p_2\beta_2, & p_3\beta_3, & p_4\beta_4, & p_5\beta_5, & \rho y \\ p_1\gamma_1, & p_2\gamma_2, & p_3\gamma_3, & p_4\gamma_4, & p_5\gamma_5, & \rho z \end{vmatrix} = 0$$

This equation has the form—

$$\rho = K \frac{Lx + My + Nz}{L'x + M'y + N'z}.$$

Let $\rho = R - H$, where R is the radius vector from the origin to the point whose co-ordinates are x, y, z, and H a disposable constant, we have—

$$R\,(L'x + M'y + N'z) = K\,(Lx + My + Nz) + H\,(L'x + M'y + N'z).$$

By a proper determination of H, and transformation of the axes, this equation reduces to—

$$Rz = ay,$$

where z and y are planes at right angles. This is the equation of the pectenoid (Art. 23). H is evidently the pitch of the cardinal screw, whose position is determined by the constant a.

96. The foregoing investigation gives the means of finding the position and pitch of the single screw, reciprocal to five given screws ; but a more direct method of solving this fundamental problem is desirable.

f. Perfect Freedom.

97. A body incapable of moving remains at rest about whatever screw a wrench be applied. The reciprocal statement is, that a body enjoying perfect freedom can be twisted about every screw in space. We shall conclude by stating for this case the theorem with reference to small oscillations really included in the discussion of Art. 87, by making $k = 6$.

Determine the six screws A_1, &c., A_6, which are conjugate screws both for kinetic and potential energy. Determine also the six screws, X_1, &c., X_6, such that X_1 is reciprocal to A_2, &c., A_5, and similarly for the others. Then X_1 is the screw, an impulsive wrench about which will cause the body to twist instantaneously about A_1. X_1 is also the screw, a wrench about which is produced by a twist of the body around A_1 against the forces. It follows, since the centrifugal wrench is evanescent, that

A must be a harmonic screw, about which the body must twist for ever, if once displaced. Also, whatever be the initial displacement, it could be decomposed into six twists about A_1, &c., A_6, and whatever be the initial twist velocity, it could be decomposed into six twist velocities about A_1, &c., A_6. It follows, that whatever be the small oscillations of a perfectly free body, they must be composed of six twist oscillations about six harmonic screws, the angle of twist about each screw being an harmonic function of the time.

DESCRIPTION OF THE PLATES.

Plate XI.—The Cylindroid.

A description of this Model is given in Art. 14. We now add the details of its construction. A boxwood cylinder, $0^{m}15$ long and $0^{m}05$ diameter, is chucked to the mandril of a lathe furnished with a dividing plate. A drill is mounted on the slide rest, and driven by overhead gear. The parameter m (in the present case $0^{m}066$) is divided into one hundred parts. By the screw, which moves the slide rest parallel to the bed of the lathe, the drill can be moved to any number z of these parts from its original position at the centre of the cylinder. Four holes are to be drilled for each value of z. These consist of two pairs of diametrically opposite holes. The directions of the holes intersect the axis of the cylinder at right angles. The following Table will enable the work to be executed with facility. θ is the angle of Art. 7:—

z	θ	$90-\theta$	$180+\theta$	$270-\theta$
0·0	0	90	180	70
17·4	5	85	185	265
34·2	10	80	190	260
50·0	15	75	195	255
64·3	20	70	200	250
76·6	25	65	205	245
86·6	30	60	210	240
94·0	35	55	215	235
98·5	40	50	220	230
100·0	45	45	225	225

For example, when the slide rest has been moved 34·2 parts from the centre of the cylinder, the dividing plate is to be set successively to 10°, 80°, 190°, 260°, and a hole is to be drilled in at each of these positions. The slide rest is then to be moved on to 50 parts, and holes are to be drilled in at 15°, 75°, 195°, 255°. Steel wires, each about $0^{m}3$ long, are to be forced into the holes thus made, and half the surface is formed. The remaining half can be similarly constructed: a length of $0^{m}066 \cos 2\theta$ is to be coloured upon each wire to show the pitch.

The plate represents the model in two different aspects. The boxwood cylinder is mounted upon a stand.

Plate XII.—The Pectenoid.

The Figure represents a Plaster Model of a portion of this surface, which has been constructed in the following manner. A drawing-board is covered with paper, ruled into squares of 1 millimeter. This paper is regarded as the plane of x, y in the surface.

$$z^2 (x^2 + y^2 + z^2) = a^2 y^2.$$

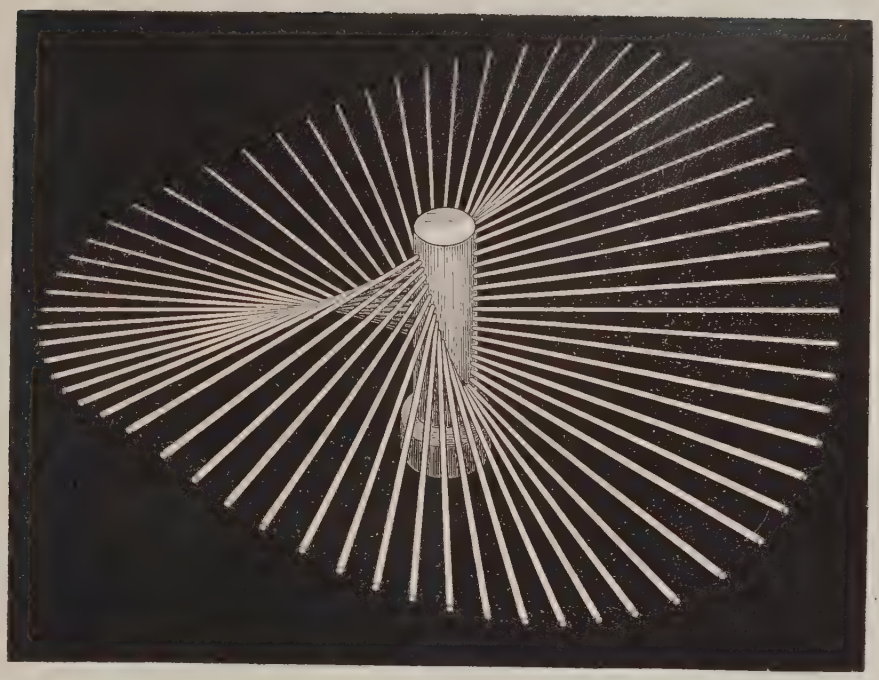
The length adopted for the parameter a is $0^m.04$.

Values of v in millimeters are successively assigned to z, namely, 5, 10, 15, 20, 25, 30, 35, 37, 38; and the hyperbolæ are plotted on the paper, which are the projections of the corresponding contour lines on the surface. A number of pins are driven into the board along these curves, each pin projecting from the surface to the height of the z, appropriate to the hyperbola which the pin intersects. (It will be found to facilitate the modelling if the projecting part of the pins exceeds z by a constant quantity which in the present case has been taken at 5 millimeters.) Sufficient pins having been adjusted to give an adequate conception of the surface, the Model is completed by filling in with modelling clay. A pair of plaster casts of the Model being adapted together, and mounted upon a board, the surface is complete. A wire should be inserted 40 millimeters from the centre, perpendicular to the plane of the paper, to show the cardinal screw; also another wire to show a radius vector.

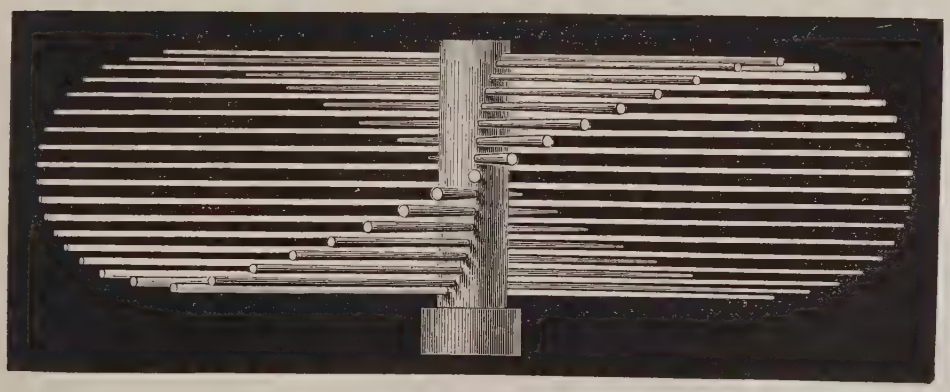
A circle is drawn on the surface in illustration of the property of Art. 26.

GLOSSARY.

Screw, Art. 1.
Pitch, Art. 1.
Twist, Art. 1.
Angle of Twist, Art. 1.
Wrench, Art. 2
Wrenching Force. Art. 2.
Wrenching Couple, Art. 2.
Instantaneous Screw, Art. 5.
Twist Velocity, Art. 5.
Cylindroid, Art. 6.
Cylindroid *contains* a screw, Art. 8.
Cylindroid (A,B), Art. 8.
Parameter of Cylindroid, Art. 9.
Reciprocal Screws, Art. 18.
Pectenoid, Art. 22.
Cardinal Screw of Pectenoid, Art. 24.
Co-ordinate Screws of Freedom, Art. 31.
Displacement Screw, Art, 50.
Restoration Screw, Art. 50.
Conjugate Screws of Potential Energy, Art. 52.
Conjugate Screws of Kinetic Energy, Art. 55.
Impulsive Screw, Art. 55.
Corresponding Screws, Art. 64.
Harmonic Screws, Art. 73.
Pitch Hyperboloid, Art. 88.
Ellipsoid of Equal Potential Energy, Art. 88.
Ellipsoid of Equal Kinetic Energy, Art. 88.
Kinematic Centre, Art. 88.

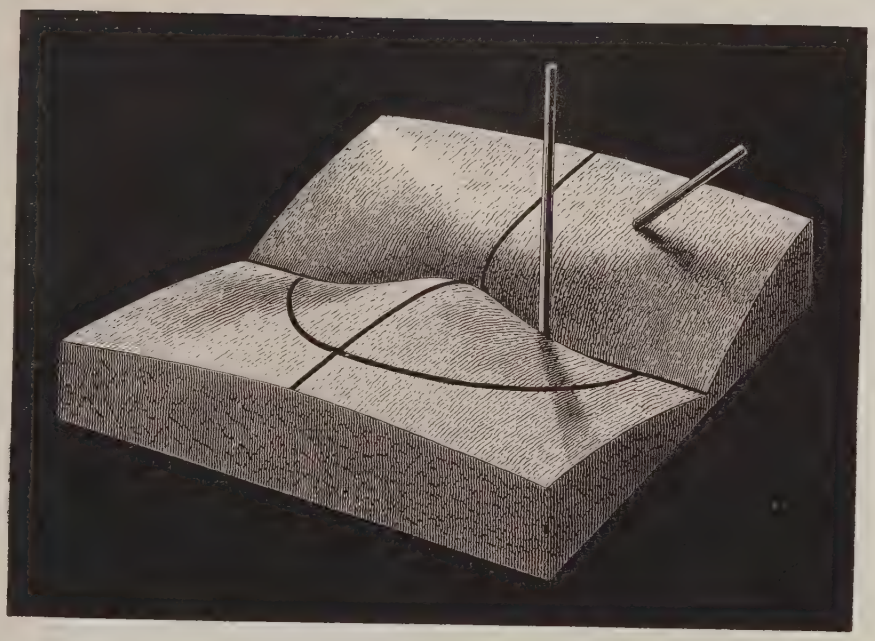


MODEL OF THE CYLINDROID.



MODEL OF THE CYLINDROID.

Viewed edgewise.



MODEL OF A PORTION OF THE PECTENOID.

IV.—*A Monograph on the Anatomy of Chlamydophorus Truncatus* (Harlan), *with Notes on the Structure of other Species of Edentata.* By ALEXANDER MACALISTER, M. B., T. C. D.; President of the Royal Geological Society of Ireland; Professor of Zoology, and Director of the Museum in the University of Dublin.

Read April, 22, 1872.

AMONG the Placental Mammals, the Order Edentata occupies a singular and aberrant position. It does not seem to be connected with the neighbouring groups, and appears, as it were, an intruder, or an unconnected outlier of the mammals. In itself it is an exceedingly compact order, even though its component animals vary within wide limits in form, size, and geographical distribution; yet they are all closely linked together. Having had recently the opportunity of examining a large number of rather rare species of this order, I have compiled the following description of the points of interest in the anatomy, especially in the myology of the order. I was fortunate enough to procure from Mr. Gerrard, the well-known taxidermist, a very beautiful specimen of one of the rarest of the group, Chlamydophorus truncatus, and this prize I have dissected with microscopic minuteness. For purposes of comparison, I have incorporated with my description a record of the dissection of Tatusia peba and hirsuta, Manis multiscutatus, and Dasypus sexcinctus, of which latter I have dissected five specimens. I have been also able to procure parts of Tolypeutes tricinctus, Tamandua bivittata, Cyclothurus didactylus, Orycteropus capensis, Pholidotus Javanicus, Bradypus tridactylus and torquatus, and Cholœpus didactylus. And thus my record will form a tolerably complete account of the myology of the entire order. In order to indicate what has been done in the way of recording the dissections of these animals, I subjoin in a note a complete record of

the bibliography of the muscular anatomy of the Edentates, as far as I am acquainted with it.*

Of all the species examined, the rarest by far is the little Chlamydophorus; and as I have had an exceedingly good specimen for examination, I have made this as perfect a monograph as I could. Some remarks on the history of this animal will form a suitable introduction to the description. In 1825, Mr. Colesberry, of Philadelphia, presented to the Philadelphia Museum of Natural History a specimen from Mendoza. This was described by Dr. Harlan, of Philadelphia ("Annals of the New York Lyceum of Natural History," vol. i., p. 235). This Paper was republished in the "London Zoological Journal," 1825, p. 154. It is very incomplete, having been prepared from a mutilated specimen, from which the viscera had been removed. From this specimen Professor Harlan deduced the following as the generic and specific characters:—"Corpore, suprà testâ coriaceâ, postice truncatâ, squamis rhomboideis, lineis transversis dispositis, conflatâ, subtus capillis albis sericeis obtecto; capite, suprà squamis testâ dorsali continuis, adoperto; palmis, plantisque pentadac-

* The three-toed Sloth has been dissected and described by Meckel ("System der Vergleichenden Anatomie"); Cuvier ("Leçon's d'Anatomie Comparée," 1835); Süsemuhl ("De Musculis in extremitatibus Bradypodis tridactyli," Berlin, 1815); Humphry ("Journal of Anatomy," 1869), and myself ("Annals of Nat. Hist.," July, 1869.)

The Cholœpus didactylus has been examined but very partially published by Mr. Galton ("Trans. Linn. Soc.," vol. xxvi., 1868, p. 524); and by Professor Humphry ("Journal of Anatomy and Physiology," 1869).

The Tamandua has been described by Rapp ("Anatomische Untersuchungen über die Edentaten," Tübingen, 1852).

The Great Ant-eater was dissected and described by Pouchet ("Mémoire sur le grand Fourmilier," 1ere livraison, 1867, and by Owen ("Trans. Zool. Soc.," 1854).

The Orycterope's muscles are described by Humphry ("Journal of Anatomy," vol. ii., 1868); Galton ("Trans. Linn. Soc.," vol. xxvi., p. 567), and figured by Cuvier and Laurillard ("Planches d'Anatomie Comparée," 254–6, Paris, 1855).

The Armadilloes have been described by Cuvier (loc. cit.), Meckel (l. c.), Galton (l. c.) The myology of the Chlamydophorus has been described by Hyrtl ("Denkschrift. der k. k. Akad. der Wissenchaft. in Wien," 9ter Band, p. 1).

The Cyclothurus has been described by Meckel ("Archiv. V.," 1819); Galton ("Annals of Nat. History," 1869), and figured by Cuvier (l. c., pl. 257).

tylis ; unguibus anterioribus longissimis, compressis ; marginibus externis, mucronibus acutis ; caudâ rigidâ, sub abdomine inflexa."

The next recorded specimen is described in the third volume of the "Zoological Journal," p. 544, by Mr. Yarrell. This specimen was also from Mendoza, and sent by Sir Woodbine Parish to the Zoological Society. From this—also a mutilated specimen—additional information was derived, and its Dasypine affinities ascertained. Yarrell's figure, with its mutilated pelvis, was copied into Todd's "Cyclopædia" by Professor Bell (vol. ii., p. 53, fig. 38). No further specimen was minutely described until 1855, when Professor Hyrtl's monograph appeared, in which, with his characteristic accuracy and minuteness, the eminent Vienna anatomist has given a most admirable account of the dissection of a female specimen, in the "Denkschriften der kaiserliche Akademie der Wissenschaften" (Neunter Band, s. 1). This elegant Latin monograph has added so much to our knowledge of the animal, that it would be presumptuous to attempt a redescription of the creature's anatomy, were it not that some points were still left unsettled, which the dissection of one specimen was not sufficient to solve. To this work we will have frequent occasion to refer.

Another specimen, in a mutilated condition, was sent to Great Britain, and its pelvis was described by Dr. J. E. Gray ("On the Pelvis of Chlamydophorus truncatus," Proc. Zool. Soc., 1857, p. 8); and still another has been more recently described by Mr. Edward Atkinson, of the Leeds Museum ("Journal of Anatomy and Physiology," vol. v., p. 1). This author gives a list of eleven specimens recorded in Museums, which is as follows :—

1. Museum of Natural History, Philadelphia.
2. British Museum.
3. Museum of the Academy of Sciences, Berlin.
4. Museum of Buenos Ayres.
5 and 6. University Museum, Halle (one imperfect)
7 and 8. Imperial Museum, Vienna (one immature).
9. British Museum.
10. Hunterian Museum.
11. Leeds Museum.

To these we have to add the two which I have now to describe :—

12. Museum Royal College of Surgeons Ireland.
13. Museum of Dublin University.

The Dublin University specimen measured 4·4 in. long; it was 1·9 in. high at the shoulder, and 1·7 at the posterior end of the body before being dissected. When stuffed, and placed in as natural a position as possible, the dimensions are as follows :—length, 4·45 ; height at shoulder, 2·1 ; height at posterior part of the body, 1·8. The leading measurements are as follows :—

	T.C.D.	R.C.S.I.	Prof. Harlan's.	Leeds Specimen.
Length from top of pelvic shield to top of snout,	4·30	5·0	5·20	
Breadth of pelvic shield,	1·82	1·80	1·80	
Height of do.,	1·25	1·10	1·30	
Length of tail,	1·33	1·45	1·20	
Length of fore limb,	1·50			
Length of hind limb,	1·32			
Length of head,	1·00	. .	1·60	1·5
Greatest width,	1·90	2·20		
Length of innermost claw (fore foot),	0·50			
Length of second do.,	0·55			
Length of third do.,	0·67	. .	0·75	
Length of fourth do.,	0·60			
Length of outer do.,	0·57			
Breadth of outer do.,	0·20			
Greatest width of spatular expansion of tail,	0·30			
Breadth between the eyes,	0·70	. .	0·80	
Girth posterior to the shoulders,	5·30	. .	4·00	
Length of the sole including the nails (hind foot),	1·00	. .	1·20	
Breadth of the sole,	. .	. .	0·30	
Length of nails (hind foot),	0.30	. .	0·20	
Length of fore foot,	0·90	. .	1·40	

The general description of the appearance of this animal is so well known as not to require a special notice here. The dorsal mantle or cloak from which the generic name is derived, is closely bound down to the anterior portion of the cranium, and to the border of the great pelvic disk. This mantle

is also fastened along the spines of the vertebræ by vincula of integument. This cloak consists of twenty-five rows of scales (twenty-seven in R. C. S. I.; in Hyrtl's twenty-four), each row overlapping its successors from before backward; of these the two hindmost are narrow, and they gradually widen to the middle, and each row consists of from twelve to twenty-three scales; the central ones are the longest, and possess about twenty-three each; the hindermost consist of about twenty, and the anterior gradually contract to twelve; the supra ocular row consists of five large scales. These scales are fulvous, darker in the centre than at their edges. The continuation of the dorsal shield over the head consists of seven irregular rows of shield-like scales, extending as far forward as the point of the muzzle. This cloak is margined around its entire free border by rows of stiff hairs or bristles. The surface of each of the scales is composed of a dense series of polygonal flattened epithelial cells, as represented in the figure (Pl. XIII., fig. 5). The marginal hairs, on microscopic examination, appeared subulo-cylindrical, rigid, with a smooth uniform cortical layer, and a faintly cellular slender medullary matter. The body hairs agree in general characters, but are more slender, longer, and more flexible.

The very singular truncation of the hinder part of the body is due to the development of the pelvic shield, which is covered by five concentric rows of scales, of which the outer is placed between two rows of bristles, one set emanating from the hinder margin of the dorsal shield, the other from the interval between the first and second row of pelvic scutes. This row contains eighteen scales; and the second row, separated from the former by the setæ, has the same number. The other rows are more irregular, and the lowest, which surrounds the aperture for the tail, has but eight. The position of the tail is interesting, it being curved downwards and backwards, the flat spatular end being placed horizontally. The tail is thick at the base, where it emerges from the notch in the shield, being 2 in. from before backward, and ·18 in. in transverse width; still lower, just before it flattens, it becomes ·14 in. in anterior posterior width, and the same transversely. The flattened oval end of the tail is ·45 in. long, and ·25 in. at its greatest width. The entire tail is encased in twelve rings of cuticular hardened series of scales, of which five embrace the spatular extremity. The entire length of the tail is 1·33 in., and its under surface, as well as its upper, is enclosed in scales similar to those of the dorsal

shield. Singular, enough, Mr. Atkinson, in his description of this animal ("Journal of Anat." vol. v., p. 13), mentions that the tail is, above, covered with scales; below, smooth. In many of the described specimens, the tail is placed curiously. In Harlan's original specimen, it is curved under, and lying close against the belly. Yarrell has adopted the same idea regarding its place, and in Hyrtl's specimens it was similarly bent. Atkinson's specimen had lived for some time in captivity, and had its tail extended exactly as in mine. The arrangement of the mantle of scales, and the structure of the vincula along the spines, are very beautifully delineated by Professor Hyrtl (Pl. I., fig. 2).

Many of the hitherto published figures do not agree with my specimen in detail. Professor Harlan's has a larger, brighter, and more exposed eye than mine, and a far greater variety of dorsal curves. Its hind legs are thicker, and the conformation of the hind foot represented, does not at all agree with the very remarkable form which that part has in nature. The tail of Mr. Yarrell's specimen is much more raised, and less sharply curved; the foreclaws are smaller and rounder than in my male specimen. He figures also on the left foot an outer scale which looks like a sixth claw, and which has no existence in my specimen; the hinder claws he also makes much thicker and shorter than they are in nature (loc. cit., Pl. XVI.).

The Dasypine affinities of Chlamydophorus are strengthened by the discovery of an allied animal in which the dorsal cloak is attached for its entire extent. The exceedingly rare and beautiful species thus characterized was discovered by Dr. Burmeister, in Santa Cruz, at the foot of the Bolivian Cordilléra, and named by him Chlamydophorus retusus. Gray has, however, erected this into a genus under the name Burmeisteria. In this species the body is longer, and the diagnostic characters are given as follows:—

"C. retusus; major, chlamyde dorsali lateribus corporis adnata; subtus cum artubus intus vellere undulato sublanuginoso indutus; cauda thecaque anali imperfectè cataphractis (Burmeister 'Ein neuer Chlamyphorus' in 'Abhandlungen d. nat. Gesellsch., zu Halle,' 7ter Band, S. 171)."

Dr. Harlan, in his original specimen, compared the texture of the dorsal cloak to sole leather, than which he says it is denser; the rows of scales in his specimen included from fifteen to twenty-two scutes, and the external ears are described as hidden in the long silky hair, and situated directly behind the

eye, furnished with a patulous opening. The eye, he also describes as concealed in the hair, small and black. He also describes the rictus of the mouth as small, a point in which his specimen does not agree with mine ; as far as can be judged from his profile, if it be accurate, the length of the mouth opening from the middle to the angle seems to be three lines ; in mine it is five lines ; in Professor Burmeister's C. retusus, it seems about the same. Professor Hyrtl objects to the use of the word *chlamys* for the dorsal scaly schield, as that name implies a loose flowing drapery. He substitutes *lorica* or *theca*, perhaps the English word *shield* would most accurately express the true relations of this structure. There is no confusion in speaking of the cloak as the dorsal shield, the posterior downward continuation of it as the pelvic shield, and the bony support of the latter as the sphaeroma ischii.

The extent to which the dorsal shield covers the body varies ; at the head on the level of the eye it covers three-elevenths ; at the posterior border of the scapula it covers four-elevenths of the entire circumference ; at the pelvis in front of the hind limbs five-elevenths.

As I wished to obtain as preparations, a skeleton as well as a skin from this individual, I took great pains in the skinning of it, and found it exceedingly difficult ; the extremely close connexion of the pelvic shield to the sphæroma, the close connexion of the skin to the hands and feet, and the inward extension of the skin beneath the sphæroma, rendered the task of skinning one of extreme tedium and trouble. The sixth row of dorsal scutes corresponds, as Hyrtl notices, to the occiput. As before noticed, my specimen agrees with Hyrtl's in having a wide mouth, his being nine lines from the truncated snout to the angle of the mouth. The eyelids are narrow, and strongly guarded with hairs ; the eye is placed four lines above the angle of the mouth ; in Professor Hyrtl's specimen, it is six lines ; the external ear is ·09 in. behind the external canthus, and in Hyrtl's half a line. The orifice has a posterior cartilaginous border covered with hairs. This cartilage is not quite a ring, and in my specimen can scarcely be called cup-like (Atkinson, p. 5). I could not find in my male specimen any rudimental mammæ ; in the female they are pectoral, and two in number. In the following description, I have not quoted Professor Hyrtl's account of the female reproductive organs, as it has been already translated by Mr. Atkinson, and will be found in his Paper (l. c., p. 11).

SKELETON.

The labours of Yarrell, Hyrtl, Harlan, Gray, and Atkinson, on this part of the subject, leave little to be desired.

The scapula is well described by Hyrtl, with its long down-curved acromion, and its tricipital spine. It has a wide coraco-humeral, and a strong spino-glenoid ligament; the costo-coracoid ligament is also strong; there is no separate mesoscapular segment, unless the long tip of the acromion is such. The bicipital groove in the humerus is completely surrounded by bone, making a bony canal. The lower end of the humerus is very wide, ·4 of an inch; the entire length of the bone being ·7; both the brachial artery and median nerve pass through the supra-condyloid foramen. The radius is small, and its motion limited; the ulna is much larger, and the extremity of the olecranon is strongly incurved. The sigmoid cavity is in the centre of the length of the ulna—an equal length being in front and behind (·45 in.); there is no lateral motion in the elbow-joint. The carpus consists of the elongated pisiform which articulates with the cuneiform and unciform, of a scapho-lunar, cuneiform, multangular major and minor, os magnum and unciform; the first metacarpal is slender, the second also slender, but longer; the third the strongest; the fifth very short and very thick. The other parts of the skeleton are exactly as described by Hyrtl and Atkinson.

MYOLOGY.

The myology of the Chlamydophorus is of extreme interest, and though it is so closely allied to the Armadillo that there is some truth in Professor Hyrtl's remark that "qui dasypodum myologiam noscit, in Chlamydophoro nostro paucissima insolita atque nova reperiet," yet there are some exceedingly curious deviations from the Dasypine type which renders the study of these muscles of interest. We will consider the cutaneous muscles first, then those of the trunk, and lastly those of the extremities.

PANNICULUS CARNOSUS.

The remarkable method of arrangement of the dorsal shield renders the arrangement of this muscle different from that in the Armadilloes. In Chlamydophorus, this muscle is divisible into two parts, which, however, are more or

less connected ; there is no portion connected to the sphæroma ; a weak and exceedingly thin slip from the abdomen to the outside of the thigh ; on the middle of the back, and from this to the pelvis it is scarcely demonstrable, but over the scapula a series of fibres appear, which pass up to the cephalic shields into which they are inserted. This portion is perfectly separable from the trapezius, but the uniting fibres described by Hyrtl (p. 29), I did not find.

No trace of muscular fibre existed in the vincula of the dorsal shield along the line of the spines of the vertebræ.

Professor Hyrtl found in his specimen a lateral cervical band going to the side of the head ; this I did not see.

It is surprising that this system of muscles should be so feeble in an animal with a dermal indurated mantle, while the true Armadilloes should show such a strongly marked series of skin-muscles. In Tatusia peba, this series is very well developed, and consists of the following parts :—1st. A distinct abdomino-femoral sheet passing from the middle line of the abdomen to the front edge of the femur ; 2nd. Abdomino-tergal, from the upper part of the middle line of the abdomen as a distinct fascicle, crossing the forearm near the elbow, to be inserted into the dorsal shield on its deep surface; this slip crosses in front of the large lateral band to be referred to presently ; 3rd. Ischio-tergal, a wide flat muscular sheet arising from the surface of the tuber ischii, passing upwards and inwards, to be inserted into the deep surface of the pelvic shield ; 4th. Pectoro-brachial from the middle line of the right pectoral integument to the fascia on the inner side of the arm ; this is very weak ; 5th. Dorso-pectoral, a very strong flat slip from the integument over the clavicular pectoral, running upwards, and forwards, and outwards, to the anterior angle of the dorsal shield ; 6th. An exceedingly strong and definite slip from the angle of the mouth, and from the skin over the side of the jaw to the lateral border of the dorsal shield, as far as the upper border of the elbow ; 7th. A deeper, longer, similar slip above and behind the last, and continuous with the great back sheet ; this runs along the margin of the mantle as far as the posterior end of the shield ; the dorsal fibres in connexion with this run outwards from the mesial line, and are connected with the borders of the central dorsal bands. These are powerful enough partly to roll up the animal, and to draw in for protection the side of the head ; the seventh slip is connected to the

inferior trapezius, and can only be separated from it with difficulty. This description will be found to differ slightly from that of Allesandrini in the same species ("Nov. Commentar. Acad. Bonon.," 1869), but it agrees with what I have found as the arrangement of the dermal muscles in other Armadilloes. In Cyclothurus, the abdomino-femoral is fused with the external oblique, and its femoral fibres are strongly marked, keeping the thigh semiflexed (Galton, p. 2, Cuv. and Laurillard, Pl. 237); the part corresponding to the second or abdomino-tergal is very closely connected to the pectoralis quartus; the sixth, or the platysmal portion, is very feeble, and there is no trace of the third, fourth, or fifth slips mentioned above; the seventh portion is also very feeble.

In Orycteropus and Pholidotus the platysmal portion is very large; in the former it extends "from the labial commissure to the lower extremity of the thorax," especially thick in the latter region, and constituting a true *sternalis brutorum* (Galton, p. 568). The posterior margin of the cervical portion is thinner, and is stretched over the enormous salivary cervical glands; this muscle below is connected to the middle line of the sternum from the sterno-mastoid back for two-thirds the length of the bone, sending fibres to the abdominal fascia, and others to the pectoro-brachial portion. The outer part of this great platysma ascends to the zygomatic arch, and stretching outwards over the shoulder, can be traced into the fascia over the radial margin of the forearm.

This part of the muscle is represented by Cuvier as a special sheet under the name "dermo-humerien," while he calls the former part "thoracico-fascien;" they are not, however, truly separate, but are to be regarded as two partly separate elements of the one platysmal sheet. Mr. Galton follows Cuvier in separating them, but they are always connected, and often the fibres interlace. In Tolypeutes the panniculus is arranged as in Tatusia, but all its bands are much stronger.

MUSCLES OF THE FACE

Are neither well developed nor numerous in Chlamydophorus; they are the following:—1. Orbicularis oris, a fine well-marked band around the mouth, entirely unconnected with any part of the platysma; it is especially well marked at the angles; Depressor anguli oris, a thin radiating muscle from the part of the mandible in front of the masseter overlapping the buccal gland, and in-

serted into the angle of the mouth ; the facial artery pierces this muscle ; 3. Zygomaticus, a single muscle arises from the zygomatic arch directly below the eye, and passing obliquely forward is inserted into the angle of the mouth and the upper lip ; 4. Levator labii superioris, the largest of all, lies in front of the zygomatic, and arises between the down-turned part of the zygoma and the side of the nasal ridge as far up as within ·1 of an inch of the orbit ; it is inserted into the lip in front of the last ; beneath it the large infra-orbital nerve passes. Hyrtl only found three of these muscles, and does not notice the first. He also describes a special levator anguli oris from the inferior border of the orbit to the angle of the mouth ; this I could not separate from the zygomatic. Hyrtl found no depressor of the angle of the mouth.

The buccinator is a large thin muscle, forming the substance of the cheek, and overlying the buccal salivary gland.

In Tatusia peba the facial muscles are, orbicularis palpebrarum, a thin, wide, muscular ring, scarcely represented in Chlamydophorus—a single zygomatic—a levator anguli oris, arising rather lower than the muscle described above by Hyrtl. I found no depressor labii, nor special levator labii superioris. In the Orycteropus the orbicularis is present, but feeble, connected to the platysma (Galton, p. 568). Levator labii superioris is described by Humphry as arising rather in the locality of a levator anguli oris, below the supra-maxillary (infra-orbital) foramen. This animal has likewise a depressor labii superioris, a levator labii inferioris, a depressor labii inferioris, a corrugator supercilii, and an orbicularis palpebrarum ; the best series of facial muscles in the entire group is in this species. The muscles of the face in Pholidotus I could not examine. In Manis there were only the two orbicular muscles, and a levator of the upper lip.

The ear muscles, as might be expected, are exceedingly feeble. The attrahens aurem, from the zygomatic arch to the anterior and inferior part of the ear tube, is the largest, and it is far more cellular than muscular ; there is a thin areolar expansion posteriorly, in place of the retrahens ; but I saw no muscular fibres in it. Hyrtl says nothing about these rudiments.

The eye-ball is exceedingly small, barely ·1 of an inch in diameter ; its orbital cavity is small and very oblique, and there are four slender recti muscles. I could not detect any obliques. The supra-orbital nerve is

slender, and ramifies under the frontal shield. The nasal is as usual. I could not recognise anything but a feeble trace of an orbicularis palpebrarum.

In Tatusia peba there is a short, thick attollens aurem, arising from the back of the temporal ridge. Retrahens aurem is very large, and overlaps the trapezius. An attrahens also exists, but it is very small; the ear meatus in this species runs downwards, inwards, and forwards, not like that in Chlamydophorus, which passes downwards and backwards.

In Orycteropus there is a triple attrahens arising from the mandible beneath its condyle, from the posterior margin of the orbit, and from the upper border of the same cavity. This animal has also a depressor aurem from the sterno-mastoid to the inferior part of the auricle (Humphry). The retrahens is large, and arises from the middle line of the nucha, and converges to the back of the ear. The attollens aurem is a continuation of the last, from the mesial line of the upper surface of the head, and it converges to the top of the ear.

The muscles of mastication are small; the masseter consists of its usual bilaminar series of fibres, tendinous at origin; it is crossed by the very rudimentary Steno's duct. I did not find it intersected by tendons, as Hyrtl describes, nor did it extend beyond the zygoma to the hinder part of the maxilla; its nerve is as usual from the fifth pair.

The temporal is very small, and otherwise is as usual. The internal pterygoideus is small, and inferior in bulk to the external.

Professor Hyrtl says, "Musculus digastricus omnino nullus adest;" and at first sight I was inclined to agree with him; but a careful examination with a lens enabled me to find an exceedingly small and interesting variety of this muscle. This arises from the bulla tympani by a thin expansion, and passes forwards over the ninth nerve. This belly ends in a tendon, from which a singularly small, short anterior belly is continued to the mandible; the anterior portion is only one-third the length of the posterior, and not one-sixth its bulk.

There is another singular provision for the depression of the jaw—a straight, central muscle from the front of the sternum to the centre of the mandible. This is described by Hyrtl: "Deprimitur maxilla ope musculi, a manubrio sterni recta linea ad menti symphysim advecti, qui cum socio

vicino in unum coalescit fasciculum, ideoque Depressor Azygos seu Sterno-maxillaris nuncupari possit."

This muscle is present in Tatusia peba, and there it is distinctly connected to the platysma; it arises over the sternum, and covers the sterno-hyoid and sterno-thyroid muscles; it is expanded above, and attached to the entire centre of the mandible; it was crossed above from side to side by a transverse band of muscle, which passed from the angle of the mandible on one side to its fellow of the opposite; it is similar to the central slip of the platysma in Orycteropus.

MUSCLES OF THE NECK.

On removing the salivary glands (to be afterwards described) the first muscle brought into view is the Sterno-mastoid, which arises by a pointed fleshy slip from the anterior surface of the sternum, extending down ·3 of an inch from the top of the manubrium; here it is contiguous and slightly connected to its fellow of the opposite side; it ascends outwards and backwards, and, crossed by the spinal accessory nerve, becomes tendinous for about ·3″, and inserted behind the bulla tympani into the par-occipital bone. The entire muscle measures 1·15 inches in length. This muscle was not confluent with its fellow, as Hyrtl describes, but it agreed with his description in being wholly unconnected to the cleido-mastoid. The same description will suit the muscles in Dasypus sexcinctus; but the fibres prolonged on the sternum are feebler in that species. In Tatusia peba there is no prolongation downwards, and the sterno- joins the cleido-mastoid at its par-occipital end; the connexion between the sternal origins of the muscles of each side is much closer in this species. In Cyclothurus the corresponding muscles are unconnected, and the same is the case in Orycteropus and Pholidotes; in the former of these two there is a deeper sterno-mastoid lying external to the main body of the muscle, separated from it by the jugular vein (Galton, p. 471).

Cleido-mastoid in Chlamydophorus underlies the sterno-mastoid, and is once and a half thicker than it; its origin is from the clavicle, extending ·2 of an inch from the sternal end; external to the last muscle, ascending it passes beneath it, and is inserted tendinous into the same place. This muscle

is only ·8 of an inch long. Hyrtl has not referred to the existence of this muscle. In Tatusia peba this muscle arises from the sternal half of the clavicle, and is inserted as in Chlamydophorus. In Dasypus it is attached to the sternal third of the clavicle. In the Orycterope it occupies one-half of the bone, while in Cyclothurus it is only attached to the middle third. In the sloths the two muscles are confluent, and the sternal factor is much larger than the clavicular, which in B. tridactylus is exceedingly feeble, but is rather stronger in B. torquatus. "Cleido-mastoideus enim non ab extremitate sternali claviculæ, sed a fine opposito, acromiali dicto, exoritur;" but the muscle thus described by Professor Hyrtl (p. 31) goes to the ex-occipital, posterior to the true cleido-mastoid, and thus is a true cleido-occipital; it arises from ·15 of an inch of the middle of the clavicle, ascends to be inserted below and behind the long ear-tube by a very wide tendon. A few fibres appeared to be attached to the latter. Its origin is $\frac{1}{4}$ from the acromial, and $\frac{1}{2}$ from the sternal ends. This muscle is ·7 of an inch long. The portio dura nerve emerges from under the anterior border of the insertion of the sterno- and cleido-mastoids, passing beneath, not through, the parotid gland, then crossing the neck of the mandible, it sends a vertical fibre upwards to the ear-tube; rising over the masseter, it sends a large branch above the eye, another forward to the levator labii superioris, and a third, and largest twig, to the zygomatic muscle. The spinal accessory nerve is seen crossing the cleido-mastoid and the cleido-occipital. In no other Edentate does this muscle seem to exist distinctly.

Professor Hyrtl describes a third muscle external to the cleido-mastoid, but much stronger, which ascends with it, and is inserted into the lambdoid suture, and into the temporal aponeurosis; this he regards as the separated clavicular trapezius. This muscle was inseparately joined to the rest of the trapezius in mine, as will be subsequently described. The omohyoid is absent in all the Edentata. Sterno-hyoid is slender and simple in Chlamydophorus, Tatusia, Dasypus, Orycteropus, Tamandua, Cholæpus, and Bradypus. I could not make out any tendinous inscription in any of these.

Sterno-thyroid is three times wider and thicker than the sterno-hyoid; but I could not make out the two strata mentioned by Hyrtl. It is similar in the other species examined.

The peculiarity of the digastric in this animal I have before referred to; it is monogastric, and attached to beneath the mandible in Orycteropus, Bradypus, Cholœpus, Dasypus, and Tatusia.

Styo-hyoid arises from the outside of the stylo-hyal bones, and is inserted by a tendon into the hyoid behind the *cerato-hyal* (lesser cornu.). I did not find the slip of this muscle attached to the fascia between the mylo-hyoid and the genis-hyoid, which Professor Hyrtl mentions. This muscle is similar, and present in all the species examined.

Stylo-glossus is not referred to by Professor Hyrtl; it is twice the size of the last, and arises a little in front of it; passing upwards under the digastric, and the ninth nerve, it is inserted into the side of the tongue; it lies above the stylo-hyoid. These muscles exhibit no interesting varieties in any of the species examined.

Stylo-pharyngeus is very slender, and placed in its usual position.

Genio-hyoid, linguales, genio-glossi are not abnormal in any respect.

Hyo-glossus is very large, and passes upwards from the side of the body, and cornua of the hyoid bone, beneath the stylo-glossus. Mylo-hyoid is thin and cellular.

The deep anterior muscles are—Rectus capitis anticus major, from the front of the transverse process and side of the body of the seventh cervical vertebra to the basi-occipital; it had an attachment, but a slender one only, to the five vertebræ above this; the rectus capitis posticus minor was slender, and arose from the transverse process of the atlas, and was inserted under the major, passing upwards and inwards. The longus colli sprang from the front of the seventh cervical and first dorsal transverse processes, and was inserted into the anterior border of the transverse process of the atlas by a tendon. This muscle is thick and short. This group is similar in Tatusia and Dasypus, as well as in Cyclothurus and the Orycterope, except that in Tatusia, the r. c. a. minor is attached to two vertebræ.

Two scalenes exist in Chlamydophorus, but as they are both behind the subclavian artery and the brachial plexus, they must be regarded as medius and posticus. They arise from the second, third, fourth, fifth, and sixth cervical vertebræ, and are inserted into the first rib under the brachial plexus. The medius is much larger than the posticus, and the latter extends to the second rib. Hyrtl gives the second rib as the insertion of the scalenus medius, and

the second and third as that of the posticus. I could not trace them so far down; but in Tatusia peba the medius goes to the second, and the posticus to the first, second, and third ribs.

MUSCLES OF THE BACK.

Trapezius is distinctly divided into two parts in the Chlamydophorus, a superior and an inferior, separated by a triangular interspace. The superior arises thick and fleshy from the shield over the occipital bone, rising above the occipital superior curved line, and extending from the middle line outwards for ·34 of an inch; it also arises from the mesial line of the neck for ·9 of an inch; its fibres run downwards, backwards, and outwards, to be inserted into the outer ·29 of an inch of the clavicle, the upper edge of the acromion and spine of the scapula. The clavicular part is exceedingly thin, and is supplied by the spinal-accessory nerve. This muscle differs from that in Hyrtl's specimen in these points, that the clavicular part was not separate, was weak, and the rest of the muscle distinct and strong. This muscle is very strong in Tatusia peba, arises from the whole length of the occipital curved line, as well as from the ligamentum muscle. This muscle has no clavicular slips in the Orycteropus, and is not divided into upper and lower parts, either in that animal nor in the Cyclothurus. In none of the sloths is it split, nor has it a clavicular insertion (Humphry and Meckel, however, found some fibres attached to the clavicle; I only found this in B. gularis). In Cyclothurus it is attached to the outer fourth of the clavicle, but in Dasypus it is not connected to this bone at all. In Pholidotus the muscle is partly divided, and this portion is continuous with the deltoid, flowing over the spine of the scapula and shoulder, with a faint superficial clavicular inscription.

Trapezius inferior arises from the spines of the upper dorsal vertebræ, commencing ·9 of an inch behind the occipital ridge, and extending for 1·1 in.; its upper fibres run transversely to the root of the scapular spine; the middle and inferior fibres run forward to ·6 of an inch of the posterior border of the scapular spine, especially strong at its middle part; these hinder fibres take origin likewise, as Professor Hyrtl notices, from the mammillary process of the lumbar vertebræ. In my specimen the two parts of the trapezius were not inserted by a common tendon, as Professor Hyrtl found. In Tatusia, the inferior trapezius is attached to the uppermost eight dorsal spines, and its inser-

tion is tendinous, quite distinct from the superior part. In Dasypus, this muscle has an origin from the mammillary lumbar processes, and is inserted into the posterior third (Galton says half) of the spine of the scapula. Mr. Galton found in his Armadillo a difference of colour between the superior or inferior trapezius, the former being paler; this was not the case in any of the specimens dissected by me. The dorsal part of the undivided trapezius in Orycteropus is attached to nine spines, and is inserted by a tendon; it is attached to eight in Cyclothurus.

Rhomboideus capitis (Occipito-scapular, Wood), is a very large muscle arising from the occiput beneath the trapezius, extending ·5 of an inch from the middle line; it passes longitudinally backwards to the posterior margin of the prescapula, and to the edge of the spine of the scapula; its fibres extend at their origin nearly as far out as the ear-tube. This muscle in Professor Hyrtl's animal was divisible into two distinct parts, which he calls respectively extensores capitis, internus et externus. The former he describes as being attached to the anterior angle of the prescapula, and to the testa of the head; the latter to the bulla tympani, and occipital crest, and to the supraspinous fossa. I could not separate naturally my rhomboideus capitis into these elements without violence. In Pholidotus, this muscle is very strong, and is mesoscapular in its insertion, being attached for nearly the whole length of the spine. It is also cleft in this animal into two segments named by Professor Humphry, in Manis Dalmanni, masto-scapular and occipito-scapular. These correspond to Hyrtl's externus and internus respectively. No corresponding muscle exists in Cyclothurus, Bradypus, or Cholœpus, but it is enormous and indivisible in Dasypus, and similar in Tatusia peba, hirsuta, and minuta, in none of which is there a masto-scapular slip. In Orycteropus, the occipital rhomboid is weak, and joined to the rest of the muscle.

Rhomboideus minor is only separable from the major in Chlamydophorus and Tatusia; in the first I found it in the upper part of the cervical region behind the occiput arising from the mesial line of the cervical vertebræ, and inserted into the base of the mesoscapula. In Tatusia, this muscle arises from the lowermost pair of cervical spines, and is inserted as in Chlamydophorus. Rhomboideus major is attached to the four or five anterior dorsal spines, extending for ·7 of an inch; its fibres run outward and backwards to the poste-

rior margin of the post-scapula. In Tatusia peba, it arises from five dorsal spines, and its lower border is overlapped by the latissimus dorsi. In Dasypus, it is attached to six dorsal spines, and closely connected to the latissimus. In Cyclothurus, it arises from five dorsal and two cervical spines, and is neither overlapped nor joined to the latissimus dorsi. In Pholidotus it arises from four vertebræ. In Orycterope, it arises also from four.

Latissimus dorsi is in Chlamydophorus very remarkably arranged, and may be conveniently divided into two portions; the uppermost of these arises from the spines and backs of the transverse processes of three posterior dorsal, and from the spines and mammillary processes of two upper lumbar vertebræ; from these sources the fibres converge to the posterior inferior angle of the post-scapula; it extends for ·7 of an inch behind the rhomboideus major. This portion is accurately described by Prof. Hyrtl. It is unrepresented in Tatusia peba and in Dasypus sexcinctus, or indeed in any others of the Edentates.

The second portion of the latissimus is easily distinguished from the last described, but with difficulty separated from the pectoralis quartus, which it underlies. Its origin is from the five lower ribs, and from the lumbar fascia (Hyrtl found it from six). As this portion is narrowing to its insertion, it has a very wide dorsi-epitrochlear attached to it, and passing this it narrows to a flat ribbon-like tendon, which, as usual, is attached below the inner humeral tubercle, above, in front, and separate from that of the teres major. These tendons are said to be connected in Myrmecophega and Tamandua; but I have not found them so in any Edentate. The latissimus in the other Edentates does not display any noteworthy features; it is only costo-fascial in Tatusia peba (4 ribs).

In connexion with the latissimus dorsi, the dorsi-epitrochlear muscle demands notice. This muscle is constant in all the Edentates, and specially well-developed in Chlamydophorus; its origin is ·35 of an inch wide, and it extends as a broad thin sheet over the inner side of the arm to the inner border of the elbow-joint; and it is inserted into the inner condyle, the internal lateral ligament, and the olecranon, as well as into the fascia of the forearm. This muscle is thin, but enormously wide in Tatusia peba, and, as in Chlamydophorus, the point of junction of the dorsi-epitrochlear and the latissimus dorsi is marked by a tendinous inscription. In the Tatusia, one slip of the dorsi-epitrochlear arose from the teres major. This muscle is very large in

Dasypus, and is well described by Mr. Galton (p. 531). In it the insertion is prolonged even beyond the condyle into the forearm fascia, as in Chlamydophorus; it is similar in the Orycteropus and Tamandua, in which latter, according to Rapp, its origin is, as I have described, in Tatusia. It is even more extensive in Cyclothurus and Pholidotus, going to the palmar fascia, and in the sloths is smaller, passing only to the inner humeral condyle.

Serratus posticus superior I could not find represented; but it is present in all the other Edentates, very feeble in Tatusia, and attached to the second and third ribs; stronger in Dasypus, very thin in Cholœpus and Bradypus.

Serratus posticus inferior arises from the mammillary processes of the lower pair of dorsal and the upper lumbar vertebræ, and is inserted into the four lowermost ribs, that to the eighth being very feeble. Hyrtl's description agrees with mine; it is similar in Tatusia, still larger in Dasypus.

Splenius capitis in Chlamydophorus is very small and weak, and there is no splenius colli. The S. capitis springs from the tip of the spine of the last cervical, and is inserted into the mastoid process; both colli and capitis exist in the Ai, Unau, and Dasypus. In Tatusia there is a single muscle, which, from its origin, represents both the capitis and colli, but it is only inserted into the occiput by a tendon.

Trachelo-mastoideus arises external to the complexus from the transverse process of the upper dorsal vertebræ, and is inserted into the para-mastoid. In Tatusia and Dasypus the insertion is under cover of the splenius. Complexus in these three genera is an enormous muscle; but it is largest proportionally in the Tatusia. In Chlamydophorus it arises from the transverse processes of the lowest six cervical vertebræ, and one or two of the dorsals, and, widening as it ascends, is inserted beneath the occipital rhomboid.

Levator anguli scapula, which is more a "Fixator" than a levator in function (Hyrtl), is inseparable from the Serratus magnus, as it is in Tatusia, Cyclothurus, Orycteropus, Unau, Ai, Pholidotus, and Tamandua, as well as in Dasypus; hence I have described it under the latter muscle.

Rectus capitis posticus major is wide and single. Rectus minor is small; the former did not extend below the axis (Hyrtl, 2 and 3), nor did the latter spring from any bone below the atlas (Hyrtl, 1 and 2). Rectus capitis lateralis arises from the transverse processes of the atlas and axis, and passes to the

exoccipital. Obliquus inferior is wide and fan-like; but I found no obliquus superior. This latter muscle in Tatusia is exceedingly feeble and thin. It is a little stronger in Dasypus, and much more distinct in the sloths.

Sacrolumbalis is perfectly normal, and sends slips to all the ribs. It has a strong series of musculi accessorii. The longissimus dorsi is flat and broad, not so strong as the last, and it terminates below the complexus. The spinalis dorsi is thin and flat, and normal. Beneath these the multifidus spinæ, rotatores spinæ, and semispinales colli et dorsi are inseparable, and form a mass of extremely great size and power, larger proportionally than in any other of the Dasypodidæ—"tanta evolutio est tanquam substernaculo incedere posse." This deep mass is particularly strong in the lumbar region, where its origin extends to the ischiatic fulcra.

Acromio-trachelian is absent, as it is in the Tatusia, Cyclothurus, Bradypus, Cholæpus; but is present and atlantic in origin in the Orycterope. In Dasypus it is supra-occipital in origin. It is united to the occipital rhomboid (outer factor) in the Pholidotus. (The exact relationship of this muscle to the other shoulder muscles is not very clear; in the largest proportion of mammals it is present, and is atlantic in origin, and acromial in insertion, occupying a position beneath the front edge of Trapezius, but most probably, as suggested by many authors, it is a slip of the superficial muscular sheet, intermediate between the trapezial and cleido-occipital factors of that expansion. The elements of this sheet seem thus to be trapezius, acromio-trachelian, cleido-occipital [to which category Galton's levator claviculæ in the Armadillo is to be referred], cleido-mastoid, sterno-mastoid, &c.)

MUSCLES OF THE ABDOMEN.

The entire abdominal wall in Chlamydophorus is so thin that the detection of the separate muscles is a matter of great difficulty. The external oblique is attached to the six lowest ribs; the internal oblique arises from the iliac crest, and is strong below; the transversalis seems to be connected to the lateral processes of the lumbar vertebræ, and is only fleshy for a short distance. The rectus abdominis ascends to the first rib, beneath the pectoral muscle. I could only find a single trace of a tendinous intersection, on the level of the ensiform cartilage; there is an exceedingly delicate sheath enveloping this muscle.

There is no pyramidalis. Prof. Hyrtl describes these muscles, but they were not distinct in his specimen, and he concludes his description by saying—"An musculi recti vagina adsit, et quonam modo ex irretilis aponeurosibus musculorum latorum enascatur, id seriorum investigatorum lynceis oculis inveniendum relinquo." The origins of the recti, as in the bats, are separated below from each other by a hiatus corresponding to the interpubic space.

The abdominal muscles in Tatusia are only remarkable for the great strength of the lower border of the internal oblique, which is at its lower edge more than double the thickness of any other part of the abdominal wall. There is in these all a wide inguinal canal for the reception of the testis. In Dasypus the origin of the external oblique ascends to the third rib; it is not so high in Tatusia.

The quadratus lumborum consists solely of one part of the muscle, the pars ilio-lumbalis; it is elongated and slender, and occupies the side of the lumbar vertebræ external to the psoæ.

Prof. Hyrtl has described, under the name "novus columnæ vertebralis flexor," a strong muscle, which lies inside the thorax, on the side of the spine, arising from the transverse processes of the upper two lumbar and last dorsal vertebræ, and inserted into the six or seven (Hyrtl says six) posterior dorsal vertebral bodies. This muscle seems a continued quadratus lumborum, and will be a powerful agent in bending the dorsal spine. I have found this muscle in no other species of the Edentata.

The caudal muscles form two thick lateral swellings inside the pelvis, and they are grouped by Hyrtl into three sets—superior, inferior, and lateral. I have nothing to add to Hyrtl's description, they are similarly arranged in all the armadilloes and are well developed, especially the superior in Pholidotus and Manis multiscutatus.

MUSCLES OF THE UPPER LIMB.

This extremity is enormously powerful, and its muscles are well developed. The great pectoral is a thick flat muscle, divided into three parts, which are easily separated. The first of these, the clavicular part, arises external to the sterno-mastoid from the lower and inner third of the clavicle; passing outwards it joins the sternal part at its insertion, which is into the most prominent part of the pectoral crest of the humerus; its insertion is ·15 of an inch long, and

it lies below and internal to the clavicular deltoid. This portion was evidently taken by Hyrtl as part of his sternal pectoral, and did not seem to be separate from the latter, nor to arise from the clavicle in his specimen.

The sternal portion arises for ·5 of an inch from the anterior part of the sternum; its fibres pass upwards and outwards to join the last at its insertion; into its deep surface passes the anterior thoracic nerve. The fibres of this part of the muscle average ·6 to ·9 of an inch long; those of the clavicular ·5. The third or abdominal portion of the muscle springs from the abdominal linea alba, as far as 1·5 inches from the top of the sternum; ascends and is inserted partly with the last, and higher up, partly into a fibrous band, which arches from the lesser tuberosity to the deltoidean crest. This portion is described by Hyrtl. Its outer border is close to the fourth pectoral.

In Tatusia peba and minuta the great pectoral is also trifid; but the three portions are separate; the clavicular is smaller, and the third arises from one-half of the abdominal linea alba. In Dasypus there is no clavicular pectoral, and the second and third parts are inseparable; of the former, Galton describes a slip from the ensiform cartilage; but this I never saw in any of the four. This ensiform slip is present as part of the rest of the muscle in Chlamydophorus, but not a separate band. Orycterope, Cyclothurus, Tamandua, Manis, and Pholidotus, have no clavicular origin; but the first-named has an abdominal portion, as also have the last-named pair. The sloths have an undivided pectoral, cleido-sternal in origin, and normal in insertion. The portio-clavicularis of Hyrtl I regard as a deltoid, and his pectoralis minor is the fourth pectoral.

I found no pectoralis minor in Chlamydophorus, nor in Tatusia peba, nor Dasypus sexcinctus. It does not exist in Cyclothurus, Tamandua, Pholidotus, nor in Myrmecophaga (Meckel). I could not separate a pectoralis minor in Ai (B. tridactylus and torquatus); but Prof. Humphry found a distinct one in that animal. Meckel found it as I did, and Cuvier found it like as in Prof. Humphry's.

Pectoralis quartus I found perfectly distinct from the pectoralis major, as well as from the latissimus dorsi, over the costal origin of which it lies. Its origin was from the thorax margin, ·8 of an inch out from the middle line, and extended for ·25 of an inch, occupying two ribs; it is inserted along with the abdominal part of the great pectoral. Its posterior margin is closely applied to the anterior edge of the costal slips of the latissimus, which it overlaps; its

tendon crosses the brachial vessels and nerves; its upper fibres are 1·5 of an inch long. This portion is described by Hyrtl as "quædam adventitia pectoralis majoris portio." I have found this muscle in Tatusia peba arising from six ribs, and ascending under the post-sternal part of the pectoral it is inserted along with the latissimus dorsi. I have also found it inserted with the pectoral, and arising from the sixth to the ninth ribs in Dasypus. I have found it also present in the Ai. It is also present in Orycteropus, and very large; also in Pholidotus, Unau, and Tamandua.

Subclavius is a very large muscle, and takes a wide origin from the first rib at its sternal end; passing under the clavicle, it is inserted into the coracoid process, and the acromion, as well as into the extreme acromial end of the clavicle. Hyrtl did not find any clavicular fibres; but though exceedingly few were attached to that bone in my specimen, yet some were so inserted; he did not find any coracoid insertion. The description of this muscle given by him, as exceedingly well developed, fleshy at its origin, with a special transverse cervical artery, and a large sub-acromial bursa between it and the head of the humerus, is strikingly accurate. Of the three strata into which he says the muscle is divisible, there are two closely united, making up what I recognise as the true subclavius; and the third I have described below separately.

In Tatusia peba the subclavius is also very large, but falls short of the comparative degree of development attained in Chlamydophorus. It is divisible into two parts, an upper costo-acromial, with no clavicular attachment, but otherwise very like the middle part of Hyrtl's three divisions; the inferior part, which arises in front of the last, sent a slip to the head of the humerus.

The subclavius is entirely absent in Cyclothurus didactylus, Manis, Tamandua tetradactyla, Myrmecophaga jubata (Pouchet), and Pholidotus.

This muscle is large in Ai, as I have described, and is attached, as in Chlamydophorus, but is unsegmented; it is much smaller in Cholæpus, which is singular, as the clavicle is better developed in the latter than in the former.

In Dasypus sexcinctus it is strong, large, and costo-acromial in its attachments. In Orycteropus the large subclavius extends inwards to the manubrium, and outwards to the acromion, and supraspinous fascia. Mr. Galton

found in the aard-vark, dissected by him, a sesamoid bone in its insertion, which has not been described or found in any other specimen.

Retro-clavicularis is very distinct in Chlamydophorus, and is the uppermost of the three strata of subclavius, described by Prof. Hyrtl; so distinct, however, is it that it well deserves a separate name, and as, in position, it resembles the muscle described by M. J. Weber in man, under the name retro-clavicularis (Handbuch der Anatomie, 1839, Bd. I., p. 560), I have preferred naming it so. It arises from the upper surface of the first rib at its sternal edge, behind the clavicle, and closely joined to the scalenus medius; passing outwards, free from the clavicle, it is inserted into the acromion process and supraspinous fascia, as well as into the coraco-acromial ligament. I did not find any comparable slip in any other of the Edentates.

There is a distinct circular discoidal interarticular cartilage in the sterno-clavicular joint, concave on the clavicular surface, rather convex on its sternal.

Rectus thoracicus lateralis is very slender and feeble, and arising from the side of the sternum is inserted into the first rib. Prof. Hyrtl does not allude to this muscle at all; it is present in all the Edentates examined. In Tatusia it is very large, and from the manubrium sterni runs downwards and outwards to the cartilage of the lowest four true ribs.

Serratus magnus is also a large muscle in Chlamydophorus, and is inseparably united to the levator anguli scapulæ; its origin is from the seven uppermost ribs by seven digitations (Hyrtl found eight in his specimen), and from the lower third of the cervical vertebræ; the fibres all converged, and were inserted into the usual site along the scapular border. In Tatusia it is divisible into two parts—a superior—including the levator anguli scapulæ, and the upper part of the serratus, arising from the three uppermost ribs, and from the lower half of the cervical region; this part is inserted into the base of the pre-and mesoscapula; the second or posterior part arises from the ribs from the fourth to the eighth inclusive; it is inserted into the inferior angle of the post-scapula. I could not separate this muscle into parts in Bradypus tridactylus or torquatus. However, Prof. Humphry found it divisible in B. gularis into three parts, a levator anguli, a superior and an inferior serratus; it is similarly trifid in the Cyclothurus, and extends down to the seventh rib. In Dasypus it is

bifid, as in Tatusia ; but does not extend below the sixth rib. It is undivided also in Cholœpus, Manis, Orycteropus, and Pholidotus.

Deltoideus is as usual a three-cleft muscle. The first part or deltoideus scapularis is described by Prof. Hyrtl under the name of Portio clavicularis pectoralis majori, and characterised by him as "parva et brevis, ab extremo claviculæ fine, qui cum acromio articulationem efformat, oritur, et in ipsa brachii spina, cum reliqua Pectoralis majoris carne conciliatur." I found it to arise from nearly ·3 of an inch of the outer extremity of the lower edge of the clavicle, and to be inserted, as Prof. Hyrtl describes, into the front of the deltoid crest above the pectoral and rest of the deltoid, but separate from both. I could not detect from what nerve it derived its supply. Its fibres averaged ·34 of an inch long. Loose cellular tissue underlies it, and separates it from the outer part of the subclavius muscle ; but there is no definite costo-coracoid membrane. In Tatusia peba the clavicular deltoid is very similar, with short fibres and perfectly separate, both from the acromial deltoid and from the presterno-clavicular pectoral ; it arises from the sternal half of the clavicle, and is inserted into the top of the deltoid crest of the humerus. There is no clavicular deltoid in Bradypus tridactylus, and in Myrmecophaga, Tamandua, and Pholidotus this portion of the muscle is continuous with the clavicular trapezius. In Cyclothurus there is a very large separate clavicular deltoid occupying the outer half of the clavicle, consisting of short fibres, and inserted high up. In Orycteropus it is large and inserted along with the biceps. In Dasypus it is small, only attached to the outer fourth of the clavicle, consisting of short fibres, and inserted high up, united at its insertion to the upper part of the great pectoral. The analogies of Tatusia and Dasypus render any determination of the nature of this muscle tolerably certain, even although it is against the opinion of so high an authority as Prof. Hyrtl, who describes this muscle as part of the pectoral, and says of the deltoid that it is destitute of a clavicular head (p. 36).

The acromial deltoid arises from the acromion process and from the coraco-acromial ligament. It is decidedly extended to, but unconnected with, the last ; its fibres are longer than are those of the clavicular part, than which it is much smaller. It is not quite separate from the scapular deltoid at its insertion, and its average length is ·5 of an inch. Prof. Hyrtl apparently did not

find these parts separable. In Tatusia it is more separate, but otherwise its arrangement is identical with that of the muscle in Chlamydophorus.

In Dasypus this muscle is, at origin, much more separate, and is fused at its insertion with the clavicular factor. In Cyclothurus it is large, and inserted behind the last and above the supinator longus. In Ai its area of origin is very small, and it is inseparable from the next muscle. In Orycterope it is larger, and separately inserted. In Pholidotus it is partly inserted into the deltoid crest, and partly into the supinator longus. This curious fusion I have elsewhere described as an anomaly in Human Anatomy (Trans. R. I. A., vol. xxv., p. 100).

The scapular deltoid proper is a thick muscle, whose origin is posteriorly tendinous, anteriorly fleshy from the spine of the scapula ; it is inserted rather beneath, but united to the last muscle, and the circumflex nerve as usual pierces into its deep surface ; its posterior border is ·9 of an inch long ; its entire origin 1 inch ; its anterior edge ·5 of an inch. In Tatusia this muscle is exceedingly thin, its lower fibres run nearly horizontally, and it is inserted behind the clavicular and acromial parts. In Dasypus this portion is very large and overlaps the last ; it is at first beneath the margin of the trapezius inferior, and has a fascial as well as humeral insertion. In Cyclothurus this part is weak, and only occupies the middle third of the scapular spine. It is larger and somewhat similar in Orycteropus ; small and inseparable from the last in the Sloths, and, according to Humphry (l. c., p. 36), absent in Pholidotus Dallmanni. Prof. Humphry also describes the scapular deltoid in Bradypus gularis as arising from the coracoid. I did not find this in B. tridactylus. Supraspinatus is very thin behind, thicker in front, and about once and a quarter the size of the infraspinatus. The suprascapular nerve sends a large branch over it. It is separated from the subscapularis by a wide suprascapular ligament, ·2 of an inch long, and from the infraspinatus by a wide strong spino-glenoid ligament. In Tatusia peba the supraspinatus is nearly double the infraspinous muscle. They are nearly equal in Dasypus, being to each other in the proportion of S : I :: 1·2 : 1; In Bradypus tridactylus they are S : I :: 1·3 : 1.

Infraspinatus consists of a single set of fibres only, and is attached as usual ; and in the other Edentates it is very similar.

Teres minor is a fine distinct muscle, whose origin is from the post-scapular

ridge, and this part is tendinous as usual; its fibres are ·55 of an inch in length, and it is inserted beneath and below the infraspinatus. It is large, and more fleshy in Tatusia peba; it is still larger in Dasypus and in Pholidotus, and its insertion is into a tubercle below the external tuberosity. In Ai I could not detect it, nor is it a separable muscle in the Cyclothurus; it is also inseparable from the infraspinatus in Unau. Humphry found it in B. gularis, and describes it as arising in the Cyclothurus from the middle of the scapular spine, between the deltoid and the teres major, quite separate from both, and inserted into the outer supra-condyloid ridge of the humerus, behind the deltoid. This very curious muscle seems to me more like a posterior slip of the scapular deltoid, as the part described by Professor Humphry as scapular deltoid is properly recognised by Mr. Galton (l. c.) as acromial factor.

The subscapularis in Chlamydophorus is a thick muscle, inserted immediately above the teres major, to which, indeed, it is closely related. It is intersected by two tendinous strata, and its tendon does not perforate the capsule of the shoulder. Professor Hyrtl was not able to separate this muscle from the teres major, and describes both as the subscapularis. They were, however, much more readily separable in my specimen. In Tatusia peba, the subscapularis is exceedingly thin. It is much thicker in Orycteropus, and is united at its posterior part with the teres major. It is small in Dasypus, Cholæpus, Bradypus, Manis, and Cyclothurus, and separate from the teres. In none of these does its tendon pierce the shoulder-joint capsule.

There was no subscapulo-humeral present in Chlamydophorus or Tatusia; I found this muscle in Bradypus tridactylus. Humphry describes it in B. gularis, and mentions a corresponding slip in Pholidotus, separated from the rest of the muscle by the circumflex nerve.

Teres major is a very large muscle, whose origin occupies an extensive area of the posterior margin of the postscapula; its insertion is directly below that of the subscapularis, to which it is closely attached. It is similar in Tatusia, more separated, and proportionally smaller in Dasypus, in all three being inserted behind and above, and quite separate from the tendon of the latissimus dorsi. Some fibres of the dorsi-epitrochlearis arise

from it in Orycteropus and Tatusia. This muscle is enormous in Cyclothurus (Humphry, p. 35; Galton, p. 5). In Pholidotus, Tamandua (Rapp. p. 47), and in Myrmecophaga jubata (Pouchet, p. 6), the teres and latissimus are united at their insertions.

The biceps is a slender muscle, which arises by one head, from the root of the coracoid process on its inner side, at the margin of the glenoid cavity. This corresponds to the glenoid long head of human anatomy. It passes in a bony canal between the tuberosities, and beneath a tendinous sheath from the deltoidal crest to the inner tuberosity (to which a part of the pectoral muscle is attached). Below this it joins slightly the brachialis anticus, and is inserted along with it, into a pit in front of the coronoid process. They are not exactly inserted together; however, a careful dissection shows that the brachialis is inserted rather in front of the biceps. The biceps is similar in Tatusia, and is, likewise, coronoid in its insertion, but separate from the brachialis anticus. In Dasypus this muscle is inserted partly into the neck of the radius, and partly into the ulna, in front of the coronoid process, slightly posterior and internal to the brachialis anticus. There has been some difference of opinion expressed regarding the origin of the biceps in Dasypus sexcinctus. Mr. Galton describes it as one-headed, but mentions, that it receives a second head from the coraco-brachialis (p. 536). I have found in two Armadilloes of this species the muscle exactly as described by him, with scarcely any accessory fibres from the coraco-brachialis. In another instance there was a well-marked second coracoid head; and in a specimen of this species, examined by Professor Haughton and myself, in order to decide this point (January, 1872), the muscle was *distinctly bicipital on both sides*, the second head being the ordinary coracoidal one, quite separate from the coraco-brachialis; this variation is of very great interest.

In Bradypus tridactylus, I found the biceps radial in its insertion, and with two heads, one humeral, separate from the deltoid, and one glenoidal. Meckel found a third slip from the deltoid. Humphry found this third slip arising from the coracoid process. In Pholidotus it is a simple gleno-ulnar muscle. In Cyclothurus it is glenoid in origin, and is inserted, partly with the brachialis, to the ulna, partly into the radial tubercle. In Myrmecophaga

jubata it is two-headed as usual. In Tamandua, Rapp describes coracoid, glenoid, and humeral portions. In Orycteropus it is single, gleno-radial, but receives the clavicular deltoid, which is inserted along with it.

Coraco-brachialis is only represented in Chlamydophorus by a very diminutive "short variety," and was not absent in my specimen, as it was in Professor Hyrtl's. It is, however, absent in Cyclothurus and Pholidotus. In Tatusia peba, I find both the short and long varieties, the latter being inserted into the supra-condyloid process; it is quite separate from the biceps. The long and short variety are present in Orycteropus (the latter only in rudiment—Galton; Humphry did not find it); the long variety only is present in Myrmecophaga jubata, and Tamandua tetradactyla; both varieties are present in Dasypus sexcinctus, and the medial form in Bradypus tridactylus, torquatus, gularis, and Cholœpus didactylus.

Brachialis anticus is a very long muscle, and arises by a large fleshy mass from a deep pit on the external side of the humerus, ascending to the head; descending and passing inwards it is separated from the biceps by the deltoidal tuberosity. Still lower, the biceps becomes connected to the brachialis and they are inserted in company, the brachialis being anterior and external at their pre-coronoid insertion. In Tatusia they are separate, but the brachialis is still large and strong. This muscle in Bradypus tridactylus is radio-ulnar in insertion, the radial tendon being behind the radial tendon of the humeral biceps. Meckel does not notice its radial insertion in his sloth; neither does Humphry in B. gularis. It is simple in Pholidotus and Manis; it receives a slip from the biceps in Cyclothurus and Orycteropus (Humphry), and sends a slip to the radius (Galton). In Tamandua Rapp describes this muscle as absent; it is single and ulnar in Myrmecophaga (Pouchet).

Triceps is an enormous muscle in Chlamydophorus, and is easily divisible into four heads; the first, or the posterior long head, occupies a space on the dorsum of the scapula, below the post-scapular crest, which is ·5 of an inch long; ·3 broad at the top, and extending down to the posterior angle; this part is inserted into the tip of the olecranon. The second part arises fleshy in front of the first and external to it, extending for ·35 of an inch forwards;

this part is tendinous at its insertion, and is attached to the olecranon, in front of the last, and inseparably united to the following parts. The third part arises tendinous from the inferior margin of the scapula, for ·2 of an inch, widens and becomes fleshy until it reaches the olecranon, where it is inserted. The fourth part, or humeral origin, was indivisible into parts, and is the smallest of all the elements. The inner side of these heads is covered by the dorsi-epitrochlear, which extends along the inner half of the ulna.

In Tatusia peba the scapular triceps only consists of two parts, corresponding to Nos. 1 and 2 of those above described; the second, or most anterior occupies five-sixths of the axillary scapular margin; the first, or posterior occupies about one-sixth.

Professor Hyrtl very clearly characterises this triceps as follows:—"Triceps pondere Bicipitem trigesies superat. Nec mirum, si ad usum extremitatum anteriorum animadverteris, quæ antibrachiorum extensione rapida et energetica, effossam cuniculorum subterraneorum terram evertunt, radices viam obstruentes convellunt truncantæ, saxa, aliaque progressus impedimenta commovent et evolvunt." Hyrtl proceeds to describe *two* scapular heads, and one humeral; and he describes the inner part (dorsi-epitrochlear) as a tensor fasciæ anti-brachii. In Bradypus the triceps has one scapular and two humeral heads. In Pholidotus there are four heads, an inferior post-scapular, a meso-scapular, an ecto-brachial, and an endo-brachial. In Orycteropus it is as in Chlamydophorus, only the humeral heads are separate from each other. The posterior or angular scapular head exists in Tamandua (Rapp, p. 48). In Dasypus there are two scapular heads, the angular slip, and the main body of the scapular head. In Cyclothurus there is but one long head. In Myrmecophaga there are two.

Epitrochleo-anconeus is very large, and is related s usual. Anconeus externus is only represented by a deep slip under the outer and lower margin of the humeral head of the triceps. The latter is described by Hyrtl. These muscles are exactly similar in Tatusia peba, but the internal slip is smaller. In Dasypus sexcinctus, the anconeus externus is extremely small, and, I think, only constitutes a part of the muscle described under this name by Mr. Galton (p. 539), the rest of this muscle being a part of the triceps. Epitrochleo-

anconeus is strong in Dasypus, Orycteropus, Tamandua, Myrmecophaga, Tolypeutes tricinctus Cyclothurus didactylus, Pholidotus Dalmanni, Cholœpus didactylus, Bradypus tridactylus, torquatus and gularis.

Pronator radii teres is very large and separate, arising from the supracondyloid process; it is inserted into the lower third of the anterior surface of the radius; there is no coronoid slip. In Tatusia peba it is attached to the lower half of the front of the radius, and is also large and round, overlying the median nerve. In both these it is fleshy, but in the Dasypus sexcinctus it is tendinous at its insertion, which is as extensive as in Tatusia; it is equally large and more fleshy in Orycteropus. In Cyclothurus it is inserted by a flat tendon in the lower third of the radius. In Bradypus it has a single origin, and splits to be inserted into two parts, one above the other, into the lower half of the radius, extending to the anterior ligament of the wrist joint. It is similar, but undivided, in Cholœpus, Tamandua, Pholidotus and Myrmecophaga.

Flexor carpi ulnaris is a slender, separate muscle, with two scarcely separate heads, one from the extreme tip of the inner condyle, and one from the olecranon; the ulnar nerve is between them, and the muscle is inserted into the pisiform; this muscle lies to the *radial* side of the deep ulnar flexor of the digits. In Tatusia its origin is almost entirely olecranal, and scarcely at all condyloid; the muscle is likewise proportionally larger. In Dasypus there is even less of a condyloid origin; in fact, in some of the specimens of D. sexcinctus, there was not a vestige, in others a fibrous band. In Orycterope the muscle is double, one part from the middle of the inner border of the ulna, to the *ulnar* side of the flexor profundus, and inserted into the pisiform; the larger part, however, is olecrano-pisiform in attachment. In Pholidotus it is very like the arrangement in Chlamydophorus, and lies to the *radial* side of part of the deep flexor. In Bradypus it is not very large, is truly bicipital, springing from the condyle and olecranon, and inserted into the pisiform, unciform, and third metacarpal. In Cholæpus it is larger, and more perfectly cleft, one slip being humero-pisiform, the other olecrano-pisiform. In Cyclothurus it is enormous, and four-cleft, condylo-pisiform, posterior ulno-pisiform, anterior ulno-pisiform, and olecrano-pisiform. Some little variety exists as to the arrangement of these, as may be seen by comparing the

descriptions of Meckel (Band V. p. 47), Cuvier (pl. 257), Galton (p. 9), and Humphry (p. 41).

Flexor carpi radialis arises under cover of the pronator radii teres; it is a fine, flat, muscular slip, the edge of which only comes to the surface, arising below the supra-condyloid foramen; it has the median nerve internal and anterior to it. Its tendon is slender and inserted into the base of the first metacarpal as usual. In Tatusia peba this muscle is small, simple, and fleshy, and has the same insertion. In Pholidotus it is inserted into the third metacarpal (Humphry); but it also sends a distinct band to the first and second, especially to the latter. In Orycteropus it is inserted into the second metacarpal, with the intervention of a sesamoid bone (Galton, p. 584). Both this author and Humphry found a recurrent radial insertion in this species. In Bradypus it is attached to the metacarpal bone of the index finger and the multangular bone, and it is attached to the latter alone in Dasypus. In Cholœpus it is inserted into scaphoid and os multangulum. In Cyclothurus, and the other Anteaters, it is as in Chlamydophorus.

Professor Hyrtl merely names, without describing, these three forearm muscles in the Chlamydophorus; he states that the fourth element in the superficial muscle-groups on the anterior surface of the forearm, is doubtful in nature, being either the palmaris longus, or the flexor digitorum sublimis; a very careful examination of this indicated to me that these were both present, the elements of these muscles being closely united together. The palmaris longus is superficial, close to the condyloid head of the flexor carpi ulnaris; it is a large, thin muscle, fleshy down to near the wrist, arising fleshy from the tip of the inner condyle; the tendon lies superficial at the wrist, and expands over the palmar ossicle, and ending in four superficial slips, is lost in sheathing the tendons on the digits—"Origine, decursu, et tendinis sui aponeurotica expansione, Palmarem longum coram esse crederes, At quum aponeurosis ejus palmaris in quatuor conflavatos tendines discedat, qui ad digitorum, excepto pollice phalanges primas abeunt, ibidemque furcata divisione tendini flexorio ad ultimum phalangem producto, viam aperiunt de flexore digitorum sublimis obvia suspicio est."

But there is, besides this, a separate flexor sublimis, and the digital expansions, I find to partake more closely of the nature of digital tendon-sheaths, than

regular split tendon. In Tatusia peba this muscle is larger, and more clearly a palmaris. In Dasypus it is intimately connected to the flexor digitorum, and purely palmar in its insertion. In Orycteropus the fusion is still closer, as it is also in Cholœpus. In Pholidotus it is combined also with the flexor sublimis, but is separate in Cyclothurus and Bradypus. In the former, however, Meckel says it is fused with the flexor carpi radialis.

Flexor digitorum sublimis in Chlamydophorus is very small, and placed internal to the last, arising from the outside of the inner condyle, and inserted into the inner side of the palmar ossicle. In Tatusia peba this muscle covers, and is nearly free from the ossicle, and lies external and superficial to the other tendons ; a very small band goes to the ossicle directly. In Pholidotus the flexor sublimis is large, and has a slip to each toe, pollex included. In Cholœpus it is with difficulty separable from the deep flexor, and from the palmaris in Bradypus it is inseparable ; in Cyclothurus it is separate from the profundus.

Three muscles coalesce to be attached to the central ossicle of the palm ; the first of these is the flexor sublimis digitorum, which arises from the external part of the inner condyle, and is attached to the palmar bone, on its ulnar side ; this is the second smallest of the three flexors. Flexor pollicis longus is the smallest of the flexors, and arises from the front of the radius. Flexor profundus digitorum is in two nearly separate parts, one olecranal, and the other from the middle of the shaft of the bone. The palmar ossicle is small, and rather irregularly ovate in shape ; from it pass off five tendons, one exceedingly fine, to the pollex ; a very large one to the median digit, and three others of intermediate strength ; the little-finger-slip arises nearest to the wrist, from the outer side of the bone.

There are three lumbricales, which are exceedingly slender, and arise from the *front* of the tendons ; one passing from the middle-finger-tendon to the first phalanx of that digit ; one from the ring-finger-tendon to the corresponding digit ; and one from the little-finger-tendon, very slender, to the ulnar side of the ring finger. Professor Hyrtl describes the flexor sublimis as a middle stratum in the forearm ; and the flexor pollicis and flexor profundus he groups together as one ; he notices the absence of an annular ligament at the wrist, and the curious trochlear form of the deep surface of

the palmar bone. He did not find the muscular lumbricales which I have above described, but found four accessory tendons "qui solito apparatu vaginali inclusi, sed additamentis lumbricalibus orbi, ad phalangis unguicularis tuberculum inferius, uti flexor profundus hominis sese præcipitant, et trajecto suo trochleis innituntur osseis quæ latus flexorium articulationis interphalangeæ ultimæ (vel, ubi hæc deest, articulationis metacarpo-phalangeæ) muniunt." From the sesamoid bone arises seven fine fleshy bundles, which are inserted into each side of the second phalanges of the fingers, except the thumb, and the ulnar side of the little digit, forming short flexors. The flexor sublimis in Tatusia peba, is nearly free from the ossicle, arises from the inner condyle, and is inserted by a slip into the pollex; another to the index, and a third goes into the palmar bone.

In Ai there is no separate flexor sublimis, and no sesamoid ossicle. In Cyclothurus it is separate and condyloid, and supplies the index and third digits; but the slips to these toes are separate from each other. In Cholœpus the muscle is single, and has two tendons, which are strengthened by two accessory slips from the surface of the flexor profundus. In the Orycterope there is a distinct flexor sublimis, giving off four perforated tendons, which also receives a supplement from the profundus. In Dasypus there are three tendons for index, medius, and annularis, the first of which often gives off an accessory slip to the pollex; this is free of the sesamoid bone. In Orycteropus there is a large flexor profundus, ascending even to the condyle. There is no sesamoid bone, and the anterior edge of the expansion gives off four tendons. In Ai there is one flexor only, radio-ulno-humeral in origin, digital in insertion, and with no sesamoid bone. In Cholœpus the flexor sublimis is separate from the profundus without any sesamoid bone. In Cyclothurus and Pholidotus, the sublimis is similar in origin, and has a sesamoid. The ossicle in the palm exists in all the Dasypodidæ.

There are no lumbricales in Ai. In Unau there are two on the radial sides, and inserted into the extensor tendons. In Cyclothurus there are also two; in Pholidotus there are four from the sesamoid bone to the second phalanges of the digits; and they are the same in Orycteropus. The flexor profundus of the Dasypus sexcinctus has three factors, a radial, an ulnar, and a humeral, all attached to the sesamoid bone; all the lumbricales in this

animal begin in a central mass, which gives off two to the index, and one to the radial side of each other finger, exclusive of the pollex. Galton found one for the middle digit; and he found a slip from the first lumbricalis, passing to the thumb. In Tatusia there is but one inseparable, deep, palmar flexor.

The pronator quadratus is only represented by an exceedingly feeble fibrous band. Hyrtl did not find any trace of it; it is also excessively rudimentary in Tatusia, and absent entirely in the Pholidotus and Dasypus. It is large and well developed in Cyclothurus, occupying the entire forearm (Galton), or absent (Meckel, Cuvier). It is also large in the Orycterope (small, Humphry), Tamandua tetradactyla, and Myrmecophaga.

Flexor carpi radialis brevis (radio-carpeus), is absent in the Chlamydophorus, Tatusia, and Dasypus; it is present in the Tamandua, constituting Rapp's Spannmuskel des ligamentum annulare (Anatomische Untersuchungen über die Edentaten, p. 48). This is represented in the Cyclothurus, but absent in the Sloths.

The short muscles of the pollex in the Chlamydophorus, are—Abductor pollicis, a slender cylindrical muscle from the scaphoid bone to the first phalanx of the pollex; opponens pollicis from the os multangulum to the entire length of the metacarpal bone; flexor brevis pollicis, and adductor pollicis, inseparable, from the os magnum, and the head of the third metacarpal to the first phalanx of the pollex. These polliceal muscles are absent in Cyclothurus, Bradypus, Cholœpus. In Pholidotus there is an abductor pollicis from the sheath of the flexor carpi radialis; a flexor brevis pollicis internal to the last; Adductor pollicis from the scapho-lunar bone to the inside of the first phalanx of the pollex. In Dasypus there is an abductor, an opponens, and a flexor brevis pollicis. In Tatusia they are very similar; the flexor is rather larger, as there is no flexor tendon to the pollex in this animal, from the sesamoid ossicle.

The short muscles of the thumb in Chlamydophorus, named by Hyrtl, are—Abductor, adductor, and flexor brevis; the last of these I believe to be truly an opponens.

For the little finger there is but one short muscle, an abductor; it is the same in Tatusia, Orycteropus, and Tamandua.

Pholidotus has a flexor as well as an abductor ; so also has Dasypus ; Ai, Unau, and Cyclothurus have none. In Chlamydophorus there is a slip which may, perhaps, be looked on as a second muscle or abductor, which arises at the base of the third metacarpal, and running transversely outwards, is inserted into the radial side of the metacarpal of the little finger. Meckel describes in Cyclothurus a large series of muscle bundles, arising from the enlarged pisiform of this anteater, and inserted into the rudimentary outer metacarpals ; there are rudiments of the flexor and opponens minimi digiti.

The interossei in Chlamydophorus are not divisible into dorsal and palmar, but are all palmar, and in one series. They are—

1. Abductor indicis, from the fourth metacarpal to the second digit.

2. Abductor tertii digiti, from the third metacarpal to the third digit.

3. Abductor tertii digiti, a slip of the first, from the same origin, inserted into the third finger.

In Tatusia they are—

1. Abductor indicis, arising from the pisiform, and inserted into the first phalanx of the index. This muscle receives an accessory slip from the heads of the third and fourth metacarpal bones, and acts along with the ulnar lumbricalis of this digit.

2. Abductor indicis, from the first and second metacarpal ; this acts along with the outer lumbricalis of this digit, which is on the radial side.

3. The third digit has on its radial side a fine rudimental slip, which is interesting, as there is likewise no lumbricalis for the digit.

4. There is an abductor quarti digiti from the third metatarsal, which conjoins with the radial lumbricalis of the digit.

Cyclothurus possesses five interossei, an abductor medii digiti, an abductor quarti metatarsi (both from the unciform), an adductor indicis ; an unciform slip to this appears as a fourth ; and lastly an abductor indicis. I found no palmar interossei in Bradypus tridactylus, and could not separate the dorsals from the extensor brevis digitorum manûs. Professor Humphry, however, in Bradypus gularis, resolves this muscle into the following components :— A radial interosseous for the second digit ; an ulnar for the fourth ; a radial and ulnar muscle between the second and third, and between the third and

fourth digits; in these he remarks the origins absolutely extended to the phalanges.

In Pholidotus there is an adductor indicis, an adductor quarti, and an adductor quinti digiti; there are also abductor indicis, abductor medii, and adductor medii, a deeper adductor quarti (Humphry), and an abductor quarti digiti. In Cholœpus there is an abductor of the index, an adductor of the medius, an adductor of the index, and an abductor medii; of these the second is described by Professor Humphry as double. In the Orycterope there is a series of eight interossei; 1. adductor minimi digiti; 2. adductor indicis; 3. abductor indicis (bicipital); 4. adductor profundus indicis; 5. adductor medii; 6. abductor medii; 7. abductor annularis, and adductor annularis. Dasypus, as I have elsewhere described (Annals of Natural History, 1868, p. 318), has the following series of interossei:—1. abductor indicis; 2. adductor annularis; 3. ulnaris medii; 4. adductor indicis; 5. radialis medii. The only muscle of this series mentioned by Hyrtl in Chlamydophorus, is the adductor indicis above described.

The supinator longus and brevis are described by Hyrtl as absent. The former certainly is unrepresented, and of the latter only a few short fibres exist, not pierced by, but underlying the posterior interosseous nerve. These supinators are absent in Tatusia peba. In Pholidotus the supinator arises from the acromial deltoid, and is inserted as usual. In Unau it is bilaminar, arises from the outer side of the humerus, high up, and is inserted into the radius, sending an expansion to the wrist. This is the case also in Bradypus and Cyclothurus. The muscle is large in Myrmecophaga, Tamandua, and Orycteropus, but absent in Dasypus; but the supinator brevis is present and pierced by the nerve in the last-named species. In Orycteropus it extends for half the radius; it is large in Tamandua, Myrmecophaga, Cyclothurus (for two-thirds), Pholidotus (three-fourths), with a sesamoid bone in its tendon of origin, as I found in a human female (Journal of Anatomy, III., p. 108). In Ai it is small, as in Unau.

Extensor carpi radialis is a single muscle, with a double tendon, arising high up from the external condyloid ridge, and inserted by two tendons into the metacarpal of the index and that of the radius. In Hyrtl's specimen it was only inserted into the index (p. 38). It is exactly the same in Tatusia

peba, Bradypus tridactylus (inserted into the first and second metacarpals), Cholœpus didactylus, Pholidotus, Dasypus (Humphry found it in the last but one to be inserted only into the third). In Cyclothurus it is only attached to the third. In Orycteropus Professor Humphry found it only attached to the third, Mr. Galton to the second and third metacarpals.

Extensor digitorum communis is a small muscle arising below and behind the last, and inserted into the second, third, and fourth digits. I did not find the fibro-cartilaginous nucleus in the fourth finger tendon, described by Professor Hyrtl; but his description of the tendon of the muscle, passing in an unbroken sheath beneath the annular ligament, and then dividing into four, and going to the four fingers, is absolutely accurate. This is succeeded by an extensor annularis, which, on the right side, went to the fourth and fifth, and on the left, only to the fourth digit; after this is the extensor minimi digiti proprius, which is inserted into the back of the second phalanx of the little finger. These two last-named muscles pass through separate sheaths in the annular ligament.

In Cyclothurus the extensor digitorum is attached to the base of the last phalanx (first, Meckel) of the middle digit. It has an extensor brevis or accessory portion for the lower end of the back of the ulna, which by two slips goes to the extensor longus, and to the ungual phalanx of the index. There is also co-existing a second extensor going to the ulnar side of the first phalanx of the third digit. In Cholœpus this muscle is strong, and goes to the two digits. In Bradypus it is much weaker, and passes to the three toes. In Pholidotus it is humero-ulnar in origin, and splitting into two is inserted by one part into the last phalanges of the third and fourth toes; and by another into the last phalanx of third and of the fifth. The tendons contain sesamoid cartilages at their respective joints. The extensor brevis digitorum in Cholœpus arises from the back of the hand, and is inserted by two tendons into the ordinary extensor. In Ai it is united to the dorsal interossei, inseparably or separably (Humphry). The second part of the extensor in Manis represents the extensor minimi digiti, which is otherwise unrepresented. The extensor muscle is exceedingly weak in the Ai. In Dasypus it is much stronger, and is inserted into the second, third, and fourth fingers. It has placed to its inferior side an extensor annularis, which, as in Chlamydophorus, has a groove

for itself in the annular ligament, and is inserted into the fourth and fifth fingers. A third extensor minimi digiti arises still lower, and is inserted into the base of the first phalanx, by its wide terminal expansion. Three extensors exist also in Orycteropus ; the extensor communis going to the four outer digits ; the extensor annularis to the fourth, fourth and fifth (Galton) ; third and fourth (Cuvier) ; and, thirdly, the extensor minimi digiti inserted into a sesamoid bone at the ulnar side of the proximal phalanx of the little finger, and into this bone itself. Prof. Humphry and Cuvier found this muscle with a slip to the fourth finger. In Tatusia peba the extensor digitorum has three tendons for the index, middle, and ring fingers ; and there is a special extensor minimi digiti, which supplies a tendon to the fourth digit alone.

The extensor carpi ulnaris in Chlamydophorus is exceedingly weak, and arises from the lowermost and back part of the condyle, and is inserted into the base of the fifth metacarpal bone. The tendon which passes through the outermost groove in the annular ligament is much stronger proportionally than the muscle. In Tatusia peba this muscle has two heads, one larger ulnar, and a second smaller condyloid ; it is inserted into a sesamoid bone at the base of the metacarpal bone of the outer digit. In Orycteropus it is inserted into the fourth and fifth metacarpals. In Dasypus this muscle is arranged as in Tatusia, condylo-ulnar in origin, the ulnar head being connected to the triceps (anconeus Galton). The muscle is also inserted into a sesamoid at the base of the outer metacarpal. In Cyclothurus this muscle is very similar in position and arrangement, and inserted into the rudimental fifth metacarpal. In Bradypus tridactylus it is purely condyloid in origin, and is inserted into the third metacarpal. In Cholœpus it is condylo-ulnar in origin, and inserted into the two metacarpals. In Bradypus gularis Prof. Humphry found a second muscle, which from the ulna passed to the outer metacarpal ; this I did not find in Bradypus tridactylus. In Pholidotus there is also this double muscle, one condylo-ulnar to outer metacarpal ; the second purely condyloid, and inserted into the metacarpal of fifth and the outer side of the last phalanx of the annularis.

Extensor ossis metacarpi pollicis is very large, and occupies a very large space on the ulna, stretching up nearly to the olecranon ; its insertion is into the metacarpal bone of the pollex. In Tatusia peba it is much the same. In

Dasypus sexcinctus it is very long, and extends the whole way to the olecranon ; it is inserted into the metacarpal of the pollex. In the Cyclothurus it is humeral in origin, and is inserted into the ossicle, which is the rudiment of the first metacarpal. In Orycteropus it arises from two-third of the ulna (inner ridge), and is inserted into the trapezium bone ; it is inserted into this bone, and by a few fibres into the pollex in Myrmecophaga. It arises from the middle of the back of the ulna in Bradypus tridactylus, and is inserted into the base of the inner rudimentary metacarpal. In Pholidotus it arises higher up from the upper half of the ulna, and is inserted into the os multangulum.

Extensor indicis ("quamvis etiam ad pollicis fasciolam fibrosam ablegaret, quae sola pollicis extensioni sufficere debet"), arises high up from the ulna as high as the olecranon ; it is very large, larger than its neighbour the extensor ossis metacarpi pollicis. I found its main tendon passing as usual to the dorsum of the index, but a broad slip of fascia stretches along the dorsum of the thumb to the last phalanx, justifying the words of Prof. Hyrtl before quoted. In Tatusia peba it is exceedingly large and strong, occupying the greatest proportion of the ulna ; even to the very tip of the olecranon it ends in two tendons, which are inserted into the index and middle fingers. In Dasypus sexcinctus this muscle is narrow and only attached to the lower half of the ulna ; it is indico-polliceal in its insertion, but this is not invariable, for in some of the specimens the thumb slip did not exist. This is the condition described by Meckel.

From the foregoing description it will be seen that Chlamydophorus keeps closer to Tatusia peba in Myology than to any other of the Edentates. I regret that I was not able to make more of my specimen of Tolypeutes, but its condition rendered most of my observations on it unreliable.

MUSCLES OF THE HINDER LIMB.

Tensor vaginæ femoris is a very distinct muscle, quite separate from the next ; with which Hyrtl combines it. It arises from the anterior border of the crest of the ilium for ·2 of an inch ; it runs downwards and backwards for ·6 of an inch, and is inserted into the anterior and upper part of the third trochanter at its root. A branch of the anterior crural nerve passes into it. It is only connected to the gluteus maximus at its insertion. In Tatusia peba the

origin of this muscle is directly behind that of the strong radiating tendon of the internal oblique, before described; passing downwards and expanding, it is inserted into the whole length of the outside of the thigh, into the fascia lata; a branch of the anterior crural nerve dips into its deep surface. In Dasypus this muscle is very thin and flat, and arises from the anterior superior spine, and from the lumbar fascia, as well as by some slips which are cutaneous in origin, and fused with the pelvic portion of the panniculus carnosus; it is inserted into the third trochanter, and usually I have found it inseparable from the lower border of the glutæus maximus, its lowest fibres being attached to the sling of fascia which stretches from the third trochanter to the outer condyle. This muscle is joined to the glutæus maximus also in Cholœpus, and is inserted into the fascia on the outside of the thigh. In Bradypus it is much more distinct, and is inserted into a definite line about half an inch long on the outside of the great trochanter. Cuvier calls this muscle the glutæus minimus. In Cyclothurus and Pholidotus it is inseparable from the glutæus maximus. In Orycteropus it is with difficulty separable, and rather beneath it. Glutæus maximus is a thick band of muscular fibres arising therein, and tendinous from the base of the upper ischiatic fulcrum of the sphæroma, and from the fibrous tissue below and behind it; passing outwards and downwards it is inserted by a tendon into the deep surface of the third trochanter at its outer part. The glutæal nerve pierces into its deep surface. Prof. Hyrtl describes the origin of this muscle as from "posteriori cristæ ossis ilei, et a ligamento inde ad coxendicem tenso;" but the fascial part above mentioned is an exceedingly small factor in the muscle.

This muscle in Tatusia peba differs in the direction of its fibres from the tensor vaginæ femoris, but otherwise they are not easily separated. In origin and insertion it is not peculiar, being attached to the posterior part of the iliac crest, and to the third trochanter. In Dasypus this, and the last-named muscles, are not separable, and the origin of the gluteus is enormously extensive, but it is exceedingly thin. In Orycteropus it is thicker and continuous with the biceps, but otherwise normal in its attachments. In Bradypus it is small, superficial, and triangular, arising from the posterior border of the iliac side of the sacrum, and is inserted into the outer part of the femur immediately below the third trochanter. It is much smaller than it is

in the Armadilloes, and quite separable from the biceps. In Pholidotus it lies close to the tensor vaginæ femoris, and is inserted into the outer side of the knee and edge of the tibia external to the tendo patellæ. In Cyclothurus it is moderate, and inserted by a strong tendon into the outer side of the femur, at the spot where the third trochanter should be, if it existed (Galton, Meckel says into the whole of the outer edge, and Humphry into the lower half of the outer border.)

The muscle which corresponds to the agitator caudæ of other mammals is a very large muscular structure in the Chlamydophorus, much larger than either of the others. It arises by a strong tendon from the lowest of the fulcra of the pelvic sphæroma, and is inserted by a wide fleshy mass into the outer and posterior part of the femur, below the third trochanter, and extending downwards to the lower end. Along the upper border of this muscle there was a semi-detached slip, with a wide membranous origin, and a tendinous insertion into the tip of the third trochanter.

In Tatusia peba this muscle is nearly inseparable from the glutæus on the one hand, and from the biceps on the other. Its origin is from the ischiatic tuber, and its insertion the third trochanter. In the other Edentates it is likewise confluent with the glutæus. The insertion of this, which is the third part of the glutæus maximus of Hyrtl, is correctly described by him.

Glutæus medius and minimus are fused together inseparably in Chlamydophorus, with a wide insertion, and an extensive origin extending inwards over the muscles of the spine, as far as the mesial line. This muscle is very well described by Professor Hyrtl. There is no other glutæal muscle. In Tatusia peba there is a glutæus quartus, or marginal muscle, as well as a medius and minimus combined. In Dasypus the medius and minimus are also united, as they are in the Bradypus, Cholœpus, Myrmecophaga, Cyclothurus, and Pholidotus. The only Edentates possessing a separate glutæus minimus are the Orycteropus and Tamandua; and in the latter the muscle is rather a marginal glutæus; hence a glutæus quartus than a glutæus minimus. The marginal muscles in Tamandua and Peba are represented in Dasypus by a semi-detached deep anterior slip of the glutæus medius.

Pyriformis was not found by Hyrtl, nor could I find a trace of it as a separate muscle; but there is a thick band of the glutæus along its

lower border, which is its representative. In Tatusia it is present, but not separate from the last. In Dasypus it is very large and marginal. In Bradypus it is quite separate, and also arising from the margin of the sacrum (Humphry could not separate them, Cuvier did so.) In Cyclothurus it is fused with the glutæus medius. In Orycteropus it is large and separate, with an iliac, as well as a sacral origin, and lies under cover of the glutæus medius, below which it is inserted (Galton). In Pholidotus it is inseparable from the glutæus, but it is partly separate in the Cholœpus didactylus.

The Obturators and Gemelli were not found by Professor Hyrtl, nor could I find any trace of them. They are partly developed in Dasypus, and there is a distinct obturator externus and gemelli, but no obturator internus in Tatusia peba.

In the Cyclothurus there is an obturator externus, but no internus, the gemelli, however, are largely developed, and form a single fan-shaped muscle tapering to its insertion. In Tamandua there is an obturator externus, and a gemellus, but no obturator internus, and the same is the condition in Cholœpus and Pholidotus. In Bradypus the obturator internus is displaced, and lies beneath the obturator externus.

Quadratus femoris has become a triangular muscle, as Hyrtl describes, from the inferior ischiatic fulcrum to the posterior part of the root of the great trochanter. This muscle exists in Bradypus tridactylus; in Cyclothurus it is absent. In Dasypus sexcinctus it is well marked, and normal in its attachments. In Unau there is a quadratus femoris, but smaller than in Ai. In Pholidotus, and Tamandua it is not present, nor does it exist in Tatusia peba. "Simili modo musculos in Dasypode setoso digestos vidi, ea sola differentia; quod glutæi magni portio anterior (tensor vaginæ femoris) sese usque ad processus transversos lumbales explicuerit, et glutæi minimi accuratior a medio distinctio extiterit. Verbo non eget, in Dasypodibus tertiam glutæi majoris portionem (cum secunda coalescentem) in normalis ischii tubere radices suas defigere." (p. 39).

The Biceps flexor cruris is an exceedingly weak muscle, closely attached to the lower border of the agitator caudæ, and inserted by fascia over the external surface of the leg, stretching for ·5 of an inch below the knee.

This portion, as Hyrtl describes, has no bony insertion, and its origin is from the tuber ischii, below the agitator caudæ—that is, beside the base of the external inferior fulcrum. In Tatusia peba this muscle is with care separable from the agitator caudæ, and is purely crural and fascial in its insertion, occupying the upper part of the leg. A detached long slip of this muscle (bicipiti accessorius, Haughton) arises behind and above the biceps from the tuber ischii, and is inserted into the outside of the ankle by the intervention of a fascia; this is an exceedingly long strap of muscle quite separate from the biceps. No trace of this accessory slip is present in Chlamydophorus.

In Dasypus this bicipiti-accessorius is very large, and passes from the tuber ischii to the heel. The other part of the biceps is partly attached to the agitator caudæ, and is inserted higher up into the outside of the upper part of the calf of the leg. In Orycteropus there is a twofold insertion—one fascial for the upper two-thirds of the leg, the other into the head of the fibula. In Cyclothurus there is an accessory femoro-fibular muscle, while the ischiatic part is fascial below and inserted high up. There is a similar femoral head in the Bradypus, Tamandua, Cholœpus, and Manis, as well as in Myrmecophaga. Semimembranosus in Chlamydophorus is large and fleshy, and is quite separate from the adductor magnus; its origin is from the tuber ischii, and its insertion is into the inside of the knee under the gracilis. The point of its origin is below the inferior fulcrum, and from the tissue in the hollow between the ischium and the sphæroma; it passed forwards close to the adductor; it is fleshy for nearly its entire extent. Prof. Hyrtl found this muscle incorporated with the adductor in his specimen. In Tatusia peba this muscle arises fleshy from the upper pair of caudal vertebræ, and it is inserted high up into the inner side of the tibia. In Dasypus it is very strong, and arises from the ischium; it passes to be inserted into the internal lateral ligament of the knee and the inner side of the tibial head. The adductor magnus condyloidea (add. primus) is generally attached to this muscle, and is described as part of it by Mr. Galton. In Ai this muscle arises by a common tendon with the semitendinous from the tuber, and is inserted into the upper part of the inner condyle of the tibia and fascia of the leg. There is an attempt at a femoral insertion in Ai, but in Cholœpus the whole muscle is tibial in its insertion, as it is also in Cyclothurus and Pholidotus. In the latter pair of animals this muscle is with

difficulty separated from the adductor magnus. It is, however, large and separate in Orycteropus and Tamandua.

Semitendinosus in Chlamydophorus is placed beneath the last, and arises from the root of the upper ischiatic fulcrum, then winding below the last it is inserted into the inner side of the tibia below the last. There seemed an obscure trace of an inscription in this muscle. In Tatusia peba there was a second head to this muscle from the upper pair of caudal vertebræ, and its insertion was into the middle third of the tibia. In Bradypus this muscle is small and simple (Prof. Humphry found a caudal slip to this muscle—I did not find one). In Cyclothurus it is double, having one portion from the tuber ischii along with the biceps, and one still lower also from the same place, and from the tail; these slips are inserted separately. Tamandua and Pholidotus have similar caudal origins. Sartorius is wide, thick, and fleshy, arising from ·3 of an inch of the anterior surface of the ilio-psoas anterior and internal to the femoral vessels; passing downwards and inwards it covers the front of the thigh, and is inserted into the inner side of the knee-joint, and the fascia on the inside of the knee. This muscle was not, as in Hyrtl's specimen, coalescent with the adductor magnus. In Tatusia peba the muscle is attached to Poupart's ligament, it is continuous internally with the gracilis, and inserted along with it. In Dasypus it is attached to the psoas tendon also, and its origin is narrow, the muscle being much more feeble than in either of the others. In Orycteropus it arises from the pubis at the pectinæal eminence, and is attached to the gracilis. In one specimen of this animal Prof. Humphry found it arising from the last rib, and passing down under the crural arch. In the Sloths it is very broad, and attached to the anterior superior spine, and to Poupart's ligament; it is inserted into the inner condyle of the femur and of the tibia; the femoral insertion is stronger in Unau. In Pholidotus it is iliac in origin, and inserted into the patella. In Cyclothurus it is as in Dasypus. Thus in all these Edentates, except Tamandua and Pholidotus, the sartorius is displaced inwards, as it is in so many ruminants and rodents.

Gracilis is a thin muscular lamina on the inside of the thigh, passing from the horizontal ramus of the pubis to be inserted into the inner side of the tibia below the sartorius and above the semimembranosus. In Tatusia it is also broad, and arises from the pubis; it is inserted as in the Chlamydophore. In

Dasypus this muscle is also thin, and inserted along with the gracilis. In Bradypus it is closely connected to the pectineus at its origin, and inserted into the inner tibial condyle. Humphry found a second slip going to the biceps from this muscle. In Cholœpus the muscle is arranged as in Bradypus, and there is no bicipital slip. It is also similar in Pholidotus, larger in Cyclothurus and in Orycteropus.

Pectineus is a small muscle, very distinct, but very diminutive. Prof. Hyrtl correctly states its attachments as ilio-pectineal, and infra-trochanteric. It is very similar in Tatusia and in Dasypus, similar but larger in Cyclothurus and Pholidotus. In Bradypus it is cleft into two, one part passing from the pectineal ridge on the os innominatum and inserted into the femur for its upper two-thirds; the second, continuing down along the femur for the lower third. This division is not so well marked in Cholœpus didactylus, but is very distinct in Orycteropus; the lowest fibres in that animal extend to the inner condyle of the femur. (Prof. Humphry describes this muscle as extending over the knee in front of the gracilis). A small detached upper slip of this muscle exists in Cyclothurus.

Rectus femoris is quite distinct, for the most part, in the Chlamydophore, and has an iliac and an acetabular portion as usual; at its insertion it unites with the other elements of the quadriceps. In Tatusia its origin is single, and for the largest part of its extent it lies under cover of the vastus externus which is large; beneath the region of the rectus in this animal a small muscular slip arose from the margin of the iliac bone, and was inserted into the vastus externus. In Dasypus the rectus has also a single extensive origin, but is not a very strong muscle, and in Bradypus, Orycteropus, Tamandua, Cholœpus, Pholidotus, Cyclothurus, and Myrmecophaga it is similar. The deeper part of the extensor mass is divisible into two parts distinctly, vastus externus, the smaller, and the combined cruræus and vastus internus, rather larger. These latter can be partly separated with a little care, but become inseparable near their insertion. They are similar in Tatusia (with the addition of the iliac slips above described). In Dasypus the cruræus and vastus internus form a much smaller mass than the vastus externus, and are far less separable; they are similar in Bradypus, Cholœpus, Cyclothurus, Orycteropus, and Pholidotus, as well as in Tamandua and Myrmecophaga.

I found no sub-cruræus in any of these.

Psoas parvus is strong, short, and fleshy, arising from the transverse process and from the side of the body of the three lumbar vertebæ; it forms a short prominent muscular mass, and is inserted into the pectineal eminence on the brim of the pelvis. "Ex hujus musculi quantitate concludere licet, Chlamydophorum, urgente periculo, Erinacei more, in pilam convolvi, cujus tabulata superficies aggredientium vim dolumve eludit." This muscle is very much smaller in Tatusia peba, and in Dasypus is long and slender. In Tolypeutes it is still larger than in Chlamydophorus, but very diminutive in Bradypus. It has only a short fleshy portion in Cyclothurus. In all the other caudate anteaters Orycteropus, Pholidotus, Tamandua, and Myrmecophaga psoas parvus is a well developed muscle, especially in the first pair.

Psoas magnus is very small and separate at its origin, as a slip along the inner margin of the ilium. Iliacus arises from the external margin and face of the same bone, the anterior crural nerve is interposed; uniting near their insertion these muscles are attached to the lesser trochanter. The portion corresponding to the psoas was not found by Hyrtl. They are inseparable in Tatusia, Dasypus, Cyclothurus, Bradypus, Cholœpus, Tolypeutes, Manis, and Tamandua. In Orycterope Cuvier found the iliacus split; neither Galton nor Humphry did so.

The adductor mass is sub-divisible into two parts, the first of which is the condyloid part of the adductor magnus (Adductor primus of Haughton); this is large, and arises directly external to the wide pubic attachment of the crus penis (which has a muscular investment formed by a very largely developed erector penis), and is inserted into the inner femoral condyle. The femoral vessels pass between this portion of the muscle and the second part.

Adductor secundus is a very large, fleshy mass, corresponding to the rest of the adductor magnus, the adductor brevis, and longus; it is attached to the middle and lower part of the back of the femur. Its origin is not only from the pubis, but from the inferior ischiatic fulcrum; no part descended below the femur, the tibial part of Hyrtl being the semi-membranosus.

In Tatusia peba the two parts of the adductor mass correspond to those in Chlamydophorus. In Cyclothurus there are two also, which correspond

to the same type. In the Sloths the adductors are well developed, and are four in number. One corresponds to the condyloid part of the adductor magnus; a second to the rest of the adductor magnus; a third to the brevis of human anatomy; and a fourth to the longus. In Pholidotus there is a similar four-fold division. Orycteropus has three adductors, identified as magnus, longus, and brevis by Humphry, and as longus, brevis and minimus (Henle) by Galton.

Popliteus in Chlamydophorus is very large, and arises from the outer condyle of the femur. I found no sesamoid bone in its tendon, as it lies under the external lateral ligament; nor does Hyrtl mention the existence of one; it becomes fleshy, and is inserted into one-half of the back of the tibia. It is similarly extensive in Tatusia peba. In Dasypus Sexcinctus it is similar. Mr. Galton found it with a double tendon in this species, but I did not see that arrangement in any of five specimens examined. The insertion is more extensive in Dasypus than in either of the others; none of these possess a sesamoid bone. In the three-toed Sloth there is a large sesamoid bone, and the muscle occupies more than one-third of the tibia. It is similar in Cholœpus. There is no sesamoid bone in the Orycteropus, but there is in Cyclothurus and Pholidotus. There does not appear to be one in Myrmecophega or in Tamandua. The muscle passes down parallel in Orycteropus, and is least extensive ($\frac{1}{3}$) in Cyclothurus.

Gastrocnemius has its usual two heads, neither of which possesses a sesamoid bone; these arise as usual, and the outer head is joined by the soleus, which is fibular in its origin; a distinct small plantaris, arising from the outer femoral condyle, and becoming tendinous, also joins the common tendon of the preceding "Tendo nimirum non ad calcar pedis firmatur, sed in teretem funem elongatus, per canalem calcanei privatem ad plantem effertur, ubi aponeuroseos formam induit, et paulo post in quatuor fasces discedit." I found, however, that a fleshy portion of this muscle was attached to the calcaneum, at its outer side. In Tatusia peba the outer head of the Gastrocnemius is enormous and fleshy for its whole extent. The internal is about half the size of the external, and the soleus is fibular in origin, and unites with the outer head; they are all fleshy, as far as the calcaneum, where they unite to be inserted into that bone; no slip passes onward to the sole, nor is there

a separate plantaris, the germ of that muscle being included in the large gastrocnemius externus. In Dasypus these muscles are distinct, the two heads of the gastrocnemius uniting together at the lower third of the leg, the solæus being fleshy the whole way, and inserted separately into the heel, and fibular in origin ; the plantaris being distinct, and winding under the heel, is inserted into the plantar fascia.

In Bradypus the two gastrocnemial heads are separate for their whole length, and had in the B. tridactylus, dissected by me, no sesamoid bone present. Meckel, however, found one in his specimen, but Professor Humphry found the parts arranged as in my individual. No sesamoid exists in Cholœpus didactylus. In Orycteropus there is a sesamoid bone in the outer head. A similar one exists in Cyclothurus and Tamandua. In most of these the muscle is purely calcaneal in insertion ; in all there is a fibular solæus which is united to the calcaneum also.

A separate plantaris exists in Orycteropus as well as in Chlamydophorus, and is inserted into the plantar fascia, being partly attached to the calcaneum. In Bradypus I found this muscle enormous, and inserted into the tendon of the flexor digitorum. In Cyclothurus it is not prolonged into the sole of the foot, but ends in the strigil-shaped bone, and the calcaneum (Galton). In Myrmecophaga, Cholœpus, Pholidotus, and Tamandua, this muscle is not distinct from the gastrocnemius.

Peroneus longus arises from the upper half of the fibula, and ending in a tendon which crosses the sole, is inserted into the first metatarsal (ento-cuneiform, Hyrtl).

In Tatusia this muscle arose from the outer condyle of the femur, under the outer head of the gastrocnemius, and from the head of the fibula, and was inserted into the first metatarsal ; it sent, however, a slip to the fifth. In Dasypus it ascends to the patella and head of the fibula, by two separate slips, and winding by its tendon round the outer ankle, it is inserted into the first metatarsal, and by a small slip to the fifth, as in Tatusia. In Cyclothurus this muscle is inserted into the os naviculare (Galton), to the ento-cuneiform and heads of two metatarsals (Humphry). In Bradypus this muscle is femoral as well as fibular in origin, and is inserted into the outer metatarsal bone. In Cholœpus it does not ascend beyond the fibula, and is inserted into the

bases of the three metatarsals. In Orycteropus it is arranged just as in Tatusia.

Peroneus brevis arises below the longus, and is inserted into the base of the fifth metatarsal; closely connected to it is the peroneus quinti, whose tendon is attached to the base of the first phalanx of the little toe. The fleshy portions of the muscles are inseparable; hence, Hyrtl describes the single belly as giving off two tendons. In Tatusia these muscles are exactly similar. In Dasypus they are inseparable, and ascend to the femur, and are attached to the outer condyle (Galton found them attached to the lower half of the patella), and to the whole of the outside of the fibula; the muscle gives off two tendons at the ankle, which are inserted, one into the metatarsal of the fifth toe, the other (P. quinti) into first phalanx of the same digit. Being post-malleolar this latter tendon has *no* claim to be considered as a peroneus tertius, which is an ante-malleolar muscle essentially. In Bradypus tridactylus these two are quite separate in origin, the P. quinti ascending to the mid-point on the fibula, and inserted into the base of the metatarsal bone, along with the Peroneus brevis. This muscle is similar in Cholœpus, Cyclothurus, Orycteropus, and in Pholidotus.

There is no trace whatever of a true peroneus tertius in any of the species examined, viz., Chlamydophorus, Tatusia, Dasypus, Bradypus, Cholœpus, Orycteropus, Tolypeutes, Pholidotus, Cyclothurus, or Tamandua. The muscles described as such by authors are, in reality, referrible to the last type (peroneus quinti).

Tibialis anticus is a large, fleshy muscle in Chlamydophorus, arising from the outer side and front of the tibia, not ascending to the femur, it is inserted into the scaphoid bone. In Tatusia this muscle is also tibial in origin, and stretches forwards to the metatarsal of the pollex. There is also in this animal a tibialis anticus secundus, running parallel to the ordinary muscle and inserted into the ento-cuneiform bone. In Dasypus there is but one, and it in all respects is similar to the muscle in Tatusia, only it is attached to the ento-cuneiform. In Cyclothurus it is similar, but has a fibular as well as a tibial origin. This is the case also in Bradypus, but the insertion in that animal is into the metatarsal of the inner toe; but in the specimen dissected by me, the two heads were not separate, as Meckel and Humphry found them. Humphry found a portion of this muscle detached below from the

lower part of the fibula to the ento-cuneiform. This did not exist in my specimen, and Meckel says nothing about its being present in his. It exists, however, in Cholœpus, as Humphry describes. The same author found the tibialis split in Cholœpus into a fibular and tibial portion; but these were inserted together into the tendons of the deep flexors, and into the aponeurosis over them.

In Pholidotus there are two tibiales, as in Tatusia, one to the metatarsal of the hallux, and one to the ento-cuneiform. And in Orycteropus there are also two tendons, but one belly; and the origin of the muscle extends up to the patella. The tendon is single in Tamandua and Myrmecophaga jubata.

Tibialis posticus is a very small, round muscle, with a short belly, and a long tendon. It is closely applied to, but not united with, the flexor digitorum, and is inserted into the navicular bone. A second muscle, named by Hyrtl the tibialis posticus accessorius, exists—"qui tibialem posticum viæ comitem laborisque socium legit, Retro malleolum internum in teretem fatiscit tendinem;" this muscle arises higher up than the tibialis proper, and is inserted into the ento-cuneiform; it is tibial in its origin. There is a similar tibialis secundus in the Dasypus sexcinctus; and Hyrtl says of this muscle, "In Dasypode (gymnuro) etiam prostat." And a similar slip exists in Tatusia. In Cyclothurus this second muscle has a high fibular origin, and an insertion into the strigil bone. The other portion shows nothing unusual. There is only one tibialis, and it a small one, in Bradypus. In Cholœpus the secundus is inserted into the inner part of the flexor brevis digitorum, and the other part is usual; in Pholidotus there is a second posticus inserted into the ento-cuneiform, and quite separate for its whole extent. In Orycteropus there is a double muscle also attached to the ento-cuneiform and first metatarsal.

Extensor digitorum communis is fibular in origin, slender in its belly, and becomes tendinous at the ankle, where it is bound down by a strong annular ligament, and divides into four slips, which are attached to the second and third phalanges of the outer toes. In Tatusia the muscle is similar, and in neither has it a femoral head. In Dasypus it is similar, and connected to the tendons of the short extensor. In Cyclothurus the tendons end in an aponeurotic expansion, on the dorsum of the foot. In Bradypus it has a

weak femoral origin, as well as a tibio-peroneal one; and the same is the case in Cholœpus, Orycteropus, and Pholidotus. In the Orycterope these tendons are arranged in a rather complicated way, as described by Humphry and Galton; and there are sesamoid bones in them, opposite the metacarpo-phalangeal articulations.

Extensor hallucis longus is very small, fibular, in origin fusiform, and inserted into the dorsum of the little toe. In Tatusia it arises by a membranous expansion, from the lower two-thirds of the margin of the fibula, and is inserted by a fascia into the dorsum of the inner digit. I could not trace it as a special tendon beyond the base of the first phalanx; there it was lost in the general fibrous structures of the toe. In Dasypus it is much more distinct, and sends a strong slip to the extensor tendon of the second toe. In Bradypus the muscle is single and weak. In Cyclothurus the muscle is weak, arises low down, and sends its tendon to join that of the common extensor. In Orycteropus it is very similar to the arrangement in Dasypus, above described. In Pholidotus it is large, and sends two slips to the first and last phalanges of the first toe, and one to the inner side of the second toe.

Extensor brevis digitorum is very thin, but distinctly present in the Chlamydophorus, arising from the outer side of the tarsus, particularly from the calcaneum, and inserted into the dorsum of each digit, except the internal; this arrangement Hyrtl found in Dasypus Gymnurus; but in his specimen of Chlamydophorus, the dorsum of the foot was so lacerated, that he could not satisfy himself regarding the presence of this muscle. In Tatusia this muscle sends no slip to either of the lateral digits, but its main body goes to the second and third; a separate slip arises from the outside of the calcaneum for the fourth toe. In Orycteropus it is inserted into the three middle digits, joining the tendons of the extensor communis. In Dasypus it is inserted into the extensor tendons of the second, third, and fourth toes. It is very large in the Sloths, and inserted into the ungual phalanges of the digits, separate from the fascia from the extensor longus. In Pholidotus it is equally separate, and goes to the outer digits.

Flexor hallucis longus is very large and fleshy, arising from the greatest part of the back of the fibula; it early joins the next muscle, and they are inserted together.

Flexor digitorum much smaller and tibial, joins with the last, and forms

a single tendon, which supplies all the toes. In the sole of the foot the tendon is swollen, and has a cartilaginous nodule before its division; it then splits into five slips, which pass one to each digit. In Tatusia peba the two muscles are at first separate, and the tendons join; after winding round the malleolus here, in the sole of the foot, they are attached to a sesamoid bone, which is held in its place by a flat band from the calcaneum, in place of a musculus accessorius; there are five tendons passing from this to the toes. "In Dasypode setoso ossiculum hoc sesamoideum, complanatum, ovalis ambitus, acie gemina longitudinali instructum, triplicem superficiem articularem tarsi ossibus offert. In Dasypode gymnuro ambitu et robore declinat." The two flexors are inseparable in Dasypus sexcinctus, and have but a common tendon which is inserted into all the toes. It has also a sesamoid bone, although the absence of this in Tolypeutes, Priodonta, and Dasypus, was remarked by Cuvier. No sesamoid bone exists in Tamandua, Cyclothurus, Myrmecophaga, Orycteropus, Cholœpus, and Bradypus; but the bone is possessed by Tolypeutes. The flexor hallucis forms the largest part of the conjoint mass in Orycteropus, Cholœpus, Bradypus, and Pholidotus; but in none of these are the two separable. Humphry describes the plantaris in Ai as a condyloid accessory head to this muscle.

No trace of a musculus accessorius exists in Chlamydophorus, none but a fibrous band in any other Armadillo (Tatusia, Dasypus, or Tolypeutes). A similar fibrous band exists in the Orycteropus; but in the other ant-eaters and sloths it is muscular and large (in Cholœpus, Bradypus, Tamandua, Myrmecophaga, Cyclothurus, and Pholidotus).

In Chlamydophorus the lumbricales are exceedingly fine, four in number and on the halluceal sides of the digits. They are similar in Tatusia, Orycteropus, and Tamandua. In Pholidotus there are three, inserted as usual into the halluceal side of the digit, none going to the hallux. In Cyclothurus there are also three. In Cholœpus there are also three, two of which take origin from the continuations of the tibialis anticus tendons. In Bradypus there are none; in Dasypus there are four, arising as usual, but the external and the two innermost ones divide, and are inserted into the opposed surfaces of the digits; hence there are seven insertions—one into

each side of each toe, except the outside of the little, the inside of the hallux, and the fibula side of the third digit.

The deep plantar muscles in Chlamydophorus are abductor hallucis, from the calcaneum to the first phalanx; flexor brevis hallucis from beneath the last to the first phalanx; flexor brevis minimi digiti, and abductor minimi digiti, from the os calcis to the proximal phalanx of the little toe; I found no transversalis pedis. In Tatusia I found the same arrangement, with some of the fibres of the flexor brevis hallucis attached to the metatarsal, and acting as an adductor of the hallux. In Dasypus there is an abductor hallucis, and a flexor brevis, as well as an abductor and a flexor brevis minimi digiti. In Cyclothurus the abductor hallucis is modified singularly, and there is a flexor brevis (adductor, Galton) hallucis, together with an abductor and a flexor brevis minimi digiti; in none of these is there a flexor brevis digitorum, except in the Cyclothurus. In Pholidotus, Cyclothurus, Cholœpus, Tamandua, Bradypus, it is present and weak. A transversalis pedis exists in Pholidotus and Cholœpus; the abductor pollucis in Cholœpus, arises from the palmar fascia and integument, and is inserted into the base of the first metatarsal. The abductor minimi digiti is a somewhat similar slip attached to the fifth metatarsal in this animal.

The interossei muscles in the pes of Chlamydophorus agree with those in Tatusia; for the first toe there is an adductor only on the palmar surface with a single head; for the second there is also a single adductor; for the third there is an abductor as well as an adductor, and both show traces of doubleness; for the fourth there are both the abductor and adductor developed; for the fifth (which has a special abductor) there is an adductor; this makes seven in all. In Dasypus there are six, which are—1. adductor hallucis; 2. abductor secundi digiti; 3. adductor secundi digiti; 4. adductor quarti; 5. abductor quarti; 6. adductor quinti digiti, the last being the largest.

In Cyclothurus there are seven passing to each side of each toe, except to the hallux, and the tibial side of the little toe. In Orycteropus there is one to each side of each of the three middle toes. In the Sloths there is one on each side of each digit, except the outer sides of the lateral digits; and the same occurs in Pholidotus and Tamandua.

VISCERAL ANATOMY.

The description of the mouth, salivary glands, pharynx, teeth, vocal and respiratory organs given by Professor Hyrtl are so admirable, that no supplement is required here. His description, likewise, of the angeiology of the animal is also as perfect as need be ; and the same is true of the neurology and of the description of the female reproductive organs ; I will, therefore, confine the remarks which I make under this head to the following :—1st, observations on the folds of peritoneum ; 2nd, on the liver ; 3rd, on the digestive canal ; and 4th, on the male generative organs.

1. PERITONEUM.

The liver has on its upper surface a falciform ligament, stretching, as usual, from before backwards, for ·3 of an inch from the free edge ; there are also for this organ two lateral ligaments, and a coronary whose layers are closely united ; from the transverse fissure, the large lesser omentum descends, enclosing the stomach, which, from its great curvature, sends off a greater omentum ; this, on the left side, passes downwards, and is then reflected upwards and backwards, towards the spine, from whence its inferior layer passes again forwards, forming a transverse mesocolon ; on the right side the ascending part of the great omentum is adherent to the transverse mesocolon. This arrangement of peritoneum certainly is strong evidence in favour of the view of the development of the omentum and mesocolon, supported by Meckel and Johannes Müller. The gastro-splenic omentum was an offshoot from the left end of the great omentum, and contained three spleniculi. Hyrtl found one much-lobed spleen.

2. LIVER.

This organ is very large, 1-2 inches long, and one inch broad ; it is divisible into five lobes. The upper surface is convex, and grooved for the fasciculi of the diaphragm ; the under surface is very much divided ; the under aspect of the left lobe is concave, and this lobe itself is very thin

and wide ; the right is much thicker, and has a small sub-pyriform gall bladder with a straight duct in a sulcus on its lower surface. The quadrate lobe is extensive and thick ; the Spigelian connected to the rest by a narrow pedicle, deeply concave on its lower surface, for the right supra-renal capsule ; the vena cava passes in a tunnel, not in a groove, in the hepatic substance. The ducts of the liver and pancreas open into the duodenum, half an inch below the pylorus.

3. DIGESTIVE CANAL.

The tongue is pointed, thirteen lines long, and free beneath for eight lines ; it is thickly coated above, with re-curved filiform papillæ, which are either simple and acuminated, or cleft. The fungiform papillæ are hemispherical, and covered with small epithelial eminences. The intestine contained fragments of the elytræ of beetles, and of the legs of insects ; an accumulation of these materials filled up the two remarkable cæca ; some softer animal matter existed in the stomach, consisting of fragments of larvæ, and the remains of several myriapods ; I found no parasites.

4. MALE REPRODUCTIVE ORGANS.

These organs were remarkably similar to the better known organs of Tatusia ; the penis was longer and tapered to its extremity, the glans being bluntly cylindrical and of the same diameter as the body of the organ ; the testes were contained in the wide abdominal end of the inguinal canals, and were very small ; the vasa deferentia short, and not convoluted ; I could find no vesiculæ seminales ; the Weberian organ long, and not distinctly bifid ; the urethra was short and straight, and the bladder sub-pyriform.

CONCLUSION.

Mr. Atkinson has proposed the theory, that the Chlamydophorus, like Pholidotus, Manis, and Myrmecophaga, walks on the dorsum of its fore-foot, and has pictured the skeleton accordingly, and gives in support the following arguments :—

1. His specimen had the claws doubled inwards.

2. The radial half of the palm is covered with soft fur.
3. The claws are flattened and imbricating.
4. The dorsal aspect of the claws looked worn.

He also adduces the great development of the flexors, and the supposed analogy of Priodonta gigas.

My specimen does not seem to bear out this idea. I have drawn it exactly as it was when it came to me, and it will be seen that the claws are in position exactly as in Tatusia and the other Armadilloes; again, the plantar aspect and outer edge of the sole is callous, as if pressed on; so is the ulnar half of the carpus. I have, consequently, preserved the prone position in my stuffed skin, and the attitude looks extremely comfortable and natural. The dorsal aspect of the claws of the forefoot in my specimen are neither flattened nor worn, and there is nothing in the structure of the flexor muscles to indicate a permanently flexed wrist. Indeed, the presence of a palmar ossicle, is important evidence on this point.

The position of Chlamydophorus will be seen from the foregoing description to be plainly among the Dasypodidæ, and very close to Tatusia. That the relations of the animals referred to in the Myological part may be more clearly understood, I subjoin a list of the species of Edentata, arranged in order, and I have in it marked in *italics* the forms which have been dissected.

ORDER EDENTATA.

1st. Family Bradypodidæ.
Genera, Bradypus. Sp. **torquatus*, affinis.
Cholæpus, **didactylus*, Hoffmanni.
Arctopithecus cuculliger *gularis*, Blainvillei, Boliviensis, marmoratus, castaniceps, **tridactylus*.

2nd. Family Dasypodidæ.
Sub-Fam., Chlamydophorina.
Genera, Chlamydophorus, Sp. **truncatus*.
Burmeisteria, Sp. *retusus*.
Sub-Fam., Dasypina.

Genera, Tatusia, **peba*, **hirsuta*, hybrida, Kappleri.
Tolypeutes, **tricinctus*, conurus.
Dasypus, **sexcinctus*, setosus, velerosus.
Xenurus unicinctus, hispidus.
Priodonta gigas.
Euphractus villosus, minutus.

3rd. Family Orycteropidæ.
Genus, Orycteropus, **Capensis*, Senegalensis, Æthiopicus.

4th. Family Manidæ.
Genera, Manis, Sp.—*multiscutata* longicauda,* Temminckii.
Pholidotus,* Indicus, *Dallmani*, Javanus Africanus.

5th. Family Myrmecophagidæ.
Genera, Myrmecophaga **jubata*.
Cyclothurus *didactylus*,* dorsalis.
Tamandua *tetradactyla*,* longicaudata.

* I have attached an (*) to those species whose anatomy I myself have been enabled to examine, in whole or in part.

Since this Paper was read I have dissected an additional species of Arctopithecus, and one of Manis multiscutatus.

MISCELLANEOUS NOTES ON THE VISCERAL ANATOMY.

The sub-maxillary gland, when spread out, measured nine lines in length, was thickest below and in front, nearly meeting its fellow in the middle line in front; it is enveloped in a fascial capsule. The parotid is separated from it by a fascial fold, which is not attached to the mandible, and by the external jugular vein. The portio dura nerve crosses beneath its upper part, and sends off four facial branches.

The skin being removed, the eyeball is found ·8 of an inch behind the anterior end of the snout; it is ·15 of an inch in transverse diameter, and ·135 in vertical, and is placed ·4 of an inch above the angle of the mouth.

From the posterior canthus a fibrous band stretches for ·15 of an inch to the anterior or cartilaginous end of the auditory meatus, which ends ·09 of an inch above the zygoma, ·14 behind the angle of the eye, and an inch behind the point of the snout.

EXPLANATION OF THE PLATES.

Pl. XIII.—Fig. 1. Sketch of the specimen before dissection.

Fig. 2. View of pelvic shield and scutes, and upper surface of tail.

Fig. 3. Palmar view of fore foot.

Fig. 4. Lateral view of fore foot.

Fig. 5. Inner side of hind foot.

Fig. 6. Epidermal scales of Chlamys × 300.

Fig. 7. Sole of hind foot.

Fig. 8. Outer view of hind foot.

Fig. 9. View of abdominal viscera *in situ*. *a*. liver; *b*. stomach; *c, d, e*. great omentum; *f*. transverse mesocolon; *h*. small intestines; *i. j*. wires passed under the great omentum.

Fig. 10. Submental region. *k*. masseter; *l*. parotid gland; *m, n*. submaxillary glands.

Fig. 11. Salivary glands dissected out. *a*. Stenon's duct; *b*. termination of duct opened; *c*. parotid; *d*. mandible; *e*. submaxillary duct (ampullated); *f*. submaxillary gland.

Fig. 12. Hyoid bone. *a*. basihyal; *b*. thyrohyal; *c*. ceratohyal.

Fig. 13. Heart. *a*. right auricle; *b*. left; *c*. right ventricle; *d*. left ditto; *e*. pulmonary artery; *f*. right subclavian; *g*. right carotid; *h*. left carotid; *i*. left subclavian; *j*. aorta.

Fig. 14. *a*. one spleniculus; *b*. pancreas.

Fig. 15. Liver. *a, k*. left lobe; *b*. quadrate; *c–e*. right lobe; *d*. gall bladder; *f*. vena portæ; *g. h*. Spigelian lobe; *i*, depression for kidney; *j*. fissure for umbilical vein

Pl. XIV.—Fig 16. Dermal muscles of Tatusia. *a*. pectoro-brachial; *b*. platysma myoides; *c, e*. facio-lateral portion of musculus orbicularis; *d*. dorso-pectoral; *e*. abdomino-femoral; *f*. dorso-axillary fold; *g*. dorsal shield.

Fig. 17. Side view of Tatusia. *g*. musculus orbicularis; *h*. ischio-tergal; *c*. facio-lateral slip of orbicularis.

Fig. 18. Stomach and cæcum. *a*. œsophagus; *b*. cardiac end of stomach; *c, e*. pylorus; *d*. pre-antral constriction; *f*. duodenum; *g*. commencement of jejunum; *h*. ileum; *i*. cæcal vermiform appendage; *j*. twin cæca; *k*. colon.

Fig. 19. Lungs. *a*. trachea; *b*. bifurcation; *c*. upper lobe of right lung; *d*. middle ditto; *e*. inferior ditto; *f*. inner mesial lobe; *g*. upper lobe of left lung; *h*. fissure; *i*. inferior lobe.

Fig. 20. Sternum of Chlamydophorus. *a.* presternum; *b.* 1st sterneber of mesosternum; *c, d, e.* successive sternebers; *f.* xiphisternum; *g.* first rib.

Fig. 21. Tarsal bones.

Fig. 22. Under surface of abdomen, showing position of anus, penis, and plantæ.

Pl. XV.—Fig. 23. Upper limb. *a.* supraspinatus; *b.* deltoid; *c.* clavicle; *d.* infraspinatus; *e, f, g.* triceps; *h.* biceps; *i.* pronator teres; *j.* extensor carpi radialis; *k.* extensor ossis metacarpi pollicis; *l.* extensor digitorum communis; *m.* extensor minimi digiti; *n.* extensor carpi ulnaris.

Fig. 24. *a.* supraspinatus; *b.* spine; *c.* infraspinatus; *d.* teres minor; *e, g.* origin of triceps longus; *h.* teres major; *i.* brachialis anticus; *j.* deltoidal crest; *k.* ecto-condyloid ridge; *l.* olecranon.

Fig. 25. Insertion of biceps (*a*) and brachialis (*c*); *b.* humerus; *d.* olecranon.

Fig. 26. *a.* flexor sublimis digitorum; *b.* olecranal head of flexor profundus; *c.* ulnar head of same; *d.* flexor pollicis; *e.* palmar ossicle; *f.* tendon to pollex; *g.* to index; *h.* lumbricalis of middle finger; *i.* tendon of middle finger; *j.* lumbricalis of ring; *k.* tendon of ring finger; *l.* lumbricalis of ulnar side of ring finger; *m.* tendon for little finger.

Fig. 27. Muscles of manus.

Fig. 28. *a.* Brachialis; *b.* brachial artery; *c.* supracondyloid process; *d.* inner condyle and origin of pronator teres; *e.* extensor carpi radialis; *f.* flexor carpi radialis; *h.* palmaris longus; *g.* flexor sublimis digitorum; *i.* flexor carpi ulnaris.

Fig. 29. Front view of carpus.

Fig. 30. Back view of carpus.

Fig. 31. Inferior view of the muzzle, showing nostrils.

Fig. 32. Hyoid arch, showing the lingual (*a*) and vagus (*b*) nerves, united at their point of exit; *d, g.* stylohyal; *h.* thyroid cartilage; *i.* crico-thyroid.

Fig. 33. Axillary muscles. *a.* subclavius; *b.* acromion; *c.* latissimus dorsi; *d.* teres major; *e.* biceps; *f.* anconeus; *g.* humerus; *h.* subscapularis.

Fig. 34. Trunk muscles. *a.* psoas accessorius; *b.* psoas parvus; *c.* tendons of psoas; *g, h.* parts of pelvic shield.

Fig. 35. Digastric muscle. *a.* mandibular belly; *b.* origin; *c.* tendon; *d.* posterior belly.

Fig. 36. Neck muscles. *a.* masseter; *b, c.* clavicular trapezius; *d.* cleido-occipital; *e.* cleido-mastoid; *f, g.* sterno-mastoids; *h.* clavicle; *i.* clavicular deltoid; *j.* acromial deltoid; *k.* clavicular pectoral; *l. v.* abdominal pectoral; *m.* pectoralis quartus; *n.* latissimus dorsi; *o.* brachial plexus; *p.* dorsi epitrochlear; *q.* anconeus internus; *t. u.* sternal pectoral.

Fig. 37. Papillæ of tongue magnified.

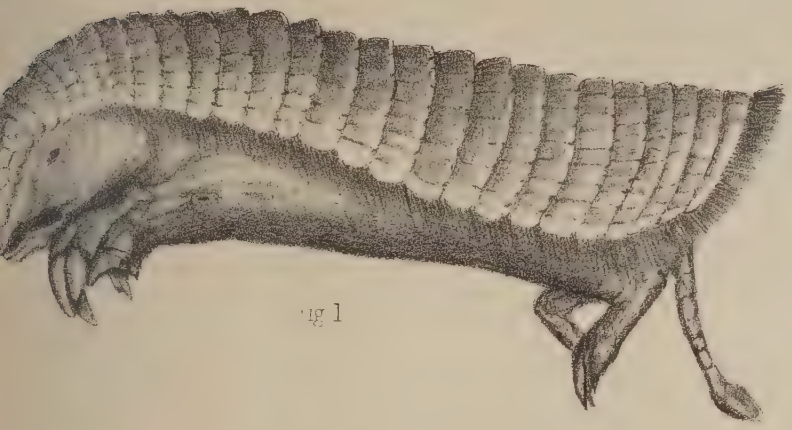


Fig 1

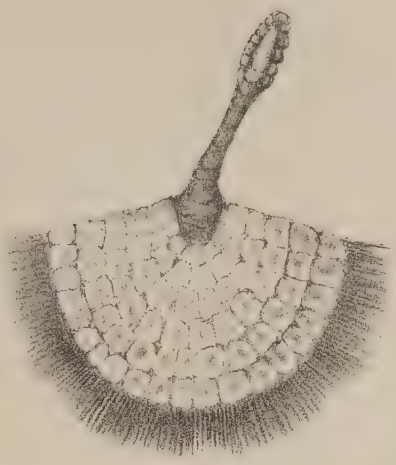


Fig. 2

Fig 3

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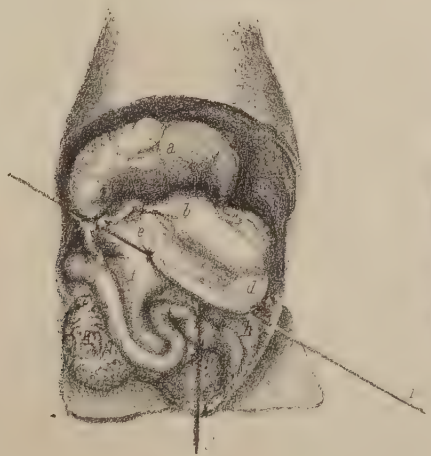

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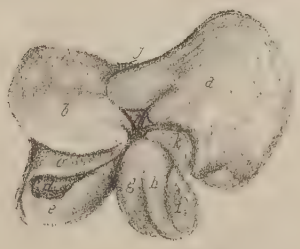


Fig 15

Fig 16

Fig 22

Fig 17

Fig 20

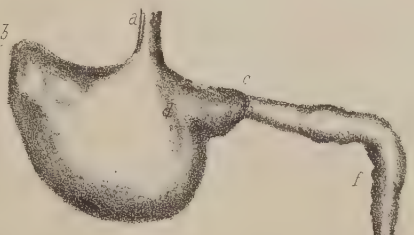


Fig 18

Fig 21

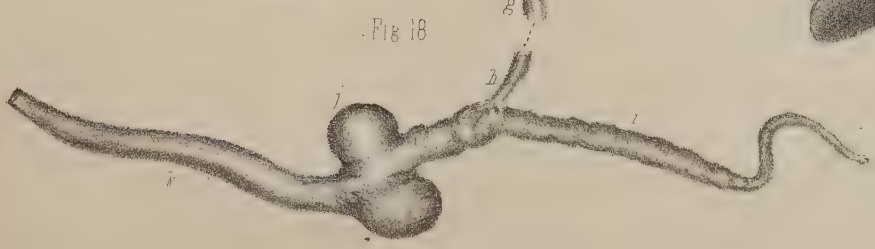


Fig 19

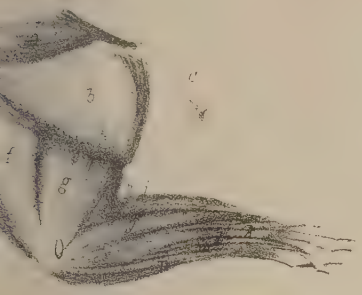


Fig. 23

Fig. 24

Fig. 25

Fig. 26

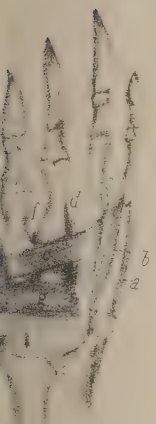


Fig. 27

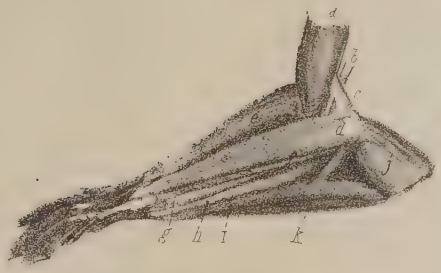


Fig. 28

Fig. 29.

Fig. 30

Fig. 36

Fig. 37

Fig. 34

Fig. 35

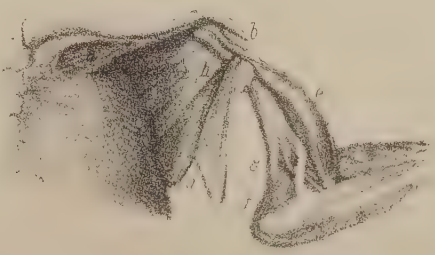


Fig. 33.

Fig. 2.

V.—*Some Theorems in the Reduction of Hyper-elliptic Integrals.* By John C. Malet, A. M.

Read, January 12, 1874.

In the following paper I shall use the expression $R(x)$ to denote a rational algebraic function of x, though not always the same function.

Let $\quad X = (x-x_1)(x-x_2)(x-x_3)(x-x_4)(x-x_5)(x-x_6)$,

then, if in the integral,

$$\int \frac{R(x)\,dx}{\sqrt{X}}$$

we substitute for x, $\dfrac{l+my}{p+qy}$ we get

$$\int \frac{R(y)\,dy}{\sqrt{Y}}$$

where Y denotes the same function of y that X does of x and

$$y_1 = \frac{px_1 - l}{m - qx_1}, \quad y_2 = \frac{px_2 - l}{m - qx_2} \text{ and so on.}$$

If now $\quad y_1 + y_4 = 0, \quad y_2 + y_5 = 0, \quad y_3 + y_6 = 0$, we have

$$\left.\begin{aligned} (pm+lq)(x_1+x_4) - 2pqx_1x_4 - 2lm = 0 \\ (pm+lq)(x_2+x_5) - 2pqx_2x_5 - 2lm = 0 \\ (pm+lq)(x_3+x_6) - 2pqx_3x_6 - 2lm = 0 \end{aligned}\right\} \quad . \quad . \quad . \quad (1)$$

Eliminating l, m, p, q, from equations (1), we find as the condition that the integral

$$\int \frac{R(x)\,dx}{\sqrt{X}}$$

should be reducible by a linear substitution to an integral of the form

$$\int \frac{R(y)\,dy}{\sqrt{(y^2-y_1^2)\,(y^2-y_2^2)\,(y^2-y_3^2)}}$$

the following relation connecting x_1, x_2, x_3, x_4, x_5, x_6, viz.:—

$$\begin{vmatrix} x_1+x_4 & x_1 x_4 & 1 \\ x_2+x_5 & x_2 x_5 & 1 \\ x_3+x_6 & x_3 x_6 & 1 \end{vmatrix} = 0. \quad . \quad . \quad . \qquad (2)$$

Now the integral

$$\int \frac{R(y)\,dy}{\sqrt{(y^2-y_1^2)\,(y^2-y_2^2)\,(y^2-y_3^2)}}$$

is always reducible to elliptic functions, for it is evident it may be written as the sum of two other integrals; thus

$$\int \frac{Mdy}{\sqrt{(y^2-y_1^2)\,(y^2-y_2^2)\,(y_2-y_3^2)}} + \int \frac{Nydy}{\sqrt{(y^2-y_1^2)\,(y^2-y_2^2)\,(y^2-y_3^2)}},$$

where M and N are rational algebraic functions of y^2. Now the first of these integrals may be reduced to elliptic functions by the substitution

$$y^2 = \frac{y_1^2\,y_3^2\,\sin^2\theta}{y_1^2\,\sin^2\theta - (y_1^2-y_3^2)},$$

for by this substitution

$$\int \frac{dy}{\sqrt{(y^2-y_1^2)\,(y^2-y_2^2)\,(y^2-y_3^2)}} \text{ becomes } -\int \frac{d\theta}{y_2\sqrt{y_1^2-y_3^2}\,\Delta(k,\theta)},$$

where $$\Delta(k,\theta,) = \sqrt{1-k^2\sin^2\theta},\ k^2 = \frac{y_1^2}{y_2^2}\cdot\frac{y_2^2-y_3^2}{y_1^2-y_3^2}.$$

The second integral is reduced to elliptic functions by the substitution

$$y^2 = y_1^2 - (y_1^2 - y_2^2)\sin^2\phi$$

for, by this substitution,

$$\int \frac{y\,dy}{\sqrt{(y^2-y_1^2)\,(y^2-y_2^2)\,(y^2-y_3^2)}} \text{ becomes } \frac{1}{\sqrt{y_3^2-y_1^2}}\int \frac{d\phi}{\Delta\,(\lambda,\,\phi)},$$

where
$$\lambda^2 = \frac{y_1^2-y_2^2}{y_1^2-y_3^2}.$$

Hence, if the condition given by equation (2) is true, the integral

$$\int \frac{R\,(x)\,dx}{\sqrt{X}}$$

can always be reduced to elliptic functions.

In actually performing the reduction it simplifies the work to assume $l = m = 1$, and we easily find the reduced expression in elliptic functions.

Let now
$$X \equiv (a_0, a_1, a_2, a_3, a_4, a_5, a_6 \between x, 1)^6,$$

then the skew invariant of X is composed of such factors as

$$\begin{vmatrix} x_1+x_4 & x_1\,x_4 & 1 \\ x_2+x_5 & x_2\,x_5 & 1 \\ x_3+x_6 & x_3\,x_6 & 1 \end{vmatrix}$$ (See Salmon's Modern Algebra, page 275.)

And the theorem may be stated thus—

"If the skew invariant of $(a_0\,a_1\,a_2\,a_3\,a_4\,a_5\,a_6 \between x, 1)^6$ vanish, then the integral

$$\int \frac{R\,(x)\,dx}{\sqrt{(a_0\,a_1\,a_2\,a_3\,a_4\,a_5\,a_6 \between x_1\,1)^6}}$$

depends on elliptic functions." A theorem due to M. Gordan, "Annali di Matematica," Series II., vol. 2, page 346.

If now in the expression $\dfrac{dx}{\sqrt{X}}$ we make the substitution

$$x = \frac{x_1\,(x_2-x_4) + x_4\,(x_1-x_2)\sin^2\theta}{x_2-x_4+(x_1-x_2)\sin^2\theta} \quad . \quad . \quad . \qquad (3)$$

we find

$$\frac{A\,dx}{\sqrt{X}} = \frac{2\{x_2 - x_4 + (x_1 - x_2)\sin^2\theta\}\,d\theta}{\sqrt{(1-k^2\sin^2\theta)(1-k_1^2\sin^2\theta)(1-k_2^2\sin^2\theta)}},$$

where

$$A = \{-(x_1-x_3)(x_1-x_5)(x_1-x_6)(x_2-x_4)^3\}^{\frac{1}{2}}$$

$$k^2 = \frac{(x_1-x_2)(x_5-x_4)}{(x_1-x_5)(x_2-x_4)},\quad k_1^2 = \frac{(x_1-x_2)(x_6-x_4)}{(x_1-x_6)(x_2-x_4)},\quad k_2^2 = \frac{(x_1-x_2)(x_3-x_4)}{(x_1-x_3)(x_2-x_4)}. \quad \ldots (4)$$

Hence, by substitution (3) the integral

$$\int \frac{R(x)\,dx}{\sqrt{X}},$$

may always be made to depend on Hyper-elliptic Integrals with 3 moduli, viz.,

$$\int \frac{R(\sin^2\theta)\,d\theta}{\Delta(k, k_1, k_2, \theta)},$$

where

$$\Delta(k, k_1, k_2, \theta) = \sqrt{(1-k^2\sin^2\theta)(1-k_1^2\sin^2\theta)(1-k_2^2\sin^2\theta)},$$

k, k_1, k_2, having the values given by equations (4). If now we multiply each side of the equation

$$\begin{vmatrix} x_1 + x_4 & x_1 x_4 & 1 \\ x_2 + x_5 & x_2 x_5 & 1 \\ x_3 + x_6 & x_3 x_6 & 1 \end{vmatrix} = 0.$$

by $x_1 - x_4$ we see that the condition given by (2) may be written

$$\frac{(x_1-x_2)(x_5-x_4)}{(x_1-x_5)(x_2-x_4)} = \frac{(x_1-x_2)(x_6-x_4)}{(x_1-x_6)(x_2-x_4)} \cdot \frac{(x_1-x_2)(x_3-x_4)}{(x_1-x_3)(x_2-x_4)},$$

or

$$k = k_1 k_2. \quad \ldots \quad (5)$$

Hence we have Jacobi's well-known theorem:

"All hyper-elliptic integrals with three moduli k, k_1, k_2, can be made to depend on elliptic integrals, provided the following relation between the moduli is true

$$k = k_1 k_2."$$

If now, in place of the substitution given in (3), we had reduced

$$\int \frac{R(x)\,dx}{\sqrt{X}}$$

to the standard form of hyper-elliptic integrals by the substitution

$$x = \frac{x_1(x_4 - x_2) + x_2(x_1 - x_4)\sin^2\theta}{x_4 - x_2 + (x_1 - x_4)\sin^2\theta}, \qquad . \quad . \quad . \quad (6)$$

in place of the values of k, k_1, k_2, given by (4) we should have the following:—

$$k^2 = \frac{(x_1 - x_4)(x_5 - x_2)}{(x_1 - x_5)(x_4 - x_2)},\ k_1^2 = \frac{(x_1 - x_4)(x_6 - x_2)}{(x_1 - x_6)(x_4 - x_2)},\ k_2^2 = \frac{(x_1 - x_4)(x_3 - x_2)}{(x_1 - x_3)(x_4 - x_2)}. \quad (7)$$

Now, since $$\frac{(x_1 - x_4)(x_5 - x_2)}{(x_1 - x_5)(x_4 - x_2)} = 1 - \frac{(x_1 - x_2)(x_5 - x_4)}{(x_1 - x_5)(x_2 - x_4)}$$

the moduli given by equations (7) are the complements of those given by equations (4). Hence (5) becomes

$$1 - k^2 = (1 - k_1^2)(1 - k_2^2). \qquad . \quad . \quad . \quad (8)$$

another well-known condition among the moduli for which hyper-elliptic integrals of the first class are reducible to elliptic functions.

Again, if in place of either transformation (3) or (6) we use

$$x = \frac{x_1(x_6 - x_2) + x_2(x_1 - x_6)\sin^2\theta}{x_6 - x_2 + (x_1 - x_6)\sin^2\theta}. \qquad . \quad . \quad . \quad (9)$$

We have to determine the moduli of the reduced integral

$$k^2 = \frac{(x_5 - x_2)(x_1 - x_6)}{(x_1 - x_5)(x_6 - x_2)},\ k_1^2 = \frac{(x_4 - x_2)(x_1 - x_6)}{(x_1 - x_4)(x_6 - x_2)},\ k_2^2 = \frac{(x_3 - x_2)(x_1 - x_6)}{(x_1 - x_3)(x_6 - x_2)}. \quad (10)$$

Comparing the values of the moduli given in (10) with those given in (7), we easily see that we may in condition (8) substitute for k, $\frac{k}{k_1}$, for k_1, $\frac{1}{k_1}$, and for k_2, $\frac{k_2}{k_1}$, making these substitutions and reducing, we find

$$k_2^2 = \frac{k_1^2(1 - k^2)}{1 - k_1^2}. \qquad . \quad . \quad . \quad (11)$$

A third well-known condition for which hyper-elliptic integrals of the first class may be reduced to elliptic integrals.

II.

I propose now to generalize the foregoing theorems, using the same method I have already employed in the particular case I have discussed.

Let $$X = (x - x_1)(x - x_2)(x - x_3) \ldots (x - x_{2m}),$$

then, if in the integral

$$\int \frac{R(x)\,dx}{\sqrt{X}}$$

we make the substitution

$$x = \frac{l + my}{p + qy}$$

we get as before an integral of the form

$$\int \frac{R(y)\,dy}{\sqrt{Y}}$$

where $$y_1 = \frac{px_1 - l}{m - qx_1}, \quad y_2 = \frac{px_2 - l}{m - qx_2},$$

If now, we have

$$\left.\begin{array}{l} (pm + lq)(x_1 + x_{m+1}) - 2pqx_1x_{m+1} - 2lm = 0. \\ (pm + lq)(x_2 + x_{m+2}) - 2pqx_2x_{m+2} - 2lm = 0. \\ \quad . \quad . \quad . \quad . \quad . \quad . \quad . \\ \quad . \quad . \quad . \quad . \quad . \quad . \quad . \\ (pm + lq)(x_m + x_{2m}) - 2pqx_mx_{2m} - 2lm = 0. \end{array}\right\} \qquad (12)$$

then Y will be of the form

$$Y = (y^2 - y_1^2)(y^2 - y_2^2) \ldots (y^2 - y_m^2).$$

If now m be even the integral

$$\int \frac{R(y)\,dy}{\sqrt{(y^2 - y_1^2)(y^2 - y_2^2)(y^2 - y_3^2) \ldots (y^2 - y_m^2)}}$$

is immediately reduced to the form

$$\int \frac{R(\sin\theta)\,d\theta}{\Delta_{m-1}(k,\theta)},$$

where

$$\Delta_{m-1}(k,\theta) = \sqrt{(1-k^2\sin^2\theta)(1-k_1^2\sin^2\theta)(1-k_2^2\sin^2\theta)\ .\ .\ .\ (1-k^2_{m-2}\sin^2\theta)}.$$

by the substitution

$$y = y_1 \sin\theta.$$

In this case

$$k = \frac{y_1}{y_2}, \quad k_1 = \frac{y_1}{y_3}\ .\ .\ .\ k_{m-2} = \frac{y_1}{y_m}.$$

If m be odd the integral

$$\int \frac{R(y)\,dy}{\sqrt{(y^2-y_1^2)(y^2-y_2^2)\ .\ .\ .\ (y^2-y_m^2)}}$$

may be written as the sum of two other integrals

$$\int \frac{Mdy}{\sqrt{(y^2-y_1^2)(y^2-y_2^2)\ .\ .\ .\ (y^2-y_m^2)}} + \int \frac{Nydy}{\sqrt{(y^2-y_1^2)(y^2-y_2^2)\ .\ .\ .\ (y_2-y_m^2)}}$$

where M and N are rational functions of y^2.

The first of these integrals can be reduced to the form

$$\int \frac{R(\sin^2\theta)\,d\theta}{\Delta_{m-2}(k,\theta)} \text{ where } \Delta_{m-2}(k,\theta) = \sqrt{(1-k^2\sin^2\theta)(1-k_1^2\sin^2\theta)\ .\ .\ .\ (1-k^2_{m-3}\sin^2\theta)}$$

$$k^2 = \frac{y_1^2}{y_2^2}\cdot\frac{y_2^2-y_m^2}{y_1^2-y_m^2},\ k_1^2 = \frac{y_1^2}{y_3^2}\cdot\frac{y_3^2-y_m^2}{y_1^2-y_m^2},\ .\ .\ .\ k^2_{m-3} = \frac{y_1^2}{y^2_{m-1}}\cdot\frac{y_{m-1}^2-y_m^2}{y_1^2-y_m^2}.$$

By the substitution

$$y^2 = \frac{y_1^2\,y_m^2\sin^2\theta}{y_1^2\sin^2\theta - y_1^2 + y_m^2},$$

for in this case we have

$$\int \frac{dy}{\sqrt{(y^2-y_1^2)(y^2-y_2^2)\ .\ .\ .\ (y^2-y_m^2)}} =$$

$$\frac{-1}{y_2 y_3\ .\ .\ .\ y_{m-1}(y_1^2-y_m^2)^{\frac{m-2}{2}}} \int \frac{(y_1^2\sin^2\theta - y_1^2 + y_m^2)^{\frac{m-3}{2}}}{\Delta_{m-2}(k,\theta)}.$$

The second integral is reduced to the form

$$\int \frac{R(\sin^2\phi)\, d\phi}{\Delta_{m-2}(\lambda, \theta)}$$

by the substitution

$$y^2 = y_1^2 - (y_1^2 - y_2^2)\sin^2\phi$$

for in this case

$$A \int \frac{y dy}{\sqrt{(y^2 - y_1^2)(y^2 - y_2^2) \ldots (y^2 - y_m^2)}} = \int \frac{d\phi}{\Delta_{m-2}(\lambda, \phi)},$$

where

$$A^2 = -(y_1^2 - y_3^2)(y_1^2 - y_4^2) \ldots (y_1^2 - y_m^2).$$

Now if we take the equations (12) in threes, and eliminate from each three the quantities $2pq$, $2lm$, $pm + lq$, we get the following extension of M. Gordan's theorem :—

"If $$X = (x - x_1)(x - x_2)(x - x_3) \ldots (x - x_{2m})$$

and $R(x)$ be any rational algebraic function of x, then the integral

$$\int \frac{R(x)\, dx}{\sqrt{X}}$$

can be made to depend on hyper-elliptic integrals with $m - 1$ or $m - 2$ moduli according as m is even or odd, provided that the following $m - 2$ relations are true, viz. :—

$$\begin{vmatrix} x_1 + x_{m+1} & x_1 x_{m+1} & 1 \\ x_2 + x_{m+2} & x_2 x_{m+2} & 1 \\ x_3 + x_{m+3} & x_3 x_{m+3} & 1 \end{vmatrix} = 0 \quad \begin{vmatrix} x_1 + x_{m+1} & x_1 x_{m+1} & 1 \\ x_2 + x_{m+2} & x_2 x_{m+2} & 1 \\ x_4 + x_{m+4} & x_4 x_{m+4} & 1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x_1 + x_{m+1} & x_1 x_{m+1} & 1 \\ x_2 + x_{m+2} & x_2 x_{m+2} & 1 \\ x_m + x_{2m} & x_m x_{2m} & 1 \end{vmatrix} = 0. \quad \ldots \quad (13)$$

If now in the expression $\frac{dx}{\sqrt{X}}$ we make the substitution

$$x = \frac{x_1(x_2 - x_{m+1}) + x_{m+1}(x_1 - x_2)\sin^2\theta}{x_2 - x_{m+1} + (x_1 - x_2)\sin^2\theta}, \quad . \quad . \quad . \quad (14)$$

we find

$$\frac{A\,dx}{\sqrt{X}} = \frac{2\{x_2 - x_{m+1} + (x_1 - x_2)\sin^2\theta\}^{m-2}\,d\theta}{\Delta_{2m-3}(k, \theta)},$$

where $\quad A = \{-(x_1 - x_3)(x_1 - x_4) \; . \; . \; . \; (x_2 - x_{m+1})^{2m-3}\}^{\frac{1}{2}}$

$$\Delta_{2m-3}(k, \theta) = \sqrt{(1 - k^2\sin^2\theta)(1 - k_1^2\sin^2\theta) \; . \; . \; . \; (1 - k^2_{2m-4}\,\mathrm{s.n}^2\theta)}$$

the moduli being given by the equations

$$\left.\begin{array}{l} k^2 = \dfrac{(x_1 - x_2)}{(x_2 - x_{m+1})} \cdot \dfrac{(x_{m+2} - x_{m+1})}{(x_1 - x_{m+2})}, \quad k_1^2 = \dfrac{(x_1 - x_2)}{(x_2 - x_{m+1})} \cdot \dfrac{(x_{m+3} - x_{m+1})}{(x_1 - x_{m+3})} \\[2ex] k_2^2 = \dfrac{(x_1 - x_2)}{(x_2 - x_{m+1})} \cdot \dfrac{(x_3 - x_{m+1})}{(x_1 - x_3)}, \quad k^2_{2m-4} = \dfrac{(x_1 - x_2)}{(x_2 - x_{m+1})} \cdot \dfrac{(x_{2m} - x_{m+1})}{(x_1 - x_{2m})} \\[2ex] k^2_{2m-5} = \dfrac{(x_1 - x_2)}{(x_2 - x_{m+1})} \cdot \dfrac{(x_m - x_{m+1})}{(x_1 - x_m)} \end{array}\right\} \; . \; . \; . \; (15)$$

Hence we see that

$$\int \frac{R(x)\,dx}{\sqrt{X}}$$

can be always reduced by substitution (14) to the form

$$\int \frac{R(\sin^2\theta)\,d\theta}{\Delta_{2m-3}(k, \theta)},$$

also from the values of the moduli given by equations (15) we see as before that the conditions (13) may be written

$$k = k_1 k_2 \; . \; . \; . = k_{2m-4}\,k_{2m-5},$$

and we have the following extension of Jacobi's theorem for hyper-elliptic integrals of the first class:—

"Hyper-elliptic integrals with $2m-3$ moduli, may always be reduced to hyper-elliptic integrals with $m-1$ or $m-2$ moduli, according as m is even or odd, provided the following $m-2$ relations among the moduli are true, viz.:

$$k = k_1 k_2 \ldots = k_{2m-4} k_{2m-5}. \qquad \ldots \quad (16)$$

If now in place of using substitution (14) to reduce

$$\int \frac{R(x)\,dx}{\sqrt{X}}$$

to normal form of hyper-elliptic integrals we used the substitution

$$x = \frac{x_1(x_{m+1}-x_2) + x_2(x_1-x_{m+1})\sin^2\theta}{x_{m+1}-x_2+(x_1-x_{m+1})\sin^2\theta}. \qquad \ldots \quad (17)$$

In place of the values of the moduli given by (15) we get the following:—

$$k^2 = \frac{(x_1-x_{m+1})}{(x_{m+1}-x_2)} \cdot \frac{(x_{m+2}-x_2)}{(x_1-x_{m+2})}, \quad k_1^2 = \frac{(x_1-x_{m+1})}{(x_{m+1}-x_2)} \cdot \frac{(x_{m+3}-x_2)}{(x_1-x_{m+3})} \ldots \Big\} \ldots \quad (18)$$

These are evidently the complements of moduli given by equations (15), we have therefore a second condition for which hyper-elliptic integrals with $2m-3$ moduli can be made to depend on those with $m-1$ or $m-2$ moduli, viz.,

$$1-k^2 = (1-k_1^2)(1-k_2^2) \ldots = (1-k^2_{2m-5})(1-k^2_{2m-4}). \qquad \ldots \quad (19)$$

Again, if in place of (14) or (17) we had used the transformation

$$x = \frac{x_2(x_{m+1}-x_1) + x_1(x_2-x_{m+1})\sin^2\theta}{x_{m+1}-x_1+(x_1-x_{m+1})\sin^2\theta}. \qquad \ldots \quad (20)$$

We would get the following values for the moduli

$$k^2 = \frac{(x_2-x_{m+1})(x_{m+2}-x_1)}{(x_{m+1}-x_1)(x_2-x_{m+2})}, \quad k_1^2 = \frac{x_2-x_{m+1}}{x_{m+1}-x_1}, \quad \frac{x_{m+3}-x_1}{x_2-x_{m+3}}$$

$$\ldots \quad k^2_{2m-5} = \frac{(x_2-x_{m+1})}{(x_{m+1}-x_1)} \cdot \frac{(x_m-x_1)}{(x_2-x_m)}, \qquad \ldots \quad (21)$$

from which it is easily seen as before that the conditions (13) may be written

$$1-\frac{k^2}{k^2_{2m-5}}=\left(1-\frac{k_1^2}{k^2_{2m-5}}\right)\left(1-\frac{k_2^2}{k^2_{2m-5}}\right)\ldots=\left(1-\frac{1}{k^2_{2m-5}}\right)\left(1-\frac{k^2_{2m-4}}{k^2_{2m-5}}\right).$$

After reduction these conditions become

$$k^2_{2m-4}=\frac{k_1^2k_2^2-k^2_{2m-5}(k_1^2+k_2^2)+k^2_{2m-5}}{1-k^2_{2m-5}}\ldots=\frac{k^2_{2m-5}(1-k^2)}{1-k^2_{2m-5}}.\ldots\quad(22)$$

A third set of conditions among the moduli for which hyper-elliptic integrals with $2m-3$ moduli can be reduced to those with $m-1$ or $m-2$ moduli.

The three sets of conditions given in (16), (19), (22) I had previously proved by a different method. See a memoir of mine on the subject (Crelle-Borchardt), t. 76, page 97.

III.

The same method may be used in many cases to reduce integrals to simpler forms when certain relations hold.

I mention a few theorems which may easily be proved.

(1). The integral

$$\int\frac{R(x)\,dx}{\sqrt[4]{(a_4+4a_3x+6a_2x^2+4a_1x^3+a_0x^4)}},$$

may be reduced to elliptic functions; for we can reduce the integral by a linear substitution to the form

$$\int\frac{R(y)\,dy}{\sqrt[4]{(y^2-y_1^2)(y^2-y_2^2)}}$$

which depends on elliptic functions.

(2). If

$$\begin{vmatrix} x_1+x_4 & x_1x_4 & 1\\ x_2+x_5 & x_2x_5 & 1\\ x_3+x_6 & x_3x_6 & 1\end{vmatrix}=0.$$

$$X=(x-x_1)(x-x_2)(x-x_3)(x-x_4)(x-x_5)(x-x_6)$$

and l and m are the roots of the equation

$$L^2 - 2L\,\frac{x_1 x_4 - x_2 x_5}{x_1 + x_4 - x_2 - x_5} + \frac{x_1 x_4 (x_2 + x_5) - x_2 x_5 (x_1 + x_4)}{x_1 + x_4 - x_2 - x_5} = 0,$$

then the integral

$$\int \frac{R\left\{\left(\frac{l-x}{m-x}\right)^2\right\} \cdot \frac{l-x}{m-x}\,dx}{\sqrt[3]{X}}$$

can be reduced to elliptic functions, for in this case if we make the substitution

$$x = \frac{l + m\sqrt{y}}{1 + \sqrt{y}},$$

the integral becomes

$$-\tfrac{1}{2}\int \frac{R(y)\,dy}{\sqrt[3]{(y - y_1^2)(y - y_2^2)(y - y_3^2)}}$$

where $\qquad y_1 = \dfrac{x_1 - l}{x_1 - m}, \quad y_2 = \dfrac{x_2 - l}{x_2 - m}, \quad y_3 = \dfrac{x_3 - l}{x_3 - m}$

which it is well known is reducible to elliptic functions.

(3). In general if

$$X = (x - x_1)(x - x_2)(x - x_3) \;.\;.\;.\;.\; (x - x_{2n}),$$

and l and m are the roots of the equation

$$L^2 - 2L\,\frac{x_1 x_{n+1} - x_2 x_{n+2}}{x_1 + x_{n+1} - x_2 - x_{n+2}} + \frac{x_1 x_{n+1}(x_2 + x_{n+2}) - x_2 x_{n+2}(x_1 + x_{n+1})}{x_1 + x_{n+1} - x_2 - x_{n+2}} = 0.$$

then the integral

$$\int \frac{R\left\{\left(\frac{l-x}{m-x}\right)^2\right\} \frac{l-x}{m-x}\,dx}{\sqrt[n]{X}}$$

may be reduced to the form

$$\int \frac{R(y)\,dy}{\sqrt[n]{Y}}$$

where $\qquad Y = (y - y_1)(y - y_2) \;.\;.\;.\;.\; (y - y_n)$

provided the following $n-2$ relations are true, viz. :—

$$\begin{vmatrix} x_1+x_{n+1} & x_1 x_{n+1} & 1 \\ x_2+x_{n+2} & x_2 x_{n+2} & 1 \\ x_3+x_{n+3} & x_3 x_{n+3} & 1 \end{vmatrix} = 0 \qquad \begin{vmatrix} x_1+x_{n+1} & x_1 x_{n+1} & 1 \\ x_2+x_{n+2} & x_2 x_{n+2} & 1 \\ x_4+x_{n+4} & x_4 x_{n+2} & 1 \end{vmatrix} = 0$$

$$\ldots \begin{vmatrix} x_1+x_{n+1} & x_1 x_{n+1} & 1 \\ x_2+x_{n+2} & x_2 x_{n+2} & 1 \\ x_{2n}+x_{2n} & x\, x_{2n} & 1 \end{vmatrix} = 0.$$

(4). The integral

$$\int \frac{\cos^2\theta\, d\theta}{(1+\sin^2\theta)\sqrt[4]{\sin^4\theta - 2a\sin^2\theta + 1}}$$

can be reduced to elliptic functions.

IV.

On the integral

$$\int_0^\theta \frac{(1-k^2\sin^4\theta)\, d\theta}{\Delta(\theta)}$$

where we have

$$\Delta(\theta) = \sqrt{(1-k^2\sin^2\theta)(1-k_1^2\sin^2\theta)(1-k_2^2\sin^2\theta)\left(1-\frac{k^2}{k_1^2}\sin^2\theta\right)\left(1-\frac{k^2}{k_2^2}\sin^2\theta\right)}.$$

If we reduce the above integral, we find it depends only on elliptic functions, in fact, after some algebraic work which is not difficult, we find the formula

$$F\left\{\sqrt{\frac{(k_1^2-k_2^2)(k_1^2k_2^2-k^2)}{k_1^2(1-k_2^2)(k_2^2-k^2)}},\ \phi\right\} = \sqrt{\frac{(1-k_2^2)(k_2^2-k^2)}{k_2^2}}\int_0^\theta \frac{(1-k^2\sin^4\theta)\, d\theta}{\Delta(\theta)} \ldots (23)$$

where

$$\sin\phi = \frac{\sqrt{(1-k_2^2)(k_2^2-k^2)}\sin\theta}{\sqrt{(1-k_2^2\sin^2\theta)(k_2^2-k^2\sin^2\theta)}}. \qquad \ldots (24)$$

From equations (23) and (24) we can find extensions of many of the properties of elliptic functions.

If we call

$$\int_0^\theta \frac{d\theta}{\Delta(\theta)}, L, \text{ and } \int_0^\theta \frac{\sin^4\theta\, d\theta}{\Delta(\theta)}, N,$$

it is evident that the transcendental equation

$$L(\theta) + L(\phi) + L(\psi) = k^2 \{N(\theta) + N(\phi) + N(\psi)\}. \quad . \quad . \quad . \quad (25)$$

has an algebraic equivalent, for this equation is equivalent to

$$F\left\{\sqrt{\frac{(k_1^2-k_2^2)(k_1^2k_2^2-k^2)}{k_1^2(1-k_2^2)(k_2^2-k^2)}}, \theta\right\} + F\left\{\sqrt{\frac{k_1^2-k_2^2)(k_1^2k_2^2-k^2)}{k_1^2(1-k_2^2)(k_2^2-k^2)}}, \phi'\right\}$$

$$+ F\left\{\sqrt{\frac{(k_1^2-k_2^2)(k_1^2k_2^2-k^2)}{k_1^2(1-k_2^2)(k_2^2-k^2)}}, \psi'\right\} = 0,$$

where $\sin\theta'$, $\sin\phi'$, $\sin\psi'$ are given by equations similar to (24). To find the algebraic equation corresponding to (25) we have

$$\sin\theta' = \frac{\sqrt{(1-k_2^2)\,k_2^2-k^2)}\,\sin\theta}{\sqrt{(1-k_2^2\sin^2\theta)(k_2^2-k^2\sin^2\theta)}},$$

from which we easily find

$$\cos\theta' = \frac{k_2\cos\theta\,\sqrt{1-k^2\sin^2\theta}}{\sqrt{(1-k_2^2\sin^2\theta)(k_2^2-k^2\sin^2\theta)}}.$$

We have also

$$\sqrt{1-\frac{(k_1^2k_2^2-k^2)(k_1^2-k_2^2)}{k_1^2(1-k_2^2)(k_2^2-k^2)}\sin^2\theta'} = \frac{k_2}{k_1}\cdot\frac{\sqrt{(k_1^2-k^2\sin^2\theta)(1-k_1^2\sin^2\theta)}}{\sqrt{(k_2^2-k^2\sin^2\theta)(1-k_2^2\sin^2\theta)}}.$$

Hence if we call

$$\sqrt{1-k^2\sin^2\theta},\ \delta(k,\theta),\ \sqrt{k_1^2-k^2\sin^2\theta},\ \delta_1(k,\theta)$$

and so on, we have by a well known formula for the addition of elliptic func-

tions the following algebraic relation corresponding to the transcendental equation (25), viz.:—

$$k_2(1-k_2^2)(k_2^2-k^2)\,\delta(k_1,\theta)\,\delta(k_1,\phi)\,\delta(k_1,\psi)\,\delta_1(k,\theta)\,\delta_1(k,\phi)\,\delta_1(k,\psi)$$

$$-k_1(k_1^2-k^2)(1-k_1^2)\,\delta(k_2,\theta)\,\delta(k_2,\phi)\,\delta(k_2,\psi)\,\delta_2(k,\theta)\,\delta_2(k,\phi)\,\delta_2(k,\psi)=$$

$$(k_1^2k_2^2-k^2)(k_1^2-k_2^2)\,k_1k_2\cos\theta\cos\phi\cos\psi\,\delta(k,\theta)\,\delta(k,\phi)\,\delta(k,\psi)\ldots\ (26)$$

This formula, though derived from the ordinary formula for the addition of elliptic functions may, in one sense, be regarded as an extension of it, for if we take the particular case $k=0$, divide by $k_1^3\,k_2^3$ and *then* make $k_2=0$, we find the algebraic equivalent of the transcendental equation

$$F(k_1,\theta)+F(k_1,\phi)+F(k_1,\psi)=0$$

namely

$$\delta(k_1,\theta)\,\delta(k_1,\phi)\,\delta(k_1,\psi)=1-k_1^2+k_1^2\cos\theta\cos\phi\cos\psi.$$

Formula (23) is an extension of the well known formula for the comparison of elliptic functions of the third kind, the products of the parameters being equal to the square of the modulus; for in (23) let $k_1^2=k_2^2=-n$ and we find the well known formula

$$\Pi(n,k,\theta)+\Pi\left(\frac{k^2}{n},k,\theta\right)-F(k,\theta)=\sqrt{\frac{n}{(1+n)(n+k^2)}}\tan^{-1}\sqrt{\frac{(1+n)(n-k^2)}{n}}\cdot\frac{\tan\theta}{\Delta(k,\theta)}.$$

If in formula (23) we make the substitutions

$$\sin\phi=\sqrt{-1}\tan\phi,\ \sin\theta=\sqrt{-1}\tan\theta$$

we find the formula

$$F\left\{\frac{k_1}{k_2}\sqrt{\frac{1-k_2^2}{1-k_1^2}\cdot\frac{k_1^2-k^2}{k_2^2-k^2}}\ \phi\right\}=k_2\sqrt{\frac{k^2-k_2^2}{1-k_2^2}}\int_0^\theta\frac{(1-2\sin^2\theta+k^2\sin^4\theta)\,d\theta}{\Delta(\theta)}\ldots\ (27)$$

where now

$$\Delta(\theta)=\sqrt{(1-k^2\sin^2\theta)(1-k_1^2\sin^2\theta)\left(1-\frac{k^2-k_1^2}{1-k_1^2}\sin^2\theta\right)\left(1-k_2^2\sin^2\theta\right)\left(1-\frac{k^2-k_2^2}{1-k_2^2}\sin^2\theta\right)}$$

and the amplitude ϕ is given by the relation

$$\tan\phi = \frac{k_2\sqrt{k^2-k_2^2}\sin\theta\cos\theta}{\sqrt{(1-k_2^2\sin^2\theta)(1-k_2^2-(k^2-k_2^2)\sin^2\theta)}}. \quad . \quad . \quad . \quad (28)$$

Formula (27) is an extension of the formula for the comparison of elliptic functions of the third kind, whose parameters are connected by the relation

$$(1+n)(1+n') = (1-k^2) \text{ for let } k_1^2 = k_2^2 = -n$$

and we find the formula

$$\frac{1+n}{n}\Pi(n,k,\theta) + \frac{1+n'}{n'}\Pi(n',k,\theta) + \frac{k^2}{nn'}F(k,\theta) =$$

$$\frac{1}{\sqrt{-nn'}}\tan^{-1}\frac{\sqrt{-n'n}\sin\theta\cos\theta}{\sqrt{1-k^2\sin^2\theta}}.$$

VI.—*Screw Co-ordinates and their Applications to Problems in the Dynamics of a rigid Body.* By Robert Stawell Ball, LL. D., F.R.S.

Read January 12, 1874.

CONTENTS.

I.—On the Nature and Use of Screw Co-ordinates (§§ 1–23).
II.—Application of Screw Co-ordinates to the Study of the Effect of an Impulse upon a free or constrained Rigid Body. (§§ 24–47.)

I.—ON THE NATURE AND USE OF SCREW CO-ORDINATES.

§ I. **Introduction.**—Of the authors who have employed the co-ordinates of a line, first suggested by Professor Cayley,* we refer to Mr. H. G. Zeuthen† for that special form of the line co-ordinates which, when suitably generalised, will be appropriate for screws. The author referred to defines the co-ordinates of a right line as follows: Resolve the unit of force along the line into forces along the six edges of a standard tetrahedron; these six components are the co-ordinates required. The six co-ordinates satisfy two equations of condition, one expressing that the six forces have no resultant couple, the other expressing that the resultant force is one unit.

We should also add that screw co-ordinates are really only an adaptation to physical purposes of Dr. Felix Klein's ‡ co-ordinates of a linear complex referred to six fundamental complexes, of which each pair are in involution. In

* "Quart. Math. Journal," t. III., p. 225. † "Math. Ann.," Band I., p. 432.
‡ "Mathematische Annalen, Band II., p. 204.

fact, we may state more generally that those parts of the theory of screws which involve only statical or kinematical, as opposed to kinetical, conceptions, will be found to consist, to a considerable extent, of physical interpretations of purely geometrical theorems, which are already familiar to those acquainted with the linear geometry of Plücker.

Many references will be made in the present paper to two previous papers* upon the "Theory of Screws." For brevity these will be referred to as No. 1 and No. 2.

As twist velocities, wrenches, and infinitely small twists are compounded by the same rules (No. 1, Art. 16), it is useful to have one common term which includes them all. We may use for this purpose the word "Dyname," introduced by Plücker† to signify either a set of forces applied to a rigid system or an infinitely small movement. Battaglini‡ adopts as the co-ordinates of a Dyname the six forces which acting along the edges of a tetrahedron are equivalent to the Dyname. The same author discusses from an analytical point of view the properties of dynames, of which the co-ordinates satisfy one or more linear equations. He shows the existence of two groups of associated dynames which are equivalent to what we have termed two reciprocal screw-complexes.

§ 2. **Notation.**—The use of screw co-ordinates is greatly facilitated by appropriate notation. The system which follows is presented with some hesitation, though it is the result of much thought.

In the following explanation of the meaning of different symbols the dimensions of every symbol which denotes a *quantity* is appended. N, A, L, W, T signify the units of number, angle, length, mass, and time respectively.

* "Transactions of the Royal Irish Academy," Vol. XXV., pp. 157–217.

"Philosophical Transactions," 1873.

† "Neue Geometrie des Raumes," p. 24.

‡ G. Battaglini, Memoria sulle dinami in involuzione. Atti di Napoli, IV. 1869.

Symbol.	Dimensions.	
t	T	This letter is always employed in its usual sense, that is to denote the interval of time between two epochs.
M	W	Denotes the mass of the rigid body.
k_i	L	The radii of Gyration of the principal axes through the centre of inertia are k_1, k_2, k_3.
α		A small Greek letter, unaccented, is never employed to denote anything except *a screw*; α is thus so far from being a single quantity that it is really a geometrical form, for the specification of which *five* quantities are necessary. Of these four are required for the determination of a straight line, while the fifth gives the pitch.
Ω		This symbol denotes the group of six co-reciprocal screws (No. 2, Art. 7), which are employed as fundamental screws of reference. It is to be carefully observed that in the present paper *any* set of co-reciprocals will serve as fundamental screws, for questions which are either kinematical or statical. When conceptions of a *kinetic* character are introduced, we invariably avail of the great simplifications which the expressions receive by employing (either expressly or tacitly) that *particular group* of co-reciprocals which we have called the *principal screws of inertia* (§ 24).
ω_n		The six individual screws which constitute Ω are severally designated ω_1, ω_2, ω_3, ω_4, ω_5, ω_6, and the symbol ω_n denotes a typical member of the group.
s_n	L	The pitch of ω_n.
a_n	N	A Dyname of one unit in magnitude, acting about α, can be resolved into six constituents about the fundamental screws Ω (No. 1, Art. 46); the constituent about ω_n is represented by a_n; the six quantities a_1, &c., a_6 constitute the co-ordinates of α.
a'	L	The pitch of α; this symbol is therefore an abbreviation for

$$s_1a_1^2 + s_2a_2^2 + s_3a_3^2 + s_4a_4^2 + s_5a_5^2 + s_6a_6^2. \quad (\S\ 10.)$$

Symbol.	Dimensions.	
α''	L^2	An abbreviation for $$s_1^2\alpha_1^2 + s_2^2\alpha_2^2 + s_3^2\alpha_3^2 + s_4^2\alpha_4^2 + s_5^2\alpha_5^2 + s_6^2\alpha_6^2.$$ $M\alpha''$ is the kinetic energy of M when twisting about α with the unit of twist velocity (§ 24).
α_0	L	An abbreviation for $$\left(k_1^2(\alpha_1 - \alpha_2)^2 + k_2^2(\alpha_3 - \alpha_4)^2 + k_3^2(\alpha_5 - \alpha_6^2)\right)^{\frac{1}{2}}$$ If the body twist about α with the unit of twist velocity, then α_0 denotes the velocity of the centre of inertia (§ 25).
$H\alpha$		This is suggested by the Hamiltonian symbol $T\rho$ expressing the tensor of a vector, ρ; α being a screw, $H\alpha$ is a Dyname about α. It is convenient to have a general symbol for a Dyname, and also to be enabled to add such marks to the general symbol as shall serve to distinguish the different descriptions of Dyname.
$H_0\alpha$	A	Denotes an infinitely small twist about α. This description of Dyname is measured by the circular measure of the angle through which the body is rotated by the twist.
$\dot{H}\alpha$	AT^{-1}	Denotes a twist velocity about α. This Dyname is measured by the angular velocity with which the body rotates about the screw. The velocity of translation parallel to the screw is $\alpha'\dot{H}\alpha$.
$\ddot{H}\alpha$	WLT^{-2}	Denotes a wrench about α. This Dyname is measured by the force along α; when α is known the wrench is completely determined by $\ddot{H}\alpha$; the wrench consists of a force $\ddot{H}\alpha$ along the screw *and* a couple, of which the moment is $\alpha'\ddot{H}\alpha$. It is important to remember that the dimensions of a wrench are the same as the dimensions of a force; in fact, a wrench is as naturally associated with the conception of a screw as a force is with a straight line.

Symbol. Dimensions.

Ka LT^{-1} By an impulsive wrench we are to understand an exceedingly great wrench, which acts for a very short time. We measure such a wrench by its effects, and for this purpose we require a new symbol. Let $\ddot{H}a$ act upon M for a time t; if we write

$$\ddot{H}a = \frac{M}{t} Ka.$$

then the new symbol Ka will be a finite quantity, which will form a convenient means of measuring the effect of the impulsive wrench. The dimensions of Ka are those of a linear velocity. Ka may be interpreted physically to be the velocity imparted to the centre of inertia of M, if, when perfectly free and quiescent, it were to receive the impulsive wrench $\ddot{H}a$ for the time t.

$\bar{a}$ If a perfectly free quiescent rigid body were to receive an impulsive wrench about the screw _, the body would commence to twist about a as an instantaneous screw; this statement defines the symbol $\bar{a}$.

(a, a) N An abbreviation for

$$a_1^2 + a_2^2 + a_3^2 + a_4^2 + a_5^2 + a_6^2.$$

(a, β) N An abbreviation for

$$a_1\beta_1 + a_2\beta_2 + a_3\beta_3 + a_4\beta_4 + a_5\beta_5 + a_6\beta_6.$$

$(a, \beta)'$ L An abbreviation for

$$s_1a_1\beta_1 + s_2a_2\beta_2 + s_3a_3\beta_3 + s_4a_4\beta_4 + s_5a_5\beta_5 + s_6a_6\beta_6.$$

$(a, \beta)''$ L^2 An abbreviation for

$$s_1^2a_1\beta_1 + s_2^2a_2\beta_2 + s_3^2a_3\beta_3 + s_4^2a_4\beta_4 + s_5^2a_5\beta_5 + s_6^2a_6\beta_6.$$

$\overline{a, \beta}$ Denotes the cylindroid drawn through a and β.

$(a, \beta)_0$ Denotes the cosine of the angle between a and β.

When the co-ordinates of a screw satisfy $6 - k$ linear equations, the screw must form one of a group, which is called a screw-complex, of the kth order

and 1st degree, or more briefly, a screw-complex $(k, 1)$. In the previous papers this was termed a co-ordinate system of screws of k degrees of freedom.

If the co-ordinates of a screw satisfy $6 - k$ equations, of which one is of the second degree, while the rest are linear, the screw must form one of a group which is called a screw-complex of the kth-order and 2nd degree, or more briefly, a screw-complex $(k, 2)$.

§ 3. **On the kinematical method of determining the position of a screw when its co-ordinates are given.**—Let α_1, &c., α_6 be the co-ordinates, and let ι be an infinitely small angle. Take a rigid body in any position P; give to the body in any order the six twists $\iota\alpha_1$, $\iota\alpha_2$ &c., $\iota\alpha_6$; let the position Q be finally attained; then the twist which would have moved the body directly from P to Q is about the required screw.

§ 4. **Lemma.**—When a number of twists are compounded, the magnitude of the resultant twist and the direction of its screw, depend only upon the magnitudes and the directions of the components, and in no respect upon their pitches or situations. This follows at once from the laws of the composition of twists by the aid of the cylindroid. (No. 1, Art. 10).

If vectors κ_1, &c., κ_6 be drawn parallel and equal to α_1, &c., α_6 then α is parallel to the vector $\kappa_1 +$ &c., $+ \kappa_6$. Since the Dyname about α is one unit the tensor of the sum of these vectors is also unity.

§ 5. **On an identical equation which is satisfied by the co-ordinates of a screw.**—Since five quantities determine a screw, the six co-ordinates must be connected by an identical relation, the nature of which is suggested by the analogy of three direction cosines.

Let a_n, b_n, c_n be the direction cosines of ω_n with reference to three rectangular directions.

Then, since the unit Dyname about α has components α_1, &c., α_6 about ω_1, &c., ω_6 we must have (from § 4).

$$(a_1\alpha_1 +, \text{\&c.}, + a_6\alpha_6)^2 + (b_1\alpha_1 +, \text{\&c.}, + b_6\alpha_6)^2 + (c_1\alpha_1 +, \text{\&c.}, + c_6\alpha_6)^2 = 1,$$

or,

$$\Sigma a_n^2 + 2\,\Sigma a_1 a_2\,(\omega_1, \omega_2)_0 = 1$$

if we denote by $(\omega_1, \omega_2)_0$ the cosine of the angle between ω_1 & ω_2.

If v_1, &c., v_6 be any six quantities, then the type

$$\frac{v_n}{\sqrt{\Sigma v_n^2 + 2\,\Sigma v_1 v_2\,(\omega_1, \omega_2)_0}}$$

represents the co-ordinates of a screw.

§ 6. **Resolution of a Dyname $H\alpha$ into its constituents about the six fundamental screws.**—Since the unit Dyname about α has constituents α_1, &c., α_6, the constituents of $H\alpha$ must be $H\alpha.\alpha_1$, $H\alpha.\alpha_2$, &c., $H\alpha.\alpha_6$.

§ 7. **Calculation of the co-ordinates of a screw when its position is given.**—The co-ordinates of a screw are to be ascertained by ordinary measurement of lines and angles. In the first place a table of triple entry is to be imagined from which the virtual coefficient (No. 2, Art. 1) of any pair of screws may be ascertained just as the cosine of an angle between two straight lines is extracted from the common trigonometrical tables. Each co-ordinate is found by dividing the virtual coefficient between the corresponding fundamental screw and the given screw by twice the pitch of the former (No. 2, Art. 9).

If s_1, &c., s_n, &c., s_6 be the pitches of the fundamental screws Ω, then $2\,s_n\,\alpha_n$ is the virtual coefficient of α and ω_n.

§ 8. **Expression of the virtual coefficient between a pair of screws in terms of their co-ordinates.**—The virtual coefficient of α and β may be denoted by $2(\alpha, \beta)'$. The work done in giving a body a small twist $H_0\alpha$ about α against the wrench $\ddot{H}\beta$ is

$$2\,H_0\alpha\,.\,\ddot{H}\beta\,.\,(\alpha, \beta)'$$

the work done by the twist $H_0\alpha$ against the constituent $\ddot{H}\beta\,.\,\beta_n$ is

$$2\,.\,H_0\alpha\,.\,\ddot{H}\beta\,.\,s_n\,\alpha_n\,\beta_n.$$

But the total work done against $\ddot{H}\beta$ must equal the sum of the works done against the six constituents of $\ddot{H}\beta$, whence

$$(\alpha, \beta)' = \Sigma s_n \alpha_n \beta_n$$

and therefore the virtual coefficient of α & β is

$$2\,\Sigma s_n\,\alpha_n\,\beta_n.$$

Since α and β enter symmetrically the reciprocal character of the virtual coefficient is at once apparent.

§ 9. **Reciprocal Screws.**—When a pair of screws α and β are reciprocal (No. 1, Art. 18), their virtual coefficient vanishes, and therefore

$$\Sigma s_n \alpha_n \beta_n = 0.$$

§ 10. **Pitch.**—The pitch of a screw can be expressed very simply in terms of the co-ordinates. The virtual coefficient of two coincident screws is double the pitch, hence if α' be the pitch of α.

$$\alpha' = \Sigma s_n \alpha_n^2.$$

If α be a right line then $\Sigma s_n \alpha^2_n = 0$: a condition amounting to this is required in every system which represents a straight line by six co-ordinates.

§ 11. **Interpretation of a linear equation in screw co-ordinates.**—If the six co-ordinates of the screw η be connected by the relation

$$\Sigma A_n \eta_n = 0$$

then η must be reciprocal to the screw of which the co-ordinates are proportional to $A_n s_n^{-1}$; the locus of η is a screw-complex $(V, 1)$; a body which had five degrees of freedom might be twisted about every screw with which it was possible for η to coincide.

§ 12. **Expression of the sexiant in terms of the co-ordinates of the six screws.**—If the sexiant (No. 2, Art. 11) vanished, then a single screw ρ could be found reciprocal to the six screws, α, β, γ, δ, ϵ, ζ, whence we must have six equations of the type

$$\Sigma s_n \alpha_n \rho_n = 0$$

eliminating ρ_1, &c., ρ_6, we have the sexiant equated to zero for the result. The determinant may be written in the abridged form

$$(\alpha_1, \beta_2, \gamma_3, \delta_4, \epsilon_5, \zeta_6).$$

The sexiant equated to zero expresses that the six screws belong to a screw-complex $(V, 1)$.

It should be observed that the sexiant is analogous to that function of three vectors, which, when equated to zero, expresses that the vectors are coplanar.

§ 13. **Construction of the single Screw reciprocal to five given Screws.**—It was stated (No. 1, Art. 96) that a construction was still required for solving the fundamental problem of the determination of that one screw which is reciprocal to five given screws. That want has been since supplied.

We have already adopted the postulate (No. 1, Art. 11), that a cylindroid may be drawn through a given pair of screws. There is only one (and therefore always one) cylindroid that can be so drawn. We proceed now to show how, by the aid of this postulate, the problem under consideration can be solved.

Let α, β, γ, δ, ϵ be the five screws, arranged with pitches in descending order of magnitude. Draw the cylindroids $\overline{\alpha, \gamma}$, and $\overline{\beta, \delta}$. Take four screws of equal pitch v on these two cylindroids, draw the two common transversals to these four screws, attribute to the two transversals the pitches $-v$ and construct a cylindroid through them. Then this cylindroid must contain all the screws reciprocal to α, β, γ, δ, because each of the transversals with pitch $-v$ is reciprocal to the cylindroids $\overline{\alpha, \gamma}$, and $\overline{\beta, \delta}$. In the same way construct a cylindroid reciprocal to the four screws β, γ, δ, ϵ. The two cylindroids thus found intersect in the screw required.

§ 14. **Co-ordinates of the Screw reciprocal to five given Screws.**—Let the five given screws be α, β, γ, δ, ϵ, and let η be the screw which is required.

Then the co-ordinates of η satisfy the equation

$$\Sigma s_n \alpha_n \eta_n = 0$$

and four similar equations. It follows that $s_n \eta_n$ is proportional to the determinant obtained by omitting the nth column from the matrix

$$\begin{vmatrix} \alpha_1, & \alpha_2, & \alpha_3, & \alpha_4, & \alpha_5, & \alpha_6 \\ \beta_1, & \beta_2, & \beta_3, & \beta_4, & \beta_5, & \beta_6 \\ \gamma_1, & \gamma_2, & \gamma_3, & \gamma_4, & \gamma_5, & \gamma_6 \\ \delta_1, & \delta_2, & \delta_3, & \delta_4, & \delta_5, & \delta_6 \\ \epsilon_1, & \epsilon_2, & \epsilon_3, & \epsilon_4, & \epsilon_5, & \epsilon_6 \end{vmatrix}$$

§ 15. **Condition that** $(m+1)$ **Screws shall form a Screw-complex** $(m, 1)$.

We can determine, *a priori*, the existence of *one* function, the evanescence of which must yield the $6-m$ conditions which are required.

Take another screw co-ordinate with the $m+1$ screws. Appropriate Dynames about the entire $m+2$ screws can neutralize; each Dyname will be proportional to a certain function of the co-ordinates, but each Dyname can only be zero if the remaining $m+1$ screws belong to a screw-complex $(m, 1)$.

We proceed to determine this function for the case of $m=2$, and the same process may be extended to other values of $m<6$.

If the Dynames $H\alpha$, $H\beta$, $H\gamma$ neutralize, the following six equations must be satisfied

$$H\alpha \,.\, \alpha_1 + H\beta \,.\, \beta_1 + H\gamma \,.\, \gamma_1 = 0,$$

$$\&c.,$$

$$H\alpha \,.\, \alpha_6 + H\beta \,.\, \beta_6 + H\gamma \,.\, \gamma_6 = 0.$$

Thus all the minors of the matrix

$$\begin{vmatrix} \alpha_1, & \alpha_2, & \alpha_3, & \alpha_4, & \alpha_5, & \alpha_6 \\ \beta_1, & \beta_2, & \beta_3, & \beta_4, & \beta_5, & \beta_6 \\ \gamma_1, & \gamma_2, & \gamma_3, & \gamma_4, & \gamma_5, & \gamma_6 \end{vmatrix}$$

must vanish.

If we make $(\alpha, \alpha) = \Sigma\alpha_n^2$, $(\alpha, \beta) = \Sigma\alpha_n\beta_n$, &c., then the sum of the squares of all the minors is*

$$\begin{vmatrix} (\alpha, \alpha), & (\alpha, \beta), & (\alpha, \gamma) \\ (\beta, \alpha), & (\beta, \beta), & (\beta, \gamma) \\ (\gamma, \alpha), & (\gamma, \beta), & (\gamma, \gamma) \end{vmatrix}$$

The evanescence of this single function will therefore afford all that is necessary, that the three screws α, β, γ shall lie upon a cylindroid.

The square root of this determinant may be called the *order-function.*

If four screws α, β, γ, δ belong to a screw-complex (III., 1), the four Dynames about these screws which could neutralize must each be proportional to the order-function of the three remaining screws.

* Salmon's Algebra, p. 13.

§ 16. **On the simplification of the order-function by judicious choice of the fundamental Screws.**—We shall compute in its most simple form the order-function of four screws, but a precisely analogous process may be applied to any number not exceeding 6. Let the four screws be $\alpha, \beta, \gamma, \delta$, and let four of the fundamental co-reciprocals be co-ordinate with $\alpha, \beta, \gamma, \delta$. We have then $a_5 = 0$, $a_6 = 0$, and similarly for β, γ, δ. The determinant of § 15 becomes a perfect square, and the order-function reduces simply to the determinant,

$$(\alpha_1, \beta_2, \gamma_3, \delta_4).$$

The order-function of six screws is the sexiant.

§ 17. **On certain identical formulæ connected with a co-reciprocal system.**—Let any screw α be inclined at the angle θ_n to the reference screw ω_n, and let d_n be the shortest distance between the two screws, then (§ 7)

$$a_n = \frac{(a' + s_n)\cos\theta_n - d_n \sin\theta_n}{2s_n}$$

but $a' = \Sigma s_n a^2_n$ (§ 10).

Substituting for a_n we obtain a quadratic for a'; this equation must vanish identically, whence

$$\Sigma \frac{\cos^2\theta_n}{s_n} = 0.$$

$$\Sigma \frac{s_n \cos\theta_n - d_n \sin\theta_n}{s_n} = 2.$$

$$\Sigma \frac{(s_n \cos\theta_n - d_n \sin\theta_n)^2}{s_n} = 0.$$

From the first of these it is manifest that three of the fundamental pitches must be + and three –, for let four of the pitches have the same sign, and take for α any screw perpendicular to the two remaining fundamental screws, then the first equation reduces to the sum of four positive terms equal 0, which is absurd. This theorem was communicated to me by Dr. Klein, as being substantially identical with his theorem, which asserts the same property for the parameters of six fundamental complexes in involution.*

* Math. Ann., Band II., p. 204.

By taking for α each one of the six fundamental screws in succession, all the conditions can be expressed which must necessarily be fulfilled by six co-reciprocal screws.

If α be one of the reciprocals, then we have

$$\Sigma \frac{\cos \theta_n}{s_n} = 0.$$

$$\Sigma \frac{\cos^2 \theta_n}{s_n} = 0.$$

$$\Sigma s_n \cos^2 \theta_n = 0.$$

§ 18. **Cosine of the angle between two Screws expressed in terms of their co-ordinates.**—Let ϕ be the angle between two screws α and β. Let ρ be a screw which makes an angle ϵ with α, and lies on the cylindroid $\overline{\alpha, \beta}$. Then, since the unit twist velocity about ρ may be decomposed into components $\sin(\phi - \epsilon) \operatorname{cosec} \phi$, $\sin \epsilon \operatorname{cosec} \phi$ about α and β, we must have,

$$\rho_n = \alpha_n \sin(\phi - \epsilon) \operatorname{cosec} \phi + \beta_n \sin \epsilon \operatorname{cosec} \phi.$$

Substituting in the formula

$$\Sigma \rho_1^2 + 2\Sigma \rho_1 \rho_2 (\omega_1 \omega_2)_0 = 1,$$

we deduce

$$\operatorname{Cos} \phi = \Sigma \alpha_1 \beta_1 + \Sigma(\alpha_1 \beta_2 + \alpha_2 \beta_1)\ (\omega_1, \omega_2)_0.$$

The conditions that α and β are parallel may be deduced by expressing that the pitch of the screw of which the co-ordinates are proportional to $\alpha_n - \beta_n$ assumes an indeterminate form.

§ 19. **Co-ordinates of a Screw upon a cylindroid.**—We may define a screw θ on a cylindroid by the angle l which it makes with one of the principal screws α. The unit Dyname about the screw θ is equivalent to components $\cos l$ and $\sin l$ about the two principal screws α, β, hence,

$$\theta_n = \alpha_n \cos l + \beta_n \sin l.$$

This enables the pitch of θ to be deduced,

$$\theta' = \Sigma s_n (\alpha_n \cos l + \beta_n \sin l)^2.$$

Expanding and observing that $\Sigma s_n \alpha_n \beta_n = 0$ since α and β are reciprocal, we have

$$\theta' = \alpha' \cos^2 l + \beta' \sin^2 l.$$

We thus see that the pitch is proportional to the inverse square of the parallel diameter of the pitch conic.

$$\alpha' x^2 + \beta' y^2 = c.$$

We can also express with facility the condition that two screws, θ and ϕ, upon the cylindroid should be reciprocal.

We have

$$\Sigma s_n(\alpha_n \cos l + \beta_n \sin l)\,(\alpha_n \cos m + \beta_n \sin m) = 0\,;$$

whence, reducing

$$\alpha' \cos l \cos m + \beta' \sin l \sin m = 0\,;$$

or θ and ϕ must be parallel to conjugate diameters of the pitch conic.

It is always possible to find one screw θ upon a cylindroid reciprocal to any given external screw η, for then

$$\Sigma s_n \eta_n(\alpha_n \cos l + \beta_n \sin l) = 0,$$

or

$$\operatorname{Tan} l = -\frac{\Sigma s_n \alpha_n \eta_n}{\Sigma s_n \beta_n \eta_n}$$

whence l is determined.

§ 20. **On the virtual coefficients between the Screws on a cylindroid and any other Screw** η.—Let the principal screws of the cylindroid be α, β, and let θ be a screw on the cylindroid making an angle l with α,

then $$(\theta,\eta)' = \Sigma s_n \eta_n(\alpha_n \cos l + \beta_n \sin l)$$

or $$(\theta,\eta)' = (\alpha,\eta)' \cos l + (\beta,\eta)' \sin l.$$

Whence the following construction. Mark off upon each screw θ of the cylindroid a distance from the nodal line equal to the virtual coefficient of θ and η; the points thus found lie upon a circular cylinder, of which the nodal axis is one of the generators.

The construction may also be made in a different manner. A certain tangent plane to the cylindroid can be found corresponding to η, such that the distance intercepted on any screw θ of the cylindroid between the nodal line and the tangent plane is equal to the virtual coefficient of θ and η.

§ 21. **Locus of a Screw which has a constant virtual coefficient with a given Screw.**—Let η be the given screw and ρ any other screw, of which the co-ordinates satisfy the equation

$$2\,\Sigma s_n \eta_n \rho_n = R$$

where R is the given virtual coefficient. Squaring this equation, so as to render it homogeneous, we have (§ 5)

$$4\,[\Sigma s_n \eta_n \rho_n]^2 - R^2[\Sigma \rho_1{}^2 + 2\,\Sigma \rho_1 \rho_2\,(\omega_1,\,\omega_2)_0] = 0.$$

The locus is a screw-complex of the fifth order and second degree (V, 2).

Two screws belonging to such a screw-complex can be found on every cylindroid, for make

$$\rho_n = \alpha_n \cos l + \beta_n \sin l,$$

and by substitution in the equation of the screw-complex we have a quadratic equation for tan l. The roots of this equation will determine the two required screws on *any* cylindroid $\overline{\alpha,\,\beta}$.

If we select from the screw-complex all those screws whose pitch is constant they will be found to form a linear complex of the second degree.*

It follows that through any point of space a cone of screws of the second degree can be drawn such that all the screws have the same pitch, and all have the same virtual coefficient with a given screw. All the screws lying in a plane which have the same virtual coefficient with one given screw may be classified into groups of equal pitch; all the screws of each group touch a certain conic. Thus we have in each plane a family of conics corresponding to the different pitches.

The physical meaning of the screw-complex here considered may be thus stated. The same twist about any one of the screws of the complex would do an equal quantity of work against a wrench about η.

§ 22. **Method of expressing the co-ordinates of a Screw belonging to a Screw-complex** (*III.* 1).—Given any three screws the entire screw-complex

* Plücker, Neue Geometrie des Raumes, p. 18.

is determined, and the pitch hyperboloid can be found (No. 1, Art. 88). Let α, β, γ be the principal screws of the system, i. e. the screws directed along the principal axes of the pitch hyperboloid. Let ρ be *any* other screw of the screw-complex which makes direction cosines f, g, h with lines parallel to α, β, γ. Since the unit Dyname about ρ has for constituents the Dynames f, g, h about α, β, γ, we must have

$$\rho_n = f\alpha_n + g\beta_n + h\gamma_n$$

whence

$$\rho' = \Sigma s_n (f\alpha_n + g\beta_n + h\gamma_n)^2$$
$$= f^2\alpha' + g^2\beta' + h^2\gamma'.$$

If the equation of the pitch hyperboloid be

$$\alpha' x^2 + \beta' y^2 + \gamma' z^2 + \alpha'\beta'\gamma' = 0,$$

and if r be the radius vector parallel to ρ, then

$$\rho' = -\frac{\alpha'\beta'\gamma'}{r^2}.$$

Thus we have the theorem (No. 1, Art. 88) expressing that the pitch of each member of a screw-complex (*III.* 1) is proportional to the inverse square of the parallel diameter of the pitch hyperboloid. A theorem equivalent to this had been already given by Plücker in his theory of the Linear Complex.*

The method of co-ordinates shows very simply that any two reciprocal screws of the screw-complex are parallel to a pair of conjugate diameters of the pitch hyperboloid. For if a second screw with direction cosines f', g', h' be reciprocal to ρ, then

$$\Sigma s_n (f\alpha_n + g\beta_n + h\gamma_n)(f'\alpha_n + g'\beta_n + h'\gamma_n) = 0,$$

whence expanding, and remembering that each pair of the principal screws are reciprocal,

$$\alpha' ff' + \beta' gg' + \gamma' hh' = 0.$$

§ 23. **On the virtual coefficient between a given Screw and each of the Screws of a Screw-complex** (*III.* 1).—Let η be the given screw, and ρ any screw of the system with direction cosines f, g, h. The virtual coefficient is

$$2(f(\eta, \alpha)' + g(\eta, \beta)' + h(\eta, \gamma)').$$

* Neue Geometrie des Raumes, p. 130.

Draw from the kinematic centre a radius vector r parallel to ρ and equal to the virtual coefficient just written ; then the locus of the extremity of r is the sphere

$$x^2 + y^2 + z^2 = 2[x(\eta, \alpha)' + y(\eta, \beta)' + z(\eta, \gamma')].$$

The tangent plane to the sphere obtained by equating the right hand of this equation to zero is the principal plane of that cylindroid which contains all the screws of the screw-complex which are reciprocal to η.

All the screws of the screw-complex which have a *given* virtual coefficient with η must be parallel to the generators of a right cone, and thus form a screw-complex (*II.* 2). Such a screw-complex consists of the generators of a ruled surface which must degrade to the cylindroid when the virtual coefficient is zero.

II.—APPLICATION OF SCREW CO-ORDINATES TO THE STUDY OF THE EFFECT OF AN IMPULSE UPON A FREE OR CONSTRAINED RIGID BODY.

§ 24. **Introduction.**—If a free quiescent rigid body receive an impulsive wrench about a screw and commence to twist about the same screw, then this is a *principal screw* of *inertia* (No. 2, Art. 26). Of such screws there are six, and they are co-reciprocal, they are therefore very appropriate screws of reference for kinetic investigations.

Let OA, OB, OC be the three principal axes through the centre of inertia. Let k_1, k_2, k_3 be the corresponding radii of gyration. Then two of the principal screws of inertia lie along OA with pitches $+k_1, -k_1$; two along OB with pitches $+k_2, -k_2$; two along OC with pitches $+k_3, -k_3$.

For convenience in writing formulæ we shall still call the pitches s_1, s_2, &c., s_6, but it is to be always remembered that

$$s_1 = +k_1,\ s_2 = -k_1,\ s_3 = +k_2, \text{ \&c.}$$

Let ω_n be one of the principal screws of inertia, the impulsive wrench $\ddot{H}\omega_n$ acting for the time t will develop a twist velocity $\dot{H}\omega_n$.

The velocity of the centre of inertia due to $\ddot{H}\omega_n$ is

$$\frac{t\ddot{H}\omega_n}{M} = K\omega_n \text{ (§ 2).}$$

whence

$$\dot{H}\omega_n \,.\, s_n = K\omega_n$$

the kinetic energy acquired is

$$M(K\omega_n)^2 = Ms_n^2\,(\dot{H}\omega_n)^2$$

§ 25. **Connexion between the impulsive Screw and the corresponding instantaneous Screw.**—Let an impulsive wrench $\ddot{H}\bar{\eta}$ act about a screw $\bar{\eta}$ for the time t upon a free quiescent rigid body of mass M. It is required to find the instantaneous screw η about which the body will commence to twist, and also the initial twist velocity $\dot{H}\eta$.

The impulsive wrench is to be resolved into constituents about the principal screws; each constituent is of the type

$$\ddot{H}\bar{\eta} \,.\, \bar{\eta}_n.$$

This constituent will in the time t produce a twist velocity about the corresponding principal screw equal to

$$\frac{K\bar{\eta} \,.\, \bar{\eta}_n}{s_n} \text{ (see § 2);}$$

this twist velocity must equal the component of the resultant twist velocity about the same screw, whence

$$\dot{H}\eta \,.\, \eta_n = \frac{K\bar{\eta} \,.\, \bar{\eta}_n}{s_n}$$

or

$$\bar{\eta}_n = \frac{\dot{H}\eta}{K\bar{\eta}}\, s_n \eta_n$$

When the six principal screws of inertia are employed as fundamental screws, the identical relation between the co-ordinates (§ 5) reduces to

$$(\eta_1 - \eta_2)^2 + (\eta_3 - \eta_4)^2 + (\eta_5 - \eta_6)^2 = 1.$$

Introducing and defining the symbol η_0 by the equation

$$\eta_0^2 = k_1^2(\eta_1 - \eta_2)^2 + k_2^2(\eta_3 - \eta_4)^2 + k_3^2(\eta_5 - \eta_6)^2.$$

we have

$$\eta_0\bar{\eta}_n = s_n\eta_n.$$

From this we deduce the principal result of this paper, namely, the relation between an impulsive screw and the corresponding instantaneous screw.

If η_1, &c., η_6 be the co-ordinates of any screw η, then an impulsive wrench, about the screw of which the co-ordinates are proportional to $s_1\eta_1$, &c., s_6,η_6 will make the body, if perfectly free and quiescent, commence to twist about η as the instantaneous screw.

§ 26. **Kinetic energy acquired by the impulse** $\ddot{H}\bar{\eta}$.—The kinetic energy produced by the component

$$\ddot{H}\bar{\eta}\,.\bar{\eta}_n$$

is expressed by

$$M(K\bar{\eta})^2\bar{\eta}_n{}^2$$

whence the total kinetic energy acquired is

$$M(K\bar{\eta})^2(\bar{\eta},\bar{\eta}) = M(\dot{H}\eta)^2\eta''\ ;\ \text{if } \eta'' = \Sigma s^2{}_1\eta_1{}^2.$$

§ 27. **If $\bar{\alpha}$ and $\bar{\beta}$ be two impulsive Screws, and α and β the corresponding instantaneous Screws, then when $\bar{\alpha}$ is reciprocal to β, $\bar{\beta}$ is reciprocal to α.**—Although this theorem has been already proved and largely employed (No. 1, Art. 56), yet its demonstration by the method of co-ordinates will be useful.

If $\bar{\alpha}$ and β be reciprocal, then

$$\Sigma s_n\bar{\alpha}_n\beta_n = 0$$

but

$$\alpha_0\bar{\alpha}_n = s_n\alpha_n$$

so that the foregoing condition can be expressed by the equation

$$\Sigma s_n{}^2\alpha_n\beta_n = 0$$

but we have also

$$\beta_0\bar{\beta}_n = s_n\beta_n$$

whence

$$\Sigma s_n\alpha_n\bar{\beta}_n = 0$$

or α and $\bar{\beta}$ must be reciprocal.

When α and β possess the property here considered, they are said to be *conjugate screws of kinetic energy.*

§ 28. **General expression for the kinetic energy of a body.**—Let α, β, γ, δ, ϵ, ζ represent any six conjugate screws of kinetic energy. Let a body

twist about a screw ρ, with a twist velocity $\dot{H}\rho$, and let ρ_1, &c., ρ_6 be the co-ordinates of ρ with reference to α, &c., ζ. The twist velocity $\dot{H}\rho$ is first to be decomposed into components $\dot{H}\rho.\rho_1$ about α, $\dot{H}\rho.\rho_2$ about β, &c., $\dot{H}\rho.\rho_6$ about ζ; each of these components is to be again decomposed into six twist velocities about the six principal screws of inertia. The twist velocity about each principal screw will thus consist of six terms, that about the screw ω_n being

$$\dot{H}\rho\,.\,(\alpha_n\rho_1+\beta_n\rho_2+\gamma_n\rho_3+\delta_n\rho_4+\epsilon_n\rho_5+\zeta_n\rho_6).$$

The total kinetic energy is therefore found by squaring this quantity, multiplying it by Ms_n^2, and adding the six expressions obtained by giving n every value from 1 to 6. It will be found that the product $\rho_1\rho_2$ contains in its coefficient the factor $(\alpha, \beta)''$; this quantity vanishes, since α and β are conjugate screws of kinetic energy. Hence the total kinetic energy is

$$M\,.\,(\dot{H}\rho)^2\,.\,(\alpha''\rho_1^2+\beta''\rho_2^2+\gamma''\rho_3^2+\delta''\rho_4^2+\epsilon''\rho_5^2+\zeta''\rho_6^2).$$

Each coefficient α'' is essentially positive, as it consists of the sum of six squares.

This expression may be written concisely in the form $(\dot{H}\rho)^2\Sigma E_n\rho_n^2$, where E_n is the kinetic energy due to the unit twist velocity about the corresponding principal screw of kinetic energy.

§ 29. **The partial differential coefficients of the pitch of a Screw with respect to the co-ordinates are proportional to the co-ordinates of the corresponding impulsive Screw.**—In this as in every other kinetical theorem the co-ordinates are referred to the principal screws of inertia.

If α be any screw then the pitch is found by

$$\alpha' = \Sigma s_n\alpha_n^2.$$

$$\therefore \frac{d\alpha'}{d\alpha_n} = 2s_n\alpha_n = 2\alpha_0\bar{\alpha}_n,$$

which proves the theorem.

§ 30. **One degree of freedom.**—Suppose a quiescent rigid body which is only permitted to twist about one screw be acted upon by an impulsive wrench, what is the initial twist velocity acquired? We may also ask, What is the kinetic energy acquired? What is the initial reaction of the constraints?

What is the impulsive wrench which would have produced the same motion in a perfectly *free* rigid body? The method of screw co-ordinates will enable these questions to be answered. Let it be noticed that the problem includes *every* case in which the body has but one degree of freedom.

§ 31. **Condition that either of two impulsive wrenches should be capable of producing the same effect upon a body which has one degree of freedom.**—Let $\ddot{H}\eta$, $\ddot{H}\xi$ be the wrenches, and a the screw about which the body twists. If the sign of one of the wrenches be changed, each will destroy the effect of the other, and therefore the resultant must be a wrench about a screw reciprocal to a.

The co-ordinates of the resultant are severally proportional to the values of

$$\ddot{H}\eta \,.\, \eta_n - \ddot{H}\xi \,.\, \xi_n$$

multiplying each of these quantities by

$$s_n a_n$$

adding the six products, and expressing that their sum is zero, we have

$$(a, \eta)' \ddot{H}\eta = (a, \xi)' \ddot{H}\xi.$$

We have thus proved that each wrench must be inversely proportional to the virtual coefficient of the corresponding screw with a. The given impulsive wrench may therefore be replaced by a wrench of appropriate magnitude about any other screw.

If the body had been perfectly free, an impulsive wrench $\ddot{H}\bar{a}$ would have made the body commence to twist about a.

We have therefore

$$\ddot{H}\bar{a} = \frac{(a, \eta)'}{(a, \bar{a})'} \ddot{H}\eta,$$

but

$$a_0 \bar{a}_n = s_n a_n,$$

or

$$(a, \bar{a})' = \frac{a''}{a_0},$$

whence

$$\ddot{H}\bar{a} = \ddot{H}\eta \,.\, \frac{(a, \eta)'}{a''} a_0.$$

It is remarkable that $\ddot{H}\bar{a}$ vanishes when a_0 vanishes, and the following is the explanation: $k_1 (a_1 - a_2)$ is the velocity of the centre of inertia parallel to

the corresponding principal axis OA, which arises from the unit of twist velocity about a. Hence a_0 is simply the velocity of the centre of inertia. This velocity can only vanish when the pitch of a is zero, and when a passes through the centre of inertia. The impulse required to produce a velocity of pure rotation about an axis passing through the centre of inertia is a couple, hence the pitch of $\bar{a}$ is infinite and, $\ddot{H}\bar{a}$ must be zero.

This reasoning may be confirmed by actual calculation of the pitch of $\bar{a}$,

$$\bar{a}' = \Sigma s_n \bar{a}_n^2 = \Sigma s_n \frac{s_n^2 a_n^2}{a_0^2},$$

whence if a_0 vanishes $\bar{a}'$ is infinite.

§ 32. **Given the impulsive wrench $\ddot{H}\eta$, determine $\dot{H}a$ when the body is constrained to twist about a.** We have

$$\ddot{H}\bar{a} = \frac{MK\eta}{t} \frac{(a, \eta)'}{a''} a_0$$

The wrench $\ddot{H}\bar{a}$ acting on a free body M for the time t would give to the centre of inertia a velocity

$$= \frac{t\,\ddot{H}\bar{a}}{M}.$$

But this must also be equal to $\dot{H}a \,.\, a_0$,

whence $$\dot{H}a = K\eta \,.\, \frac{(a, \eta)'}{a''}.$$

This expression, which is of considerable importance, may also be deduced as follows :—

The constituent of $\ddot{H}\bar{a}$ about ω_n is $\ddot{H}\bar{a} \,.\, \bar{a}_n$.

This constituent must generate the twist velocity $\dot{H}a \,.\, a_n$,

whence $$\dot{H}a \,.\, a_n = \frac{K\bar{a}}{s_n} \,.\, \bar{a}_n,$$

but $$K\bar{a} = K\eta \frac{(a, \eta)'}{a''} a_0 \text{ and } a_0 \bar{a}_n = s_n a_n,$$

whence, by substitution, we deduce the expression for $\dot{H}a$.

§ 33. **Expression for the kinetic energy acquired by a body only free to twist about a when acted upon by $\ddot{H}\eta$ for the short time t.** A body animated by the twist velocity $\dot{H}\omega_n$ has the kinetic energy.

$$Ms^2_n (\dot{H}\omega_n)^2$$

whence the kinetic energy of the body with a twist velocity $\dot{H}a$ is

$$Ma'' (\dot{H}a)^2$$

or finally (§ 32)

$$M \frac{(a, \eta)'^2}{a''} (K\eta)^2$$

If $\eta = \bar{a}$, then $\qquad a_0\eta_n = s_n a_n,$

and the expression for the energy is

$$M (\eta, \eta) (K\eta)^2.$$

§ 34. **Expression for the difference between the kinetic energy which would have been acquired by the body, if free, in consequence of the impulsive wrench $\ddot{H}\eta$, and the energy which is acquired when the body is only permitted to twist about a.** This expression is simply the difference between those of the last article, and may be written

$$M \frac{(K\eta)^2}{a''} (a''(\eta, \eta) - (a, \eta)'^2).$$

The quantity inside the bracket reduces to the sum of 15 square terms, the first of which is $\qquad (s_1 a_1 \eta_2 - s_2 a_2 \eta_1)^2.$

The expression being essentially positive shows that there is always a loss of energy by the constraints. This theorem is due to Euler.

§ 35. **On the locus of a if the kinetic energy acquired be constant.**—We must then have

$$E = M \frac{(a, \eta)'^2}{a''} (K\eta)^2.$$

The locus of a is therefore a screw complex (V, 2).

§ 36. **If a wrench $\ddot{H}\eta$ act upon a body only free to twist about a, it is required to find $\ddot{H}\rho$, the initial reaction of the constraints.** We must have

$$\ddot{H}a \,.\, \bar{a}_n = \ddot{H}\eta \,.\, \eta_n + \ddot{H}\rho \,.\, \rho_n.$$

Substituting for $\ddot{H}\bar{a}$ from (§ 31) we find

$$\rho_n = \frac{\ddot{H}\eta}{\ddot{H}\rho . a''} [(a, \eta)' s_n a_n - a'' \eta_n].$$

The co-ordinates of the reactionary screw are thus determined ; and it is easy to verify that a and ρ are reciprocal.

§ 37. **On the ellipse of equal kinetic energy in the principal plane of a cylindroid.**—It has been already shown that the twist velocity of a body which has a given kinetic energy about a screw on a cylindroid is proportional to the parallel diameter of a certain conic. The calculation of this conic can be very conveniently made by the aid of co-ordinates.

Let θ, λ, μ be three screws on a cylindroid whereof the angle between θ and λ is l, and between θ and μ is m.

We have.

$$\theta_n = \frac{\sin m}{\sin (l+m)} \lambda_n + \frac{\sin l}{\sin (l+m)} \mu_n.$$

The kinetic energy due to $\dot{H}\theta$ is $M\theta''(\dot{H}\theta)^2$ or

$$M\left(\frac{\dot{H}\theta}{\sin (l+m)}\right)^2 [\lambda'' \sin^2 m + {}^2(\lambda,\mu)'' \sin l \sin m + \mu'' \sin^2 l].$$

If we set off in the principal plane a radius vector r parallel to θ and proportional to $\dot{H}\theta$, so that if k be the unit of length

$$\frac{r}{k} = \dot{H}\theta,$$

and if x and y be co-ordinates of the extremity of the radius vector referred to axes parallel to λ, μ we have

$$\frac{r}{\sin (l+m)} = \frac{x}{\sin l} = \frac{y}{\sin m}$$

whence the expression for the energy reduces to

$$E = \frac{M}{k^2} [x^2 \lambda'' + 2xy(\lambda,\mu)'' + y^2 \mu''].$$

If E be constant, we have the ellipse of equal kinetic energy. If λ, μ be conjugate screws of kinetic energy, then we have seen (§ 27) that

$$(\lambda, \mu)'' = 0.$$

We now see that when this is the case λ and μ are parallel to a pair of conjugate diameters of the ellipse of equal kinetic energy.

§ 38. **The Screw η being given, find $\ddot{H}\eta$, which, acting for a short time t shall impart one unit of kinetic energy to a body which is constrained to twist about the Screw λ upon the cylindroid $\overline{\alpha, \beta}$.**—Let l be the angle which λ makes with α. We have

$$1 = M(K\eta)^2 \frac{(\lambda, \eta)'^2}{\lambda''}$$

or

$$1 = M(K\eta)^2 \frac{[(\alpha, \eta)' \cos l + (\beta, \eta)' \sin l]^2}{\alpha'' \cos^2 l + 2(\alpha, \beta)'' \sin l \cos l + \beta'' \sin^2 l},$$

by which $K\eta$ and therefore $\ddot{H}\eta$ are determined.

If $K\eta$ be given we have for $\tan l$ two values corresponding to two screws λ, μ. Lines parallel to λ and μ may be ascertained by joining the origin to the intersections of the ellipse of equal kinetic energy ($K = 1$, $E = 1$),

$$\alpha'' x^2 + 2(\alpha, \beta)'' xy + \beta'' y^2 = \frac{1}{M} \tag{1}$$

with the line

$$(\alpha, \eta)' x + (\beta, \eta)' y = \frac{1}{M \,.\, K\eta} \tag{2}$$

(2) is parallel to that screw ρ upon the cylindroid which is reciprocal to η.

If p be the perpendicular from the origin on (2) then

$$\frac{1}{p} = \mathrm{M}[(\alpha, \eta)'^2 + (\beta, \eta)'^2]^{\frac{1}{2}} K\eta.$$

We can express the construction for λ and μ in a more simple form by the following considerations: The perpendicular p makes with the axis x an angle of which the cosine is

$$\frac{(\alpha, \eta)'}{[(\alpha, \eta)'^2 + (\beta, \eta)'^2]^{\frac{1}{2}}},$$

whence if ϕ be the screw on the cylindroid which is parallel to p, we have

$$\phi_n = \frac{(\alpha, \eta)' \alpha_n + (\beta, \eta)' \beta_n}{[(\alpha, \eta)'^2 + (\beta, \eta)'^2]^{\frac{1}{2}}}.$$

From this we find

$$(\phi, \eta)' = [(\alpha, \eta)'^2 + (\beta, \eta)'^2]^{\frac{1}{2}}.$$

If a body twist uniformly around ϕ the work done against $\ddot{H}\eta$ in the time t is

$$2(\phi, \eta)' \dot{H}\phi \,.\, \ddot{H}\eta \,.\, t$$

If this work be one unit then

$$2\dot{H}\phi = p.$$

If therefore it be proposed to select from all the screws upon a cylindroid those two screws λ, μ such that if the body be constrained to twist about either λ or μ and receive a given impulsive wrench about η for the short time t, the body will move off with the unit of kinetic energy, the construction is as follows :—Draw the ellipse of equal kinetic energy of which the radii vectores each contain as many units of length as the number of units of twist velocity with which the body must twist about the parallel screw on the cylindroid in order to have one unit of kinetic energy. Find the screw ρ on the cylindroid reciprocal to η, and the screw ϕ perpendicular to ρ. The twist velocity $\dot{H}\phi$ is then to be ascertained with which a body should twist about ϕ for a time t in order to do one unit of work against $\ddot{H}\eta$; a straight line parallel to ρ is then to be drawn in the principal plane at a perpendicular distance $2\dot{H}\phi$ units of length from the origin. The intersections of this line with the ellipse of equal kinetic energy are to be joined to the origin, and the required screws λ and μ are parallel to the joining lines.

When the line just touches the ellipse the two screws λ, μ coincide in a screw ν which possesses the property that a given impulse about η will impart to the body constrained to twist about ν a greater kinetic energy than if the body had been constrained to twist about any other screw on the cylindroid. Euler's theorem (§ 34), tells us that if the body had been free to twist about all the screws of the cylindroid, and that it received the impulse $\ddot{H}\eta$ that ν would be the instantaneous screw.

The last theorem may otherwise be shown thus. Since ρ and ν are parallel to conjugate diameters they are therefore conjugate screws of kinetic energy, but η is reciprocal to ρ, and therefore ν must be the instantaneous screw which the body would naturally select if permitted to make a selection from all the

screws of the cylindroid. We have thus obtained a completely general solution for two degrees of freedom of the connexion between the impulsive screw and the instantaneous screw.

§ 39. **Calculation of the co-ordinates of the three principal Screws of inertia of a body which has three degrees of freedom.**—Let α, β, γ, be the three screws belonging to the system which lie along the principal axes of the pitch hyperboloid. Though it is often convenient to call these the *principal screws of the screw-complex*, they are merely kinematical, and must be carefully distinguished from the principal screws of inertia, the essential feature of which is that if the body receive an impulsive wrench about one it commences to twist about the same screw.

Let ρ be a principal screw of inertia and let f, g, h be the direction cosines of ρ with reference to α, β, γ.

An impulsive wrench $\ddot{H}\rho$ must, when compounded with a certain reactionary wrench $\ddot{H}\xi$, constitute the wrench $\ddot{H}\bar{\rho}$. We have therefore

$$\ddot{H}\bar{\rho} \,.\, \bar{\rho}_n = \ddot{H}\xi \,.\, \xi_n + \ddot{H}\rho \,.\, \rho_n \tag{A}$$

or since

$$\rho_0 \bar{\rho}_n = s_n \rho_n$$

and

$$\rho_n = f\alpha_n + g\beta_n + h\gamma_n$$

$$\ddot{H}\bar{\rho} \,.\, s_n(f\alpha_n + g\beta_n + h\gamma_n) = \rho_0 \,.\, \ddot{H}\xi \,.\, \xi_n + \rho_0(f\alpha_n + g\beta_n + h\gamma_n)\ddot{H}\rho$$

multiplying by $s_n\alpha_n$, adding the six equations, repeating the same for $s_n\beta_n$, $s_n\gamma_n$,

and making

$$v = \rho_0 \frac{\ddot{H}\rho}{\ddot{H}\bar{\rho}}$$

we have for v the equation

$$\begin{vmatrix} \alpha'' - \alpha' v, & (\alpha, \beta)'', & (\alpha, \gamma)'' \\ (\beta, \alpha)'', & \beta'' - \beta' v, & (\beta, \gamma)'' \\ (\gamma, \alpha)'', & (\gamma, \beta)'', & \gamma'' - \gamma' v \end{vmatrix} = 0$$

When the three values of v are known, the three principal screws of inertia are determined. Multiply the equation A by $s_n\rho_n$ and add the six equations of this type, we find

$$v = \frac{\rho''}{\rho'}$$

If p be the parallel radius vector in the pitch hyperboloid, and q in the ellipsoid of equal kinetic energy

$$\rho' = -\frac{\alpha'\beta'\gamma'}{p^2} \text{ (§ 22)}$$

$$\rho'' = \frac{1}{Mq^2} \text{ (§ 40)}$$

$$v = -\frac{p^2}{q^2}\frac{1}{M\alpha'\beta'\gamma'}.$$

We have also

$$\dot{H}\rho = \frac{\rho'}{\rho''} K\rho \text{ (§ 32)}$$

so that

$$v = \frac{K\rho}{\dot{H}\rho}$$

or

$$\frac{K\rho}{\dot{H}\rho} = -\frac{p^2}{q^2}\frac{1}{M\alpha'\beta'\gamma'}.$$

We have already seen that the principal screws of inertia are parallel to the common conjugate diameters of the pitch hyperboloid and the ellipsoid of equal kinetic energy, we now see that the twist velocity produced by a given impulse about one of the principal screws of inertia is determined by the same construction.

§ 40. **Calculation of the ellipsoid of equal kinetic energy.**—If a rigid body twist with a velocity $\dot{H}\rho$ about the screw ρ which is parallel to a line passing through the kinematic centre and making direction angles whose cosines are f, g, h with the principal screws α, β, γ, then the kinetic energy is (§ 26).

$$M(\dot{H}\theta)^2 \Sigma s^2_n (f\alpha_n + g\beta_n + h\gamma_n)^2.$$

If we equate this to unity and replace $\dot{H}\theta$ by a radius vector q of which the direction cosines are f, g, h, we have for the ellipsoid of equal kinetic energy

$$\alpha'' x^2 + \beta'' y^2 + \gamma'' z^2 + 2(\beta, \gamma)'' yz + 2(\alpha, \gamma)'' xz + 2(\alpha, \beta)'' xy = \frac{1}{M}.$$

The property possessed by this surface may be thus enunciated. The number of units in the radius vector from the centre to the surface is the

number of units of twist velocity with which the body must twist about the parallel screw in order to possess the unit of kinetic energy.

If θ, ϕ be two conjugate screws of kinetic energy then (§ 27),

$$\Sigma s^2{}_n\theta_n\phi_n = 0,$$

substituting for θ_n its value $f\alpha_n + g\beta_n + h\gamma_n$, and for ϕ_n its value $f'\alpha_n + g'\beta_n + h'\gamma_n$ we deduce that θ and ϕ are parallel to a pair of conjugate diameters of the ellipsoid of equal kinetic energy.

§ 41. **The reciprocal plane.**—All the screws selected from a screw-complex (*III*, 1) so as to be reciprocal to ρ lie upon a cylindroid: the principal plane of this cylindroid may be called the *reciprocal plane* with respect to ρ.

If f, g, h be the direction cosines of a line in the reciprocal plane then

$$\Sigma s_n\rho_n(f\alpha_n + g\beta_n + h\gamma_n) = 0,$$

or

$$(\rho, \alpha)'f + (\rho, \beta)'g + (\rho, \gamma)'h = 0,$$

whence the equation of the reciprocal plane is

$$(\rho, \alpha)'x + (\rho, \beta)'y + (\rho, \gamma)'z = 0.$$

The same reciprocal plane will be found for any other screw which is co-ordinate with a group consisting of ρ and three screws of the reciprocal screw-complex.

§ 42. **Kinetic energy acquired by an impulse.**—A rigid body having three degrees of freedom receives an impulsive wrench $\ddot{H}\rho$; if the body be constrained to twist about θ of which the direction cosines are f, g, h, determine the kinetic energy acquired. The general expression for the energy E (§ 33)

is

$$M\frac{(\theta, \rho)'^2}{\theta''}(K\rho)^2,$$

making $\theta_n = f\alpha_n + g\beta_n + h\gamma_n$, we have

$$E = \frac{M(K\rho)^2[f(\alpha, \rho)' + g(\beta, \rho)' + h(\nu, \rho)']^2}{\alpha''f^2 + \beta''g^2 + \gamma''h^2 + 2(\alpha, \beta)''fg + 2(\alpha, \gamma)''fh + 2(\beta\gamma)''gh},$$

whence if E be given the line drawn from the kinematic centre with direction cosines f, g, h must lie on the cone

$$\alpha''x^2+\beta''y^2+\gamma''z^2+2(\alpha,\beta)''xy+2(\alpha,\gamma)''xz+2(\beta,\gamma)''yz$$
$$=\frac{M(K\rho)^2}{E}[(\alpha,\rho)'x+(\beta,\rho)'y+(\gamma,\rho)'z]^2.$$

Draw the ellipsoid which has for equation

$$\alpha''x^2+\beta''y^2+\gamma''z^2+2(\alpha,\beta)''xy+2(\beta,\gamma)''yz+2(\gamma,\alpha)''xz=\frac{E}{M} \qquad (1)$$

and is similar to the ellipsoid of equal kinetic energy.

Draw the plane

$$(\alpha,\rho)'x+(\beta,\rho)'y+(\gamma,\rho)'z=\frac{E}{M\,.\,K\rho}, \qquad (2)$$

which is parallel to the reciprocal plane.

Construct the cone from the kinematic centre to the intersection of (1) and (2) and find all the screws of the complex which are parallel to the generators of this cone.

If the body when constrained to move about any one of these screws θ receive the impulse $\ddot{H}\rho$ for the time t, it will commence to move with the kinetic energy E, and with a twist velocity equal to the corresponding radius vector of (1).

For the kinetic energy is

$$E=M\theta''(\dot{H}\theta)^2,$$

but if q be the radius vector of (1)

$$q^2\theta''=\frac{E}{M},$$

whence

$$\dot{H}\theta=q.$$

The following is a slightly different method of constructing the cone :— Draw the plane reciprocal to ρ, and find the screw ϵ of the complex normal to the reciprocal plane. Determine the twist velocity $\dot{H}\epsilon$ with which a body must be twisted about ϵ in order to do one unit of work against $\ddot{H}\rho$ in the time t. Draw a plane parallel to the reciprocal plane at a perpendicular distance $\dot{H}\epsilon$ therefrom, then the cone from the origin to the intersection of this plane with the ellipsoid of equal kinetic energy is the cone required.

The locus selected from the screw-complex (*III*, 1), of the instantaneous screw about which, if the body were compelled to twist, it would with a *given* impulsive wrench assume a given kinetic energy E, is a screw-complex (*II*, 2). All the screws of such a complex lie upon a ruled surface of the sixth degree. This surface degrades on the one hand to a cylindroid when E is zero, on the other hand to a single screw when E is a maximum.

Four screws may be selected so that both the initial twist velocity and kinetic energy shall be of given amount.

If $\ddot{H}\rho$ be the least impulsive wrench about ρ which will impart the kinetic energy E, then the plane (2) must be as far as possible from the kinematic centre, and must therefore touch the ellipsoid (1). The screw about which the body must twist is parallel to the diameter conjugate to the reciprocal plane. It follows from Euler's theorem (§ 34), that this is the screw which the body will naturally select as an instantaneous screw if permitted to make a choice from all the screws of the screw-complex. We have therefore the following construction for the instantaneous screw corresponding to a given impulsive screw.

Draw the reciprocal plane to the impulsive screw; find the conjugate direction to this plane in the ellipsoid of equal kinetic energy. Then that one screw of the screw-complex which is parallel to this direction is the instantaneous screw about which the body will commence to twist.

§ 43. **Application of the theory of conjugate Screws of kinetic energy.** —The construction given in the last article can be demonstrated very simply from the fundamental property of conjugate screws of kinetic energy.

Any three conjugate diameters of the ellipsoid of equal kinetic energy are parallel to three conjugate screws of kinetic energy. Let ρ be an impulsive screw. Draw the reciprocal plane to ρ; take in the reciprocal plane any two conjugate diameters of the ellipsoid of equal kinetic energy, and let μ and ν be the screws of the screw-complex which are parallel to these diameters. If λ be the screw parallel to the third conjugate diameter then λ is the instantaneous screw corresponding to ρ. For, since λ, μ, ν are conjugate screws of kinetic energy, and since η is reciprocal to μ and ν it follows that an impulsive wrench about η must cause an instantaneous twist velocity about λ.

If the impulsive wrench were merely a couple, then the reciprocal plane would be parallel to the plane of the couple. If further the constraints were such that the body merely rotated about a point, then the ellipsoid becomes the momental ellipsoid, and the construction now given reduces to Poinsot's well-known theorem.

§ 44. **On the reaction of the constraints.**—Let the impulsive screw be ρ, and the instantaneous screw found by the construction just given be λ. Construct the screw $\overline{\lambda}$ and the cylindroid $\overline{\rho, \overline{\lambda}}$. Let η be the screw about which the initial reaction of the constraints acts, then η must lie on $\overline{\rho, \overline{\lambda}}$, and since η is also reciprocal to the system α, β, γ it will be defined by seeking that screw on the cylindroid which is reciprocal to α. It is somewhat remarkable that the screw thus found is also reciprocal to β and γ.

§ 45. **Determination of the impulsive Screw when the instantaneous Screw is known.**—When the instantaneous screw λ is given the impulsive screw ρ is indeterminate. Draw the plane conjugate to λ in the ellipsoid of equal kinetic energy; construct the cylindroid of the system whose screws are parallel to this plane, then any screw ρ reciprocal to the cylindroid will be an impulsive screw corresponding to λ as an instantaneous screw. Thus through any point of space a cone of screws of the second degree can be drawn, an impulsive wrench about any one of which will make the body commence to twist about λ. (No. 1, Art. 89).

§ 46. **An impulsive wrench acts about a Screw ρ upon a body which has five degrees of freedom. Find the instantaneous Screw θ.**—Let η be the screw reciprocal to the freedom (No. 1, Art. 94), we must have

$$(\eta, \theta)' = 0.$$

The kinetic energy acquired is

$$M(K\rho)^2 \frac{(\rho, \theta)'^2}{\theta''}.$$

We may determine θ from Euler's theorem (§ 34), whence

$$\Sigma s_n[\rho_n\theta'' - (\rho, \theta)' s_n\theta_n]\delta\theta_n = 0.$$

We have also $$\Sigma s_n\eta_n\delta\theta_n = 0,$$

whence $$\theta''\rho_n - (\rho, \theta)' s_n\theta_n + k\eta_n = 0,$$

multiplying by η_n and summing the six equations

$$\theta''(\eta, \rho) + k(\eta, \eta) = 0$$

whence $$s_n\theta_n = \frac{(\eta, \eta)\theta''\rho_n - \theta''(\eta, \rho)\eta_n}{(\eta, \eta)},$$

we thus see that the body will commence to twist as if it had been free, but had received an impulsive wrench about a screw whose co-ordinates are proportional to quantities of the type

$$(\eta, \eta)\rho_n - (\eta, \rho)\eta_n.$$

§ 47. **A rigid body has five degrees of freedom, calculate the five principal Screws of inertia.**—If ρ be a principal screw of inertia then (from § 25 and § 46) v being a certain constant

$$\frac{(\eta, \eta)}{v} s_n\rho_n = (\eta, \eta)\rho_n - (\eta, \rho)\eta_n,$$

whence $$(\eta, \eta)\rho_n = \frac{(\eta, \rho)}{1 - \frac{s_n}{v}}\eta_n,$$

multiplying each side by $s_n\eta_n$, and summing, we have the following equation for v.

$$\Sigma \frac{s_n}{s_n - v}\eta_n^2 = 0.$$

This equation is of the fifth degree in v; when it has been solved each root substituted in the type

$$\frac{\eta_n}{s_n - v}$$

will give the co-ordinates of the five principal screws of inertia.

§ 48. **On the general theory of the Screw-complex of the mth order and second degree.**—Construct a screw-complex S of the $(m+1)^{th}$ order and first degree, take $m+1$ screws of this complex as screws of reference, then every other screw θ belonging to S will be defined by the $m+1$ co-ordinates θ_1, &c., θ_{m+1}.

If $U = 0$ denote a homogeneous equation of the second degree between θ_1, &c., θ_{m+1} then all the screws θ which satisfy this equation will constitute a screw-complex $(m, 2)$.

If η represent any screw belonging to S then the equation

$$\eta_1 \frac{dU}{d\theta_1} + \&c. + \eta_{m+1} \frac{dU}{d\theta_{m+1}} = 0$$

gives a linear relation between the co-ordinates of θ and thus defines a screw-complex $(m, 1)$. All the screws reciprocal to the complex thus found constitute a complex $(6 - m, 1)$ which may be called the polar of η with respect to U.

The relation between η and its polar complex is of great importance in the theory of screws, and with the enunciation of one application we shall conclude the present Paper.

§ 49. **Geometrical determination of the connexion between the impulsive Screw and the instantaneous Screw for all orders of freedom.**—An imaginary screw-complex of the second degree can be found for each order of freedom, which is the locus of those screws about which the body could twist so that its kinetic energy shall be zero. This may be called the *kinetic complex.* The polar of the screw η with respect to the kinetic complex is the locus of those screws an impulsive wrench about any one of which would make the body commence to twist about η.

NOTE, *added April*, 1874.—Professor Clifford has lately published a Memoir on "Biquaternions" (Proceedings of the London Mathematical Society, Nos. 64, 65, page 382). A "Biquaternion" is defined to be the ratio of two twists or wrenches, or, more generally, of two "motors," to use the word employed by Professor Clifford, which is synonymous with (and perhaps preferable to) the word Dyname, which we have adopted from Plücker. The Biquaternions are shown to be intimately associated with the speculations of the geometry of elliptic space.

A Memoir has just appeared in the Mathematische Annalen, Vol. VII., pp. 137–228, by Dr. Ferdinand Lindemann, entitled "Uber unendlich kleine Bewegungen und über Kraftsysteme bei allgemeiner projectivischer Massbestimmung." Among many results in this Paper, which are closely related to the "Theory of Screws," we find that the cylindroid is only a degraded form of a surface of the fourth order with two double lines which is presented in the theory of the kinematics of a rigid body in elliptic space.

VII.—*On Accessory Lobes of the Human Lungs.* By Edward W. Collins, M.D., Univ. Dub., Demonstrator of Anatomy in the University of Dublin; with Plate XVI.

Read 13th April, 1874.

While engaged lately in demonstrating the thoracic viscera of a man, aged apparently about fifty years, a congenital arrangement of some of the contents of the right side of the thorax arrested my attention. A superficial examination revealing morphological peculiarities interesting, and to me novel, induced me to remove the specimen, preserving intact its relations to the surrounding structures. From what I have since learned I believe it to be one of considerable rarity. After dissection I have therefore made a preparation of the parts *in situ*, which I now submit to the Academy, with the accompanying drawings, as the only specimen of the kind yet met with in this city, intending subsequently to deposit them in the Anatomical Museum of the University.

The specimen presents a three-fold abnormality as regards the lung, pleura, and vena azygos.

Immediately above the posterior part of the root, in the angle between it and the upper portion of the right lung, springs an accessory lobe somewhat pyriform in shape, with a broad peduncular attachment. It and other portions of the lung bear traces of previous vesicular emphysema. Removed from its position, it is found to measure four inches in length by two and a half in breadth at its widest part. The breadth of the peduncle is an inch and a half. The lobe lies beside and upon the bodies of the five upper dorsal vertebræ in an accessory pleural pouch, which communicates with the general pleural cavity only around the peduncle of the lobe.

The pouch is formed by an arched duplicature which depends from the

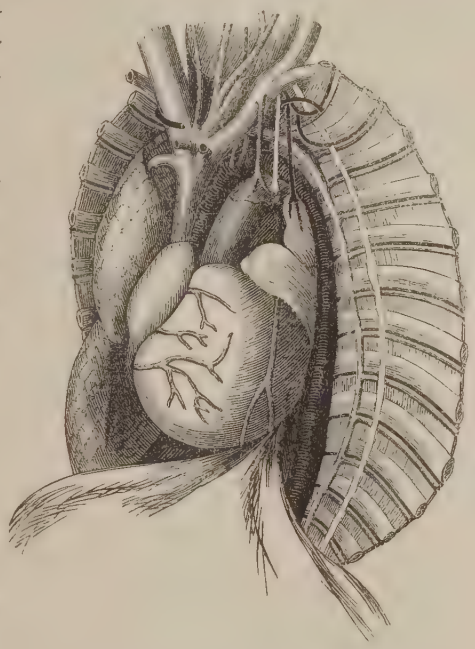
outer part of the pleural cone, immediately above the angle of the first rib, in a direction downwards and slightly inwards. Attached externally along a line corresponding to the junction of the ribs with the vertebral bodies as far as the head of the fifth rib, from this line of reflection the pleural fold arches forwards and inwards over the accessory lobe to the mesial line of the dorsal vertebræ from the fifth upwards to the first. Here it is again reflected outwards and backwards investing the side of the spine, so as to become continuous with the inner layer of the pleural fold. As well above as along its outer border the duplicature is continuous with the costal portion of the pleura: along its inner border with the mediastinal. Its lower border alone is free. Its measurements are along its outer border three inches; along its inner three inches and a half; along its lower an inch and three-quarters. The lower free margin contains the azygos vein, and directly above the vein a thickening, which on dissection is found to consist of fat enveloping a minute artery and veins, doubtless for the supply of the vena azygos and its pleural investment into which delicate vessels can be traced.

The azygos vein, as it is traced upwards, is seen to incline outwards and backwards from the body of the sixth dorsal vertebra to the head of the fifth rib. There receiving the superior intercostal vein, instead of arching over the bronchus behind the pleura, it enters the free margin of the pleural fold. Describing in it a curve at first forwards and slightly upwards, then directly inwards, over the peduncle of the accessory lobe, it reaches the extra-pericardial portion of the superior vena cava at a point corresponding to the level of the fourth dorsal vertebra.

The arched pleural fold containing the azygos vein in this manner covers in the accessory lobe, and isolates it completely, save at its peduncle, from the upper portion of the lung. Its other relations are not a little interesting. Further inwards than the upper lobe there lie upon it the venous trunks of the cava right innominate and end of the subclavian, the lower part of the trachea with the right bronchus and vagus nerve, and the right portion of the œsophagus: arching over its apex, the subclavian artery (the recurrent nerve as usual winding round it) with its branches the internal mammary and superior intercostal which arise immediately above the vertebral, the first dorsal nerve, and more intimately still the dome-like fascia from the scaleni muscles. It

will be remarked that the trachea is completely separated from the spinal column over a considerable space by the pleural pouch.

H. A. Wrisberg,* nearly a hundred years since, in 1777, laid before the Royal Society of Göttingen an example similar to the one I have detailed. It is, I believe, the earliest recorded example we possess of such a conformation. The observations which he makes are chiefly directed to the condition of the vena azygos, which passed through an accessory fissure in the lung above the right bronchus, isolating a lobe which passed up behind the vein for some distance. The azygos terminated in the highest part of the vena cava. A plate, from which the annexed woodcut is copied, accompanies the description, shewing the condition of the parts.



The next allusion to the subject is to be found in Rokitansky's Pathological Anatomy. In the article upon abnormal conditions of the pleura, he directs attention to what he styles a hitherto unnoticed duplication, depending from the apex of the right pleura, containing the arch of the vena azygos, and dividing the upper lobe into two portions. He adds that such a congenital duplication of the pleura is extremely rare, especially when contrasted with the frequency of similar duplications of the peritoneum.

Bouchaud, in 1862, laid a case of this kind before the Anatomical Society of Paris: and within the last few years four similar instances of accessory lobes above the root of the right lung have been noticed in detail by Dr. Chiene in Edinburgh, Professor Cleland in Galway, and Professor Wenzel Gruber in

* Cf. Observationes Anatomicæ de venâ azygâ duplici aliisque hujus venæ varietatibus in Novis Commentariis Societatis Reg. Scient. Göttingen, anno 1777, tom. viii. p. 14.

St. Petersburg.* Though differing somewhat in size, the accessory lobe in each instance bore the same relations to the lung, pleural fold, and vena azygos.

In all, I have been able to collect only seven cases similar to the preparation I now exhibit. Of these, two have fallen under the notice of the same observer at St. Petersburg, and with this exception they have each occurred in different parts of Europe: three only have been preserved.

Compared with other so-called accessory lobes of the human lungs, the lobe of the vena azygos, owing to its peculiar relations, is much the most interesting, and in not a few respects deserves attentive consideration from anatomists. The consideration of the cause of the constant correlation of the three structures, lung pleura and azygos vein, upon the right side has suggested to me the inquiries—Is such a conformation confined to the right side of the body?—or is it even more common upon the opposite side? I can answer each query in the negative: for Wrisberg's case, to which allusion has been previously made, is further remarkable as furnishing the only recorded instance of an accessory lobe of the left vena azygos. This vessel, which arose from the left renal vein, receiving all the tributaries from the left side of the thorax, formed a venous trunk which, curving through a fissure in the upper part of the left lung above its root, opened into the left vena innominata.† A single anastomotic branch connected the two azygos veins across the fifth dorsal vertebra.

The persistence of the fœtal condition of the left azygos vein in this case is itself remarkably suggestive, viewed in connexion with the mode of formation of the accessory lobe. Professor Cleland has with justice insisted that there must be some very explicit cause for the supplementary lobe upon the right side, and he offers an explanation of the phenomenon which appears to be consonant with the development of the venous system.

* Cf. Bouchaud, Bulletin de la Société Anatomique, 1862, p. 166. Dr. John Chiene, Journal of Anatomy and Physiology, 1869, p. 89. Prof. Cleland, Journal of Anatomy and Physiology, 1870, p. 200. Prof. Wensel Gruber, Zwei Fälle des Vorkommens eines Spitzenlappens an der rechten Lunge, Bulletin de l'Académie Imp. des Sciences de St. Petersbourg, 1870, tom. vii.

† The passage runs thus: "Ut venæ azygos sinistræ progressus versus subclaviam facilior redderetur, lobus pulmonis sinistri superior parili sulco seu potius incisura instructus erat, qualem fusius occasione venæ azygos dextri describemus.

Early in fœtal life, from some such cause as a slight adhesion of the lung to the thoracic wall, or less probably an undue curvature of the embryo, he believes that the azygos vein, as it was drawn downwards at a right angle to its original transverse direction by the descent of the heart, dragged down around it a fold of the pleura, thus deeply notching the lung instead of slipping behind both.

Embryology thus offers the true key to the solution of the difficulty regarding the mode of origin of the lobe. A further confirmation, which I would almost regard as conclusive, of the correctness of this theory is to be found in the consideration of the question regarding the occurrence of the accessory lobe upon the left side. The unique case recorded by Wrisberg, where the left cardinal vein persisted as the left vena azygos retaining its original connexion with the metamorphosed left duct of Cuvier and passed through a fissure above the root of the left lung, has hitherto escaped the observation of embryologists in this connexion. Yet regarded from the common point of similarity of relations venous and pleural, of position, and of origin, with its more frequently found fellow upon the opposite side, it sheds a flood of light upon the whole question. The cause of the greater frequency of the lobe upon the right side can no longer be doubtful; for the persistence of the cardinal vein retaining its original connexion with the duct of Cuvier while the usual arrangement upon the right side of the body upon the left is far from common. Indeed it is only under such an exceptional circumstance as the continuation of the left vena azygos into the innominate that the lobe described by Wrisberg can be expected.

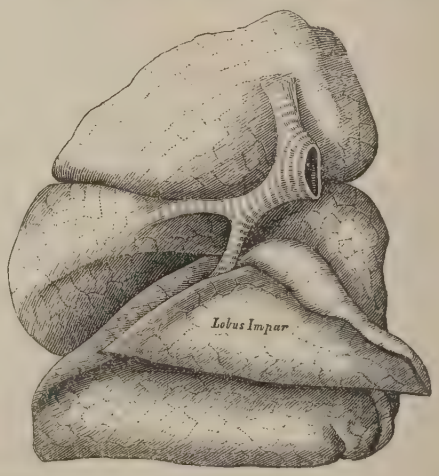
I cannot leave this part of the subject without noticing another point the due significance of which can only properly be appreciated in this connexion. It may be asked what determined the site of reflection of the duplicature externally from the pleura? As the determining limit of the lower margin of the fold was unquestionably the vena azygos, so doubtless the external limit was determined by another vein, the superior intercostal, itself the superior representative and main tributary of the azygos. This it will be seen is no mere theory in the specimen before you, and I direct attention to it here more particularly, as it is a point which has also altogether escaped observation in the seven previously recorded cases, and as in determining the mode

of origin and limits of the accessory lobe it deserves equal consideration with its coadjutor the vena azygos.

A resemblance has been traced by Dr. Chiene between the pleural fold in this case separating the two pulmonary lobes and the falx cerebri. I am inclined to think that a much closer etiological correspondence with the fold in question is to be found in the falciform duplicature of the peritoneum drawn down around the umbilical vein in the originally undivided pleuro-peritoneal cavity.

The lobes occupying the situations I have described on each side of the body are, I am forced to conclude, the only true accessory lobes as yet described in man. Yet other lobes, besides those bearing these definite and constant venous relations, have been recorded. Some of them are not without interest.

Comparative anatomists from the time of Cuvier have shown that a fourth lobe upon the right side, which makes its first appearance in many of the quadrumana, is very constantly found in other mammals occupying a defined position, between the pericardium and diaphragm in the one direction and between the inferior vena cava and spine in the other—a position due to the altered situation of the pericardium. An accessory lobe of the right lung in man lying below the root on the inner side, and bearing a tolerably strict homology to this so-called lobus impar which has been regarded by some zoologists as forming a distinctive difference between the lungs of quadrupeds and man, has been made the subject of a memoir by M. Pozzi.* Its relations to the pericardium unfortunately were not ascertained: but it had an intimate connexion with the inner part of the lower lobe, and rested on the diaphragm.



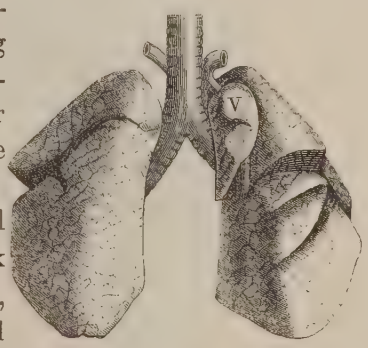
* Cf. Revue d'Anthropologie dirigée par P. Broça, 1872, p. 443.

I have copied the accompanying drawing from M. Pozzi's memoir, as it is one of interest viewed in the light of comparative anatomy, besides affording the only record of a well marked accessory lobe in this situation in man.

A similarly situated accessory lobe upon the left side has been described in detail by Prof. Rektorzik of Vienna.* He premises his remarks by dwelling upon the fact that upon both sides of the body a constant notch is to be seen at the lower part of the posterior border of the lung by which a small tongue-like process is marked off from the inferior lobe. Since my attention has been directed to the point I have not found this arrangement to be constant, though it has been clearly marked in some instances.

The case which Professor Rektorzik has placed on record occurred in a girl who had survived her birth only fourteen days. The accessory lobe was wholly isolated from the remainder of the lung, unexpanded, having no connexion with the bronchus, though it contained collapsed alveoli and bronchia lined with ciliated epithelium. It retained its pleural covering, but was very moveable, being easily displaced into the right side of the thorax. It received a vessel of considerable size from the aorta about the tenth dorsal vertebra, while its vein ran into the demiazygos. The specimen has been preserved in the Anatomical Museum in Vienna.

Yet other so called lobes, marked off by shallow fissures, have been noticed, as in one of the cases recorded by Professor Wensel Gruber, where, besides exhibiting an accessory lobe of the azygos vein, the remainder of the right lung was divided into four lobes by a cruciform fissure as shown in the drawing here copied.

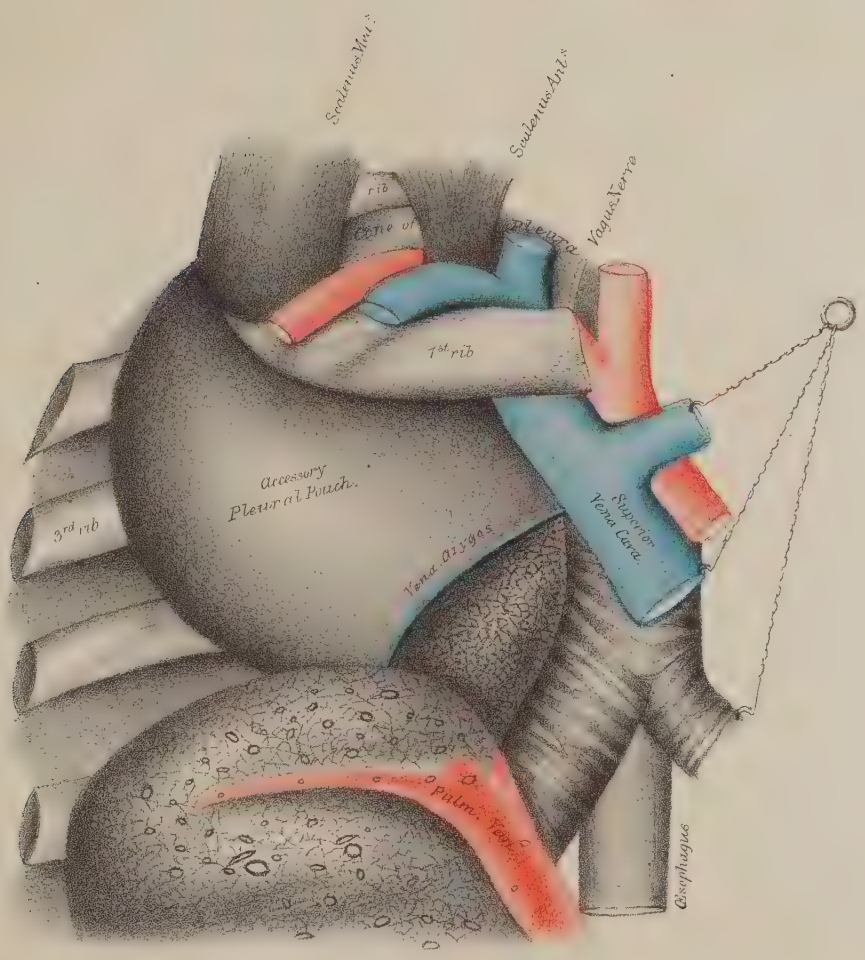


Reviewing these latter cases, I am disposed to think that those given by Pozzi and Rektorzik are but higher developments of notches which, in not a few instances, normally exist. All of them I would regard as merely examples of that redundant lobulation not unfrequently seen in other viscera, and as not meriting the title or consideration of special accessory lobes.

* Cf. Zeitschrift der K. K. Gesellschaft der Aerzte der Wein. 1861, No. I.

In conclusion, let me draw attention to a circumstance which has not escaped the observation of Professor Gruber. In those exceptional cases* where an accessory bronchus has sprung from the trachea at or above the usual bifurcation, and has gone, as it always has, to the upper part of the right lung, no mention has been made of an accessory lobe, which under those circumstances could certainly not have escaped observation. The converse of this statement is equally true in the instances which I have detailed of special accessory lobes.

* Cf. Cruveilhier, Traité d'Anat. Descript. Ed. iii., tom. iii., 1852, p. 491. Leudet, Compte rendu des scéances de la Soc. de biologie ser. 2, tom. iii., 1857, p. 54. Luschka, Die Anat. der Brust des Menschen, Tubingen, 1863, s. 304. Hasse, Henle's Handbuch der Anat. 1866, s. 268



Lung (Section of a portion with Palm.y Vein.)

VIII.—*On certain Symmetric Functions of the Roots of an Algebraic Equation.*

By John C. Malet, M. A.

Read May 25, 1874.

Let $x_1, x_2, x_3 \quad . \quad . \quad . \quad . \quad x_n$ be the roots of the equation

$$x^n + a_1 x^{n-1} + a_2 x^{n-2} + \quad . \quad . \quad . \quad . \quad + a_n = 0 \quad . \quad . \quad . \quad . \quad (1)$$

The following theorem is well known.

$$\begin{vmatrix} 1 & 1 & . & . & . & 1 \\ x_1 & x_2 & . & . & . & x_n \\ x_1^2 & x_2^2 & . & . & . & x_n^2 \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ x_1^{n-2} & x_2^{n-2} & . & . & . & x_n^{n-2} \\ x_1^{n-1} & x_2^{n-1} & . & . & . & x_n^{n-1} \end{vmatrix} = \pm\sqrt{D} \quad . \quad . \quad . \quad (2)$$

where D is the discriminant of equation (1).

If we call the determinant on the left-hand side of equation (2) Δ, I propose to find the value of the symmetric function of the roots of equation (1), formed from Δ by increasing the power of each constituent, of the last p_1 rows by any number m, of the next p_2 rows by $m - 1$, of the next p_3 rows by $m - 2$, and so on, where p_1, p_2, p_3 may be any numbers, their sum being less than n. From the value given for this symmetric function we can find the values of several others, by letting any of the ps be equal to 0. If we introduce a new quantity among the xs, and increase n by unity, we evidently get from (2) the following relation or identity :—

$$\begin{vmatrix} 1 & 1 & 1 & . & . & . & 1 \\ x & x_1 & x_2 & . & . & . & x_n \\ x^2 & x_1^2 & x_2^2 & . & . & . & x_n^2 \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ x^{n-1} & x_1^{n-1} & x_2^{n-1} & . & . & . & x_n^{n-1} \\ x^n & x_1^n & x_2^n & . & . & . & x_n^n \end{vmatrix} =$$

$$\pm \sqrt{D}\{x^n + a_1 x^{n-1} + a_2 x^{n-2} \quad . \quad . \quad . \quad + a_n\} \quad . \quad . \quad . \quad . \quad . \quad (3).$$

By comparing coefficients of x^{n-p_1} on each side of this equation we get the relation

$$\begin{vmatrix} 1 & 1 & . & . & . & 1 \\ x_1 & x_2 & . & . & . & x_n \\ x_1^2 & x_2^2 & . & . & . & x_n^2 \\ . & . & & & & \\ . & . & & & & \\ . & . & & & & \\ x_1^{n-p_1-1} & x_2^{n-p_1-1} & . & . & . & x_n^{n-p_1-1} \\ x_1^{n-p_1+1} & x_2^{n-p_1+1} & . & . & . & x_n^{n-p_1+1} \\ . & . & & & & \\ . & . & & & & \\ x_1^n & x_2^n & . & . & . & x_n^n \end{vmatrix} = \pm a_{p_1}\sqrt{D} \quad . \quad . \quad . \quad (4).$$

The determinant on the left-hand side of this equation is evidently formed from Δ by increasing by unity the power of each constituent in the last p_1 rows.

Let us now denote the symmetric function whose value we are seeking by $F\{p_1, p_2, \quad . \quad . \quad . \quad p_r, m\}$, then equation (4) may be written

$$F\{p_1, 1\} = \pm a_{p_1}\sqrt{D} \quad . \quad . \quad . \quad (5).$$

Now, as before, by introducing an additional x, and increasing n by unity, it is easily seen that $F\{p_1, p_2, 2\}$ must be the coefficient of $x^{n-p_1-p_2}$ in the expression

$$\pm\sqrt{D}\{a_{p_1-1}x - a_{p_1}\}\{x^n + a_1x^{n-1} + a_2x^{n-2} \quad . \quad . \quad . \quad + a_n\}$$

Hence we find

$$F\{p_1, p_2, 2\} = \pm\{a_{p_1+p_2+1}a_{p_1-1} - a_{p_1+p_2}a_{p_1}\}\sqrt{D}$$

$$= \pm\sqrt{D}\begin{vmatrix} a_{p_1+p_2+1} & a_{p_1+p_2} \\ a_{p_1} & a_{p_1-1} \end{vmatrix} \quad . \quad . \quad . \quad (6).$$

For convenience let us call

$$p_1 + p_2 + p_3 \quad . \quad . \quad + p_r, S_r,$$

and relation (6) may be written

$$F\{p_1, p_2, 2\} = \pm\sqrt{D}\begin{vmatrix} a_{(s_2+1)} & a_{s_2} \\ a_{s_1} & a_{(s_1-1)} \end{vmatrix}$$

In a similar manner we find that $F\{p_1, p_2, p_3, 3\}$ is the coefficient of x^{n-s_3} in the expression

$$\{(a_{s_2+1} - xa_{s_2})(a_{(s_1-1)} - xa_{(s_1-2)}) - (a_{s_2} - xa_{(s_2-1)})(a_{s_1} - xa_{(s_1-1)})\}$$
$$\times \{x^n + a_1x^n \quad . \quad . + a_n\}.$$

We easily find

$$F\{p_1, p_2, p_3, 3\} = \pm\sqrt{D}\begin{vmatrix} a_{(s_3+2)} & a_{(s_3+1)} & a_{s_3} \\ a_{(s_2+1)} & a_{s_2} & a_{(s_2-1)} \\ a_{s_1} & a_{(s_1-1)} & a_{(s_1-2)} \end{vmatrix} \quad . \quad . \quad (7)$$

or if $p_3 = 0$ we have

$$F\{p_1, p_2, 3\} = \pm\sqrt{D}\begin{vmatrix} a_{(s_2+2)} & a_{(s_2+1)} & a_{s_2} \\ a_{(s_2+1)} & a_{s_2} & a_{(s_2-1)} \\ a_{s_1} & a_{(s_1-1)} & a_{(s_1-2)} \end{vmatrix}$$

In a similar manner we find the equation

$$F\{(p_1, p_2, p_3, p_4, 4\} = \pm\sqrt{D}\begin{vmatrix} a_{(s_4+3)} & a_{(s_4+2)} & a_{(s_4+1)} & a_{s_4} \\ a_{(s_3+2)} & a_{(s_3+1)} & a_{s_3} & a_{(s_3-1)} \\ a_{(s_2+1)} & a_{s_2} & a_{(s_2-1)} & a_{(s_2-2)} \\ a_{s_1} & a_{(s_1-1)} & a_{(s_1-2)} & a_{(s_1-3)} \end{vmatrix} \quad \ldots (8)$$

from which of course we have

$$F\{p_1, p_2, p_3, 4\} = \pm\sqrt{D}\begin{vmatrix} a_{(s_3+3)}, & a_{(s_3+2)} & a_{(s_3+1)} & a_{s_3} \\ a_{(s_3+2)} & a_{(s_3+1)} & a_{s_3} & a_{(s_3-1)} \\ a_{(s_2+1)} & a_{s_2} & a_{(s_2-1)} & a_{(s_2-2)} \\ a_{s_1} & a_{(s_1-1)} & a_{(s_1-2)} & a_{(s_1-3)} \end{vmatrix}$$

and

$$F\{p_1, p_2, 4\} = \pm\sqrt{D}\begin{vmatrix} a_{(s_2+3)} & a_{(s_2+2)} & a_{(s_2+1)} & a_{s_2} \\ a_{(s_2+2)} & a_{(s_2+1)} & a_{s_2} & a_{(s_2-1)} \\ a_{(s_2+1)} & a_{s_2} & a_{(s_2-1)} & a_{(s_2-2)} \\ a_{s_1} & a_{(s_1-1)} & a_{(s_1-2)} & a_{(s_1-3)} \end{vmatrix}$$

By a precisely similar method we find

$$F(p_1, p_2, \ldots p_m, m) =$$

$$\pm\sqrt{D}\begin{vmatrix} a_{(s_m+m-1)} & a_{(s_m+m-2)} & \cdot & \cdot & \cdot & \cdot & a_{s_m} \\ a_{(s_{m-1}+m-2)} & a_{(s_{m-1}+m-3)} & \cdot & \cdot & \cdot & \cdot & a_{(s_{m-1}-1)} \\ a_{(s_{m-2}+m-3)} & a_{(s_{m-2}+m-4)} & \cdot & \cdot & \cdot & \cdot & a_{(s_{m-2}-2)} \\ \cdot & \cdot & & & & & \cdot \\ \cdot & \cdot & & & & & \cdot \\ \cdot & \cdot & & & & & \cdot \\ a_{s_1} & a_{s_1-1} & & & & & a_{s_1-m+1} \end{vmatrix} \quad \ldots (9)$$

From which we easily get the required formula, viz.,

$$F\{p_1, p_2 \ldots p_r, m\} =$$

$$\pm\sqrt{D}\begin{vmatrix} a_{(s_r+m-1)} & a_{(s_r+m-2)} & \cdot & \cdot & \cdot & \cdot & a_{s_r} \\ a_{(s_r+m-2)} & a_{(s_r+m-3} & \cdot & \cdot & \cdot & \cdot & a_{(s_r-1)} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ a_{(s_r+r-1)} & a_{(s_r+r-2)} & \cdot & \cdot & \cdot & \cdot & a_{(s_r+r-m)} \\ a_{(s_{r-1}+r-2)} & a_{(s_{r-1}+r-3)} & \cdot & \cdot & \cdot & \cdot & a_{(s_{r-1}+r-m-1)} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ a_{s_1} & a_{(s_1-1)} & \cdot & \cdot & \cdot & \cdot & a_{(s_1-m+1)} \end{vmatrix} \quad \ldots (10)$$

If $p_1 = 1$, $p_2 = p_3 = p_r = 0$, we have the formula

$$\begin{vmatrix} 1 & 1 & \cdot & \cdot & \cdot & 1 \\ x_1 & x_2 & \cdot & \cdot & \cdot & x_n \\ x_1^2 & x_2^2 & \cdot & \cdot & \cdot & x_n^2 \\ \cdot & \cdot & & & & \\ \cdot & \cdot & & & & \\ x_1^{n-2} & x_2^{n-2} & \cdot & \cdot & \cdot & x_n^{n-2} \\ x_1^{n+m-1} & x_2^{n+m-1} & & \cdot & \cdot & x_n^{n+m-1} \end{vmatrix} =$$

$$\pm\sqrt{D}\begin{vmatrix} a_m & a_{m-1} & a_{m-2} & \cdot & \cdot & \cdot & a_1 \\ a_{m-1} & a_{m-2} & a_{m-3} & \cdot & \cdot & \cdot & 1 \\ a_{m-2} & a_{m-3} & a_{m-4} & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & & & & \\ \cdot & \cdot & \cdot & & & & \\ a_1 & 1 & 0 & \cdot & \cdot & \cdot & 0 \end{vmatrix} \quad \ldots (11)$$

It is now evident that we can write down in terms of the coefficients the symmetric function of the roots of (1) formed from Δ by the following process :—

Increase the power of each constituent of the last p_1 rows by m_1

" " " " " next p_2 rows by m_2 .

" " " " " . . . .

" " " " " p_r rows by m_r

where $p_1 + p_2 + p_3 \; . \; . \; . \; + p_r \angle n$

and $m_1 \rangle m_2 \rangle m_3 \qquad \rangle m_r.$

This follows at once from formula (10), of which it is only a particular case, by the following process.

Let $p_2, p_3, p_4, \; . \; . \; p_{m_2-m_3}$, each equal 0 in formula (11), and then replace $p_{m_2-m_3+1}$ by p_2; then let the succeeding $(m_2 - m_3 - 1)$ *ps* each equal 0, and call the next one p_3, and so on. It is evident, then, that we have the required result, which can at once be written down in any particular case.

IX.—*Experiments on the Movements of Water in Plants* (Part I.).—By William Ramsay McNab, M. D., Edin., Professor of Botany, Royal College of Science, for Ireland, with Plates XVII. to XXIV.

Read February 9, 1874.

In presenting this first part of my paper recording the experiments in vegetable physiology, for which I received a grant from the Royal Irish Academy last year, it may be as well for me to make a few preliminary observations on these experiments. The chief difficulty I have had to contend with has been the impossibility of obtaining in Dublin, in the same locality, the two essentials for experimenting—namely, a laboratory and a botanical garden. The appliances of a chemical laboratory must be within easy reach of the plants to be experimented on; if not, then errors are sure to be made; and as much time would necessarily elapse between procuring the plant for experiment and the commencement of the experiment itself, the results obtained would certainly be untrustworthy. In fact, the nearer the plants are to the laboratory the better; the results will be more accurate, and the experiments much more easily performed. As it was impossible to obtain in Dublin the necessary laboratory appliances in or near to a botanical garden, I was compelled to seek for a place to experiment in which they could be thus obtained. I here take the liberty of suggesting that some steps might be taken to obtain the necessary laboratory accommodation in the Botanical Garden in this city. A large number of most interesting and valuable experiments might be made if only a few pieces of apparatus could be placed near the plants to be experimented on. A balance, a water-oven, spectroscope, and the like, are essential; while the few chemicals and small pieces of apparatus required could easily be had. There can be little doubt that the reason why so few physiological experiments are made in this country is to be looked for in the absence of the necessary laboratory

accommodation near our gardens. In Germany and France the agricultural stations supply most of the researches in vegetable physiology. Here, however, all depends on private enterprise; and when there is an observer capable of undertaking experiments, he may not be willing to incur the expense of supplying plants and apparatus.

As it was impossible to conduct the experiments I had in view in Dublin, I was compelled to look elsewhere for a suitable place. The series of experiments which I communicated to the Botanical Society of Edinburgh, and which are published in their Transactions, Vol. xi., p. 45, were made at Cirencester, where, at the Royal Agricultural College, there exists an admirable chemical laboratory, close to a small but very convenient botanical garden. I could also obtain a room for laboratory work in the Royal Botanic Garden, Edinburgh. My experiments were made in these two places. The months of August and September, 1873, were devoted to the work; the experiments being made in Edinburgh during August, and in Cirencester during September. In both places, however, my work was seriously hindered by the occurrence of bad weather. Very much of my time was lost in Edinburgh owing to the prevalence of rain, and cold, sunless weather during August. The weather in Cirencester was much more favourable, and after waiting for a fortnight, during which I could do nothing, I was rewarded with ten days' brilliant weather, which enabled me to obtain many very interesting results. Professor Balfour very kindly placed the Histology Class-room, at the Royal Botanic Garden, Edinburgh, at my disposal, and I was able, with some trouble, to convert it into a tolerable laboratory. Professor Crum-Brown, of Edinburgh, also furnished me with a chemical balance and several small pieces of apparatus. I have therefore much pleasure in thanking Professors Balfour and Crum-Brown for their kind help, as without their aid I would not have been able to perform one experiment. I am also under the deepest obligation to my friend and former colleague, Professor A. H. Church, of Cirencester, for the use of his laboratory, and for his constant advice and assistance; while to the Rev. J. Constable, M.A., Principal of the Royal Agricultural College, I am also much indebted.

The experiments here recorded I shall divide into two series. I. Experiments on the water transpired by leaves; and II., on the ascent of water in

the stem. The plants selected for both series of experiments were 1, Cherry Laurel (*Prunus lauro-cerasus*), on which my former experiments were all made; 2. Common Privet (*Ligustrum vulgare*); and 3, Common Elm (*Ulmus campestris, Sm.*)

I.—EXPERIMENTS ON THE WATER TRANSPIRED BY LEAVES.

In the experiments on transpiration recorded in the Transactions of the Botanical Society of Edinburgh, Vol. xi., only the amount of fluid given off in a short time (as an hour) was observed. In the experiments now recorded the branch was kept under observation for a whole day, and in one case for three days, the temperature being recorded every hour, the amount of fluid transpired during that time being carefully determined.

Experiment A 1. Cherry Laurel. August 7, 1873. (Plate XVII., No. 1.)

Weight of branch experimented on, 5.8 grammes.

Weight of whole apparatus at commencement of experiment, 44.87 grammes.

Branch placed in open air (south exposure) at 12 noon. Sky cloudy. Temperature 21° Cent. Removed at 1 p.m. Sun. Temp. 22.5° Cent.

Weight, 44.235 gr. Loss = 0.635 gr., or 10.9 per cent.

Branch replaced in open air (south exposure) at 1.5 p.m.

Removed at 2.5 p.m. Sun. Temp. 25.5° Cent.

Weight, 43.21 gr. Loss = 1.025 gr., or 17.6 per cent.

Branch replaced at 2.10 p.m.

Removed at 3.10 p.m. Sun. Temp. 26.5° Cent.

Weight, 42.615 gr. Loss = 0.595 gr., or 10.02 per cent.

Branch replaced at 3.15 p.m.

Removed at 4.15 p.m. Sky cloudy. Temp. 22° Cent.

Weight 42.22 gr. Loss = 0.395 gr., or 6.81 per cent.

Branch replaced at 4.20 p.m.

Removed at 5.20 p.m. Bright diffused daylight. Temp. 22.5° Cent.

Weight, 41.865 gr. Loss = 0.355 gr., or 6.12 per cent.

The total loss of fluid during this day's experiment was 3.005 gr., or 51.81 per cent. of the weight of the branch employed.

At the conclusion of this experiment the branch was placed close to an east window, where it remained during the night. Next morning it was

weighed at 10.30 a.m., and the experiment continued during the day on the same branch of Cherry Laurel.

Experiment A 4. Cherry Laurel (same branch as A 1). August 8, 1873. (Plate XVIII., No. 1.)

Branch weighed at 10.30 a.m., 40.14 gr. Loss, 1.725 gr., or 29.74 per cent.
Branch placed in open air (south exposure) at 10.30 a.m.
Sky dull. Temp., 18° Cent.
Removed at 11.30 a.m. Faint sunlight. Temp. 19.5° Cent.
Weight 39.855 gr. Loss, 0.285 gr., or 4.91 per cent.
Branch replaced at 11.32.
Removed at 12.32 p.m. Diffused daylight. Temp. 21.5° Cent.
Weight, 39.56 gr. Loss, 0.295, or 5.08 per cent.
Branch replaced at 12.37 p.m.
Removed at 1.37 p.m. Sky dull. Temp., 21° Cent.
Weight, 39.405 gr. Loss, 0.155, or 2.67 per cent.
Branch replaced at 1.39 p.m.
Removed at 2.39 p.m. Sky dull. Temp. 20° Cent.
Weight, 39.29 gr. Loss, 0.115, or 1.98 per cent.
Branch replaced 2.42 p.m.
Removed at 3.42 p.m. Sky dull. Temp. 19.5° Cent.
Weight, 39.195 gr. Loss 0.095, or 1.63 per cent.
Branch replaced at 3.45 p.m.
Removed at 4.45 p.m. Sky dull. Temp. 18.3° Cent.
Weight, 39.115 gr. Loss 0.08, or 1.37 per cent.
Branch replaced at 4.48.
Removed at 5.48. Sky dull (bright in west). Temp. 17.7° Cent.
Weight, 39.025 gr. Loss, 0.09, or 1.55 per cent.

The total loss of fluid during the day was only 1.115 gr., or 19.22 per cent. of the weight of the branch. The light and temperature were, however, very different from the preceding day. The branch was again placed close to an east window, and on being weighed next morning (August 9), at 10 a.m., was found to have lost more than 10 per cent. of water.

Experiment A 11. Cherry Laurel (same branch as A 4). August 9, 1873.

Branch weighed at 10 a.m., 38.425 gr. Loss, 0.6 gr., or 10.34 per cent.

Branch placed in open air, south exposure, at 10.5 a.m.

Sunlight: temp., 22° Cent. Removed at 11.5 a.m. Sky cloudy. Temp. 21° Cent.

Weight, 38.335 gr. Loss, 0.09, or 1.55 per cent.

Branch replaced at 11.8 a.m

Removed at 12.8 p.m. Sun. Temp. 25° Cent.

Weight, 38.235 gr. Loss, 0.1 gr., or 1.72 per cent.

The branch was again replaced, but in a few minutes a sudden gust of wind blew the whole apparatus over, and terminated the experiment.

The total decrease in weight was from 44.87 grammes to 38.235 grammes, the actual loss being thus 6.635 grammes. The branch experimented on had, however, decreased in weight from 5.8 grammes to 5.66 grammes, or lost 0.14 grammes. The total amount of water passed through the branch during the experiment was 6.495 grammes, or 114.3 per cent. of the original weight of branch.

Experiment A 3. Privet. August 7, 1873. (Plate XVIII., No. 2.)

Weight of branch experimented on, 2.52 grammes.

Weight of whole apparatus at commencement of experiment, 21.63 grammes.

Branch placed in open air (south exposure) at 11.50 a.m. Sky cloudy. Temp. 20.5° Cent.

Removed at 12.50. Sun. Temp. 22° Cent.

Weight, 21.455 gr. Loss, 0.175 gr., or 6.94 per cent.

Branch replaced at 1 p.m.

Removed at 2 p.m. Sun. Temp. 25.5° Cent.

Weight, 21.28 gr. Loss, 0.175 gr., or 6.94 per cent.

Branch replaced at 2.5 p.m.

Removed at 3.5 p.m. Sun. Temp. 26.5° Cent.

Weight, 21.145 gr. Loss, 0.135 gr., or 5.35 per cent.

Branch replaced at 3.10 p.m.

Removed at 4.10 p.m. Sky cloudy. Temp. 22° Cent.

Weight, 21.05 gr. Loss, 0.095 gr., or 3.76 per cent.

Branch replaced at 4.15 p.m.

Removed at 5.15. p.m. Bright diffused daylight. Temp. 22.5° Cent.

Weight, 20.955 gr. Loss, 0.095 gr., or 3.76 per cent.

The total loss of weight during the experiment was 0.675 gr., or 26.78 per cent. of the total weight of the branch employed.

Experiment A 6. Privet. August 8, 1873. (Plate XIX., No. 1.)

Weight of branch experimented on, 2.57 grammes.

Weight of whole apparatus at commencement of experiment, 20.75 grammes.

Branch placed in open air (south exposure) at 11.18 a.m. Sky cloudy. Temp. 20° Cent.

Removed at 12.18 p.m. Diffused daylight. Temp. 21.5° Cent.

Weight, 20.57 gr. Loss, 0.18 gr., or 7.0 per cent.

Branch replaced at 12.20 p.m.

Removed at 1.20 p.m. Sky dull. Temp. 21° Cent.

Weight, 20.285 gr. Loss, 0.285 gr., or 11.08 per cent.

Branch replaced at 1.25 p.m.

Removed at 2.25 p.m. Sky dull. Temp. 20° Cent.

Weight, 20.19 gr. Loss, 0.095 gr., or 3.69 per cent.

Branch replaced at 2.27 p.m.

Removed at 3.27 p.m. Sky very dull. Temp. 19.5° Cent.

Weight, 20.12 gr. Loss, 0.07 gr., or 2.72 per cent.

Branch replaced at 3.30 p.m.

Removed at 4.30 p.m. Sky dull. Temp. 18.3° Cent.

Weight, 20.065 gr. Loss, 0.065 gr., or 2.52 per cent.

Branch replaced at 4.32 p.m.

Removed at 5.32 p.m. Sky dull. Temp. 17.7° Cent.

Weight, 20.01 gr. Loss, 0.055 gr., or 2.14 per cent.

The total loss of weight during the experiment was 0.75 grammes, or 29.18 per cent. of the total weight of the branch employed.

Experiment A 9. Privet. August 9, 1873. (Plate XIX., No. 2.)

Weight of branch experimented on, 2.315 grammes.

Weight of whole apparatus at commencement of experiment, 21.115 grammes.

Branch placed in open air (south exposure) at 10.23 a.m. Sky cloudy. Temp. 23.8° Cent.

Removed at 11.23 a.m.

Weight, 20.89 gr. Loss, 0.225 gr., or 9.71 per cent.

Branch replaced at 11.25 a.m.
Removed at 12.25 p.m. Sun. Temp. 28.3° Cent.
Weight, 20.725 gr. Loss, 0.165 gr., or 7.12 per cent.
Branch replaced at 12.30 p.m.
Removed at 1.30 p.m. Sun. Temp. 30.5° Cent.
Weight, 20.58 gr. Loss, 0.135 gr., or 5.83 per cent.
Branch replaced at 1.32 p.m.
Removed at 2.32 p.m. Sky dull. Temp. 21° Cent.
Weight, 20.455 gr. Loss, 0.125 gr., or 5.39 per cent.

Rain began at this time, and terminated the experiment. The total loss of weight during the experiment was 0.65 grammes, or 28.07 per cent of the total weight of the branch employed.

The results of all these experiments can be very readily expressed graphically, the time, the amount of transpiration, and the temperature being indicated. This has been done, and the results are given on Plates XVII., XVIII., and XIX. It will be seen that the transpiration, as a rule, decreases with the decrease of temperature. The decrease and increase of temperature would also, to a certain extent, indicate the changes in the light, as the thermometer rose rapidly when the sun appeared, and fell at once when the sky became cloudy.

Experiment A 2. Elm. August 7, 1873. (Plate XVII., No. 2.)

Weight of branch experimented on, 2.37 grammes.
Weight of whole apparatus at commencement of experiment, 42.88 grammes.

Branch placed in open air (south exposure) at 12.10 p.m. Sky cloudy. Temp. 21° Cent.

Removed at 1.10 p.m. Sun. Temp. 22.5°
Weight, 42.54 gr. Loss, 0.34 gr., or 14.30 per cent.
Branch replaced at 1.12 p.m.
Removed at 2.12 p.m. Sun. Temp. 25.5 Cent.
Weight, 42.17 gr. Loss, 0.37 gr., or 15.61 per cent.
Branch replaced at 2.15 p.m.
Removed at 3.15 p.m. Sun. Temp. 26.5° Cent.
Weight, 41.835 gr. Loss, 0.335 gr., or 14.13 per cent.
Branch replaced at 3.20 p.m.
Removed at 4.20 p.m. Sky cloudy. Temp. 22° Cent.

Weight, 41.575 gr. Loss, 0.26 gr., or 10.97 per cent.

Branch replaced at 4.22 p.m.

Removed at 5.22 p.m. Bright, diffused daylight. Temp. 22.5° Cent.

Weight, 41.325 gr. Loss, 0.25 gr., or 10.54 per cent.

The total loss of weight during the experiment was 1.555 grammes, or 65.61 per cent. of the total weight of the branch employed.

Experiment A 7. Elm. August 8, 1873. (Plate XIX., No. 1.)

Weight of branch experimented on, 1.94 grammes.

Weight of whole apparatus at commencement of experiment, 39.955 grammes.

Branch placed in open air (south exposure) at 11.23 a.m. Sky cloudy. Temp. 20° Cent.

Removed at 12.23 p.m. Bright, diffused daylight. Temp. 21.5° Cent.

Weight, 39.53 gr. Loss, 0.425 gr., or 21.9 per cent.

Branch replaced at 12.25 p.m.

Removed at 1.25 p.m. Sky dull. Temp. 21° Cent.

Weight, 39.285 gr. Loss, 0.245 gr., or 12.68 per cent.

Branch replaced at 1.27 p.m.

Removed at 2.27 p.m. Sky dull. Temp. 20° Cent.

Weight, 39.09 gr. Loss, 0.155 gr., or 7.98 per cent.

Branch replaced at 2.30 p.m.

Removed at 3.30 p.m. Sky very dull. Temp. 19.5° Cent.

Weight, 38.915 gr. Loss, 0.175 gr., or 9.02 per cent.

Branch replaced at 3.32 p.m.

Removed at 4.32 p.m. Sky dull. Temp. 18.3° Cent.

Weight, 38.74 gr. Loss, 0.175 gr., or 9.02 per cent.

Branch replaced at 4.38 p.m.

Removed at 5.38 p.m. Sky dull. Temp. 17.7° Cent.

Weight, 38.575 gr. Loss, 0.165 gr., or 8.5 per cent.

The total loss of weight during the experiment was 1.34 grammes, or 69.07 per cent. of the total weight of the branch employed.

Experiment A 10. Elm. August 9, 1873. (Plate XIX., No. 2.)

Weight of branch experimented on, 21.15 grammes.

Weight of whole apparatus at commencement of experiment, 41.55 gr.

Branch placed in open air (south exposure) at 10.30 a.m. Sky cloudy. Temp. 23.8° Cent.

Removed at 11.30 a.m.

Weight, 41.295 gr. Loss, 0.255 gr., or 11.1 per cent.

Branch replaced at 11.32 a.m.

Removed at 12.32 p.m. Sun. Temp. 28.3° Cent.

Weight, 41.065 gr. Loss, 0.23 gr., or 10.82 per cent.

Branch replaced at 12.34 p.m.

Removed at 1.34 p.m. Sun. Temp. 30.5° Cent.

Weight, 40.87 gr. Loss, 0.195 gr., or 9.21 per cent.

Branch replaced at 1.36 p.m.

Removed at 2.36 p.m. Sky dull. Temp. 21° Cent.

Weight, 40.725 gr. Loss, 0.145 gr., or 6.85 per cent.

The experiment here terminated, as rain began to fall. The total loss of weight during the experiment was 0.825 grammes, or 39.00 per cent of the total weight of the branch employed.

The results of these experiments are also represented graphically in the same manner as the others. In the transpiration experiments it is always necessary to employ comparatively small branches, because large ones cannot be easily placed in the pan of the balance ; and I prefer the use of the balance, because in my former experiments the results so obtained were much more accurate than when the moisture was determined with calcium chloride or by sulphuric acid tubes.

In my experiments communicated to the Edinburgh Botanical Society, I found that the water in plants existed both as transpirable water and cell-sap water, and that the proportion was as follows :—

Transpirable water, . .	7, or	6 per cent.
Cell-sap water, ...	56, or	57 per cent.
Dry substance of plant, .	37.	
	100.	

In subsequent experiments I have tried to confirm this, but not altogether with success. In Experiment A 5, the lithium rose only four inches in ten minutes, the transpiration at the time, as shown by A 1, being at the rate of

4.91 per cent. per hour. This experiment is so far confirmatory, because as the transpiration was small, the lithium had not passed through the whole branch. Experiment B 3 was also confirmatory. The transpiration was at the rate of 9.06 per cent. per hour; the lithium had passed through the whole branch. In Experiments B 2, B 4, B 6, B 7, the results are not what I expected. The lithium was all through the branches, but they had transpired: B 2, 0.63 per cent.; B 4, 0.023 per cent.; B 6, 2.89 per cent.; B 7, 2.87 per cent.

In none of the experiments was the actual quantity of water present in the plant determined. This I believe will form a very important element in all experiments, because if the maximum quantity be present, then when seven per cent. has been transpired the whole of the transpirable fluid would be removed; but suppose the maximum quantity was not present, and I have reason to believe that the transpirable fluid is a variable quantity, then the result would be very different. If no transpirable fluid were present, the seven per cent. would have to be replaced before it would transpire.

In a series of experiments made in February and March, 1871, I found that the fluid in the plant (cherry laurel) had decreased. Thus, on February 28, 1871, no transpiration; the fluid in the leaves, 57.2 per cent. On March 6, 1871, no transpiration; the fluid in the leaves, 57.1 per cent. Another experiment was made on April 4, 1871, when the reserve materials in the cherry laurel leaves were passing over into the stem, and the leaves were beginning to change before falling, and the fluid found was only 47.3 per cent.

The amount of transpirable water will probably vary in different plants, and as yet it is only in the cherry laurel that I have attempted to determine its amount. If a further series of experiments were made in this direction, it would be necessary to determine the amount of fluid transpired, the distance the fluid replacing that transpired had moved (by lithium and the spectroscope), and lastly to determine accurately the amount of fluid in the plant at the different times the experiments were made. It could then be easily ascertained whether the maximum quantity of fluid was present or not, and if there was a deficiency, that quantity would be accurately determined, and its effect, if any, on the transpiration carefully noted.

II.—ASCENT OF WATER IN THE STEM.

Many experiments have been made to determine the channel by which fluid ascends in the stem. Coloured solutions of various kinds, iron solutions, &c., have all been employed. The only experiments made to determine the actual rate of ascent are those of Sachs and myself. Sachs made a single experiment on the silver poplar, and obtained a rate of nine inches per hour. In my own experiments, recorded in the Transactions of the Botanical Society, of Edinburgh, Vol. xi., p. 45, I obtained by means of lithium citrate, thallium citrate, and the spectroscope, a maximum speed of twenty-four inches per hour. Sachs, in his *Lehrbuch der Botanik*, ed. 3, page 590, considers that the results obtained are too small, and in the experiments I have now to record I have been able to obtain much higher results.*

Experiments were made on the same plants on which the transpiration experiments were made—privet, elm, and cherry laurel.

A.—Ascent of Fluid in the Privet.

Experiment A 8. Privet. August 8, 1873.

A branch of privet, 10.7 inches long, was allowed to absorb a solution of carbonate of lithium for ten minutes, at 11.28 a.m. The temperature was 20° Cent., there being no sun, but bright, diffused daylight. On examination with the spectroscope, it was found that the lithium had only gone up one inch in the ten minutes, or at the rate of six inches per hour. The lithium had not entered the leaves, as it had not reached the first node. The rate of transpiration during the time of experiment, as determined by Experiment A 6, was seven per cent. per hour.

The privet is not well adapted for the purpose of experimenting, because it very rapidly becomes flaccid. I have no doubt that the absence of root pressure is the cause of the branch becoming flaccid.

B.—Ascent of Fluid in the Elm.

Experiment A 12. Elm. August 9, 1873.

A branch of elm, 8.6 inches long, was allowed to absorb a solution of car-

* Note added June 2nd, 1874.—Since this Paper was read before the Royal Irish Academy I have seen a notice of experiments by Pfitzer, in Botanische Zeitung (17th April, 1874), p. 248.

bonate of lithium for ten minutes, at 11.33 a.m. The temperature was 24.5° Cent., the sky being sunny, with occasional clouds. The lithium was found to have gone up nearly to the first leaf, or 2.6 inches, being at the rate of 15.6 inches per hour. The transpiration during the time of experiment, as determined by Experiment A 10, was at the rate of 10.82 per cent. per hour.

The experiments made with privet and elm cannot be considered as altogether quite satisfactory. In both plants, even after a few minutes' exposure, the leaves and stem became flaccid. This was undoubtedly due to the plant giving off more fluid than it was able to take up, probably from the absence of root-pressure, which, under ordinary circumstances, does exercise a very considerable influence on the ascent of fluid in the stem. For this reason the experiments on elm and privet were very soon discontinued, and the cherry laurel alone used.

C.—Ascent of Fluid in the Cherry Laurel.

First Series.—Edinburgh Experiments.

Experiment A 5. Cherry Laurel. August 8, 1873.

A branch of cherry laurel, 5.4 inches long, and bearing six leaves, was allowed to absorb a solution of carbonate of lithium for ten minutes. The experiment was commenced at 10.57 a.m., the sky being dull, and the temperature 20° Cent. At the end of the experiment the thermometer had fallen to 18.8° Cent. On examination with the spectroscope the lithium was not observed in the leaves, but had risen four inches in the stem, or at the rate of twenty-four inches per hour. This was nearly equal to the maximum speed obtained in my former experiments (Transactions of the Botanical Society, Edinburgh, Vol. xi., p. 61), on October 10, 1870.

The transpiration during the Experiment A 5 was, as determined by Experiment A 4, at the rate of 4.91 per cent. per hour.

Experiment A 13. Cherry Laurel. August 9, 1873.

No lithium was taken up in this experiment. The sky was a little cloudy, but the wind was high, and often blew the branch out of the tube containing the lithium solution.

Experiment A 14. Cherry Laurel. August 15, 1873.

The experiments, which were completely suspended by the occurrence of wet weather, were recommenced on the 15th. A branch of laurel, with three leaves, and measuring 5.6 inches in length, was placed in solution of carbonate of lithium at 11.33 a.m. The sky was sunny, with thin clouds; the temperature during the whole experiment was 26.5. After absorbing the lithium solution for ten minutes it was tested by the spectroscope in the usual way, and was found to have ascended 2.2 inches in ten minutes, or at the rate of 13.2 inches per hour.

Experiment A 15. Cherry Laurel. August 15, 1873.

A branch measuring 4.8 inches in length, and having three leaves, was placed in solution of lithium carbonate at 11.50 a.m, and allowed to absorb the solution for ten minutes; the sky sunny, with thin clouds, and the temperature 26° Cent. The lithium was found to have risen 3.1 inches in ten minutes, or at the rate of 18.6 inches per hour.

In none of these experiments had the lithium entered the leaves, and I am not able as yet satisfactorily to explain how in some cases the lithium rapidly enters the leaves, while in others it remains in the stem, not even entering the petiole.

Second Series.—Cirencester Experiments.

These experiments I shall group under six heads:—1st. Rapidity of ascent of fluid in *a*, sun, *b*, diffused daylight, and *c*, darkness; 2nd, Rapidity of ascent in branches which had been cut in the dark; 3rd, Rapidity of ascent in a branch with the cortical tissues removed; 4th, Rapidity of ascent in branches from which the leaves had been removed; 5th, Rapidity of absorption of lithium when applied at the apex of the branch; and 6th, Rapidity of ascent when the fluid is taken up under pressure of mercury, intended to represent the root-pressure of the plant.

In these experiments many very interesting results were obtained. In the first place the greatest recorded rapidity of ascent was observed in Experiments B 8, and B 10. In the former the rate of ascent was 39.9 inches per hour; in the latter, 40.5 inches per hour. The two experiments were made on the same day, one after the other, and the close agreement in the two results

is noticeable. In the second place, a most curious and unexpected result was obtained, namely, that when two branches were cut under the same circumstances, and in bright sunshine, but allowed to absorb lithium solution, the one in sunlight, the other in darkness, the rate of ascent was nearly the same in both, while the transpiration was almost entirely stopped in the latter. In the first experiment with the branch in sunshine (B 8), the fluid transpired was 2.42 per cent of the weight of the branch employed, the lithium rising 39.9 inches; in the second (B 9), the branch only transpired 0.27 per cent. by weight of fluid, while the lithium had risen 34.8 inches in the stem and leaf. In the stem alone the results were still more remarkable. In B 8 the lithium had only risen 16.5 inches in the stem, while in B 9 it had risen 27.6 inches. The activity of the leaves in B 8 would easily account for the difference, because the lithium fluid would be taken to the leaves to supply the place of fluid transpired, and thus be prevented ascending the stem so rapidly. The influence of light and heat on the opening and closing of the stomata is already well known,* while its influence on transpiration is very well marked. It is, however, of the highest interest and importance that the great and rapid stream of fluid up the stem is not so easily influenced, and fluctuations in the amount of light do not cause corresponding fluctuations in the amount of fluid and the rapidity of the stream through the wood-portion of the stem. In the third place it was found, contrary to expectation, that after a bright warm day a branch cut in the darkness was still able to absorb lithium solution; and although the rapidity of the current was much reduced, from 7 to 12 inches per hour, still the darkness had not brought with it entire stagnation of the current. This is probably to be accounted for by the leaves parting with more water during the day by transpiration than was supplied by the rapid current. Hence the defect would have to be made up by the upward current of water through the stem to the leaves. This current would go on until the equilibrium was obtained, when it would cease only to be again set in motion when the early morning sun again caused the transpiration to become active.

* See Dr. Czech, Bot. Zeitung, 1859, Ueber den Einfluss des Lichtes auf das Oeffnen und Schliessen der Stomata: and Dr. C. Zingeler, Die Spaltoeffnungen der Carices, in Pringsheims Jahrbuch, Vol. ix., p. 144.

The removal of the cortical tissues does not influence the current, as it is through the wood-portion of the fibro-vascular bundles that the rapid current ascends. In branches deprived of leaves the lithium solution ascended in one 14.7 inches per hour, in the other 13.2 inches per hour. In both cases the fluid could only be taken up to supply the deficiency caused by transpiration before the experiment began. The stem was found to conduct lithium solution rapidly in both directions, as the solution applied at the top in sunshine moved 12.5 inches per hour.

Lastly, an experiment was made in which pressure of mercury was applied to imitate the root-pressure in plants. As a result the fluid was found to move with considerable rapidity through the stem, but not so markedly in the leaves.

1. *Rapidity of Ascent of Fluid in the Sun, diffused Daylight, and in Darkness :*

A—In the Sun.

Experiment B 3. Cherry Laurel. September 22, 1873. (Plate XX., fig. 1.)

A branch measuring 4.35 inches in length, with four leaves, and weighing 9.485 grammes, was allowed to absorb lithium, at 10.45 a.m., for thirty minutes in the sun, at a temperature of 18° Cent. On examination the lithium was found throughout the whole stem and leaves. It had thus risen 4.35 inches in the stem, and 4.75 inches in the upper leaf. This gives the rate in the stem as 8.7 inches per hour ; in the stem and leaf together at 18.2 per hour. During the experiment the transpiration was noted. At the commencement the branch, lithium solution, &c., weighed 34.18 grammes ; at the termination, 33.75 grammes. The loss of fluid was therefore 0.43 grammes, or 4.53 per cent. of the weight of the branch employed.

Experiment B 8. Cherry Laurel. September 23, 1873. (Plate XXI., fig. 4.)

A branch measuring 3.7 inches in length, and having four leaves, was allowed to absorb a solution of lithium citrate for ten minutes in the sun, at 10.5 a.m. The branch weighed 6.575 grammes, the whole, with the lithium solution, weighing, at the commencement of the experiment, 31.315 grammes.

After an exposure of ten minutes in the sun, temp. 23° Cent., the whole weighed 31.155 grammes, showing a loss by transpiration of 0.16 grammes, or 2.42 per cent of weight of branch used. On examination with the spectroscope the lithium was found to have risen in the stem 2.75 inches, or 16.5 inches per hour, while in the stem and leaf it had risen 6.65 inches in ten minutes, or at the rate of 39.9 inches per hour. If the transpiration had continued the same for a whole hour, the rate would have been 14.52 per cent. of weight of branch employed.

Experiment B 10. Cherry Laurel. September 23, 1873. (Plate XXII., fig. 5.)

A branch 26 inches in length was placed in solution of lithium citrate for twenty minutes, in the sun, the temperature being 28° Cent., the experiment commencing at 10.30 a.m. On examination with the spectroscope the lithium was found to have ascended eight inches in the stem in twenty minutes, or at the rate of 24 inches per hour, while in the stem and leaf it had gone up no less than 13.5 inches, or at the rate of 40.5 inches per hour. This was the highest result obtained, and is the most rapid ascent recorded. Taking the former experiment, B 8, which at the same time, and under similar circumstances, gave 39.9 inches per hour in stem and leaf; and comparing it with B 10, we find the results are singularly close, and I think fully warrant us in believing the method employed to be accurate, and the results obtained quite trustworthy. The mean of the two experiments would give 40.2 inches per hour.

B.—In diffused Daylight.

Experiment B 12. Cherry Laurel. September 25, 1873. (Plate XX., fig. 4.)

A branch weighing 8.695 grammes was allowed to absorb solution of lithium citrate, at 10.22 a.m., for ten minutes, the temperature being 14° Cent. The whole apparatus and branch weighed 41.975 grammes at the beginning of the experiment. At the conclusion the weight was 41.92 grammes, the loss by transpiration being .055 grammes, or 0.63 per cent. of weight of branch employed. On examination with the spectroscope the lithium was found to have ascended in ten minutes 2.85 inches in the stem, or 17.1 inches per hour, and 3.55 inches in stem and leaf, or 21.3 inches per hour.

Experiment B 14. Cherry Laurel. September 25, 1873. (Plate XXIV., fig. 1.)

A branch with ten leaves, and measuring 13.65 inches in length, was allowed to absorb solution of lithium citrate in bright diffused daylight, at 10.55 a.m., the temperature being then 14° Cent. The experiment was continued for half an hour. During the last eight minutes of the experiment, the temperature rose to 16° Cent. and then to 20° Cent. The branch was removed at 11.20 a.m., cut in pieces, and dried in the usual way in the water-oven. The lithium was found to have ascended 11.4 inches in the stem in thirty minutes, or at the rate of 22.8 inches per hour, and in the stem and leaf, 11.65 inches, or 23.3 inches per hour.

Experiment B 18. Cherry Laurel. September 30, 1873. (Plate XXIV., fig. 2.)

A branch measuring 17.5 inches in length, and having 13 leaves, was allowed to absorb solution of lithium citrate for 30 minutes, in diffused daylight. The experiment began at 9.15 a.m., with a temperature of 12° Cent., the thermometer indicating the same temperature throughout the whole experiment. On examination with the spectroscope the lithium was found to have risen 12.4 inches in the stem, or 24.8 inches per hour, while it had only risen 11.05 inches in the stem and leaf, or 22.1 inches per hour.

C.—In Darkness.

The experiments made in darkness may be divided into two series, one in which the absorption of lithium solution went on for many hours during the night; the second in which the lithium was absorbed only for a short time in artificial darkness during the day.

Experiment B 1. Cherry Laurel. September 19, 1873. (Plate XXII., fig. 3.)

A branch of cherry laurel, five inches long, with four leaves, and weighing 8.94 grammes, was allowed to absorb lithium for 17½ hours. On examination with the spectroscope the lithium was found throughout the whole branch.

Experiment B 2. Cherry Laurel. September 19, 1873. (Plate XXII., fig. 4.)

A branch of cherry laurel, 4.35 inches long, with four leaves, and weighing 5.955 grammes, was allowed to absorb solution of lithium citrate for 17½ hours. The whole weighed at the commencement of the experiment 39.555 grammes, and at the termination, 39.517 grammes, the loss by transpiration during the experiment, at a temperature of 13° Cent., being 0.038 gramme, or 0.63 per cent. of weight of the branch. On examination with the spectroscope the lithium was found throughout the whole.

Experiment B 4. Cherry Laurel. September 22, 1873. (Plate XX., fig. 2.)

A branch measuring 4.2 inches in length, with four leaves, was allowed to absorb solution of lithium citrate, at 10.50 a.m., in darkness, for thirty minutes. The branch weighed 8.665 grammes, the whole before experiment weighing 42.43 grammes, after, 42.41 grammes, the loss by transpiration during the half-hour being .02 gramme, or 0.023 per cent. of weight of branch. On examination with the spectroscope the lithium was found throughout the whole branch.

Experiment B 9. Cherry Laurel. September 23, 1873. (Plate XXII., fig. 1.)

A branch measuring 4.6 inches in length, with four leaves, was allowed to absorb solution of lithium citrate at 10.23 a.m., being placed in darkness at a temperature of 15° Cent. The branch weighed 8.975 grammes, the whole apparatus at beginning of experiment weighing 42.785 grammes. In ten minutes the whole weighed 42.76 grammes, the loss being thus .025 gramme, or 0.27 per cent. of weight of branch employed. On examination with the spectroscope the lithium was found to have risen 4.6 inches in the stem, or 27.6 inches per hour, and in stem and leaf, 5.8 inches, or 34.8 inches per hour.

Experiment B 11. Cherry Laurel. September 23, 1873. (Plate XX., fig. 3.)

A branch of cherry laurel, 5.1 inches long, with four leaves, was allowed

to absorb solution of lithium citrate for ten minutes in darkness, at 3.12 p.m. On examination the lithium was found to have risen 3.3 inches in the stem, or 19.8 inches per hour, and in stem and leaf, 4.75 inches in ten minutes, or 28.5 inches per hour.

Experiment B 13. Cherry Laurel. September 25, 1873. (Plate XXII., fig. 2.)

A branch of cherry laurel, measuring 5.4 inches in length, and having four leaves, was allowed to absorb solution of lithium citrate for ten minutes in the dark. The branch weighed 10.525 grammes, the whole apparatus at the commencement of the experiment weighing 34.73 grammes. The temperature during the experiment was 13° Cent. At the end of the experiment the whole weighed 34.71 grammes, indicating a loss of 0.02 grammes by transpiration, or 0.018 per cent. of total weight of branch. On examination by the spectroscope the lithium was found to have risen 3.1 inches in the stem, or 18.6 inches per hour, and in stem and leaf, 3.35 inches in ten minutes, or 20.1 inches per hour.

2.—*Rapidity of Ascent in Branches cut in the Dark.*

Experiment B 5. Cherry Laurel. September 22, 1873. (Plate XXI., fig. 2.)

A branch, 4.25 inches long, with five leaves, was cut in the garden at 8.25 p.m., when it was quite dark. It was placed in solution of lithium citrate at 8.30, and allowed to absorb the solution for thirty minutes. On examination with the spectroscope the lithium was found to have risen 4.25 inches in the stem, or 8.5 inches per hour, and 5.45 inches in stem and top leaf, or 10.9 inches per hour.

Experiment B 6. Cherry Laurel. September 22, 1873. (Plate XXI., fig. 1.)

A branch, 4.15 inches long, with four leaves, and weighing 7.775 grammes, was allowed to absorb lithium solution for twelve hours, in darkness. The whole apparatus at the commencement of the experiment weighed 41.33 grammes. Next morning, when weighed, it was found to be 41.105 grammes, the amount of fluid transpired being .225 grammes, or 2.89 per cent. of weight of branch.

On weighing the branch after the experiment, it was found to be 7.795 grammes, having thus gained .02 gramme for twelve hours. On examination with the spectroscope the lithium was throughout the whole, or 4.15 inches in the stem, and 7.9 inches in stem and leaf.

Experiment B 7. Cherry Laurel. September 22, 1873. (Plate XXI., fig. 3.)

A branch, 4.35 inches long, and weighing 9.9 grammes, was allowed to absorb solution of lithium citrate for twelve hours, in the dark. The whole weighed 34.39 grammes at the commencement of the experiment. After the experiment the whole weighed 34.105 grammes, the loss by transpiration being thus .285 gramme, or 2.87 per cent. of weight of branch. The branch, like the preceding, was found to have gained weight, it being at the end of the experiment 9.955 grammes, or .055 gramme heavier. On examination with the spectroscope the lithium was found throughout the whole. The gain in weight points to deficiency in the fluid in the branch, which was made up by absorption during the night, when transpiration is exceedingly slow.

3.—*Rapidity of Ascent in a Branch with the Cortical Tissues removed.*

In order to observe the effect of the removal of the cortical tissues of the stem a single experiment was made. It is generally known that it is through the wood portion of the fibro-vascular bundle that the fluid of the rapid current moves, hence it was not expected that the removal of the cortical tissues would influence the rapidity of the current.

Experiment B 15. Cherry Laurel. September 25, 1873. (Plate XXIII., fig. 1.)

A branch of cherry laurel, 13.5 inches long, and having eleven leaves, was taken and the cortical tissues and phlœm portion of the fibro-vascular bundles removed throughout the whole of the first internode. It was then allowed to absorbed lithium for thirty minutes in bright diffused light; temperature 14° Cent. On examination it was found that the lithium had ascended 7.4 inches, or at the rate of 14.8 inches per hour in the stem; while in stem and leaf it had ascended 10.55 inches, or at the rate of 21.1 inches per hour.

4.—*Rapidity of Ascent in Branches with the Leaves removed.*

Two experiments were made to determine the rate of ascent in a branch deprived of its leaves. One experiment was made in sunlight, the other when there was only diffused daylight.

Experiment B 16. Cherry Laurel. September 27, 1873. (Plate XXII., fig. 6.)

A branch measuring 16.55 inches was taken, and deprived of leaves, the buds remaining intact. It was allowed to absorb solution of lithium citrate for thirty minutes in the sun, the temperature being 27° Cent. On examination with the spectroscope the lithium was found to have ascended 7.35 inches, or at the rate of 14.7 inches per hour.

Experiment B 19. Cherry Laurel. September 30, 1873. (Plate XXII., fig. 7.)

A branch measuring 16.85 inches, deprived of leaves, but with buds intact, was allowed to absorb solution of lithium citrate for thirty minutes in diffused daylight, the temperature being 12° Cent. On examination the lithium was found to have risen 6.6 inches, or at the rate of 13.2 inches per hour.

5.—*Lithium Solution applied at the top of the Branch.*

One experiment was made to discover whether the lithium solution would move as rapidly from apex to base as from base to apex. A branch was taken, and the terminal bud removed. A test tube with solution of lithium was then applied, and on testing, the lithium was found in all the leaves.

Experiment B 17. Cherry Laurel. September 27, 1873. (Plate XXIII., fig. 2.)

A branch of cherry laurel, 7.45 inches in length, and having five leaves, was allowed to absorb solution of lithium citrate for thirty minutes in the sun, the temperature being 27° Cent., the time 10.51 a.m. The lithium was applied at the top, and on examination it was found to have descended 6.25 inches in thirty minutes, or at the rate of 12.5 inches per hour. In the upper

leaf it had risen 4 inches, in the second, 4.25 inches, in the third, 4 inches, in the fourth, 1.6 inches, and in the fifth or lowest, only 0.25 inch.

6.—*Lithium Solution supplied under Pressure of Mercury.*

It had been observed that in the experiments on privet and elm that the branches always became flaccid. This was due to the diminution in tension caused by the leaves transpiring more fluid than was supplied through the stem. In the cherry laurel the stem and leaves are so rigid that a small diminution in the quantity of water does not cause any very marked flaccidity. In experimenting on cut branches the very important element of root-pressure cannot be taken into account, and the experiments are thus, to a certain extent, incomplete.

A series of experiments were devised, in which a given pressure of mercury was to be applied to one series of branches, while no pressure was exerted on others. The weather, however, put an end to the experiments after a single one had been completed.

Experiment B 20. Cherry Laurel. September 30, 1873. (Plate XXIV., fig. 3.)

A branch, 13.6 inches in length, and having ten leaves, was taken and fixed in the short arm of a bent glass tube. Into this tube a quantity of solution of lithium citrate was introduced, and then a quantity of mercury. The mercury was poured in at 11.7 a.m., the whole exposed in the open air, on a dull day, at 11.10 a.m., the temperature being 14° Cent. The branch was removed at 11.40 a.m., as it then began to rain, the temperature being now 13.5° Cent. In all 143.25 grammes of mercury were employed, but as 32.72 grammes occupied the bend of the tube, the pressure was consequently exerted by 110.53 grammes of mercury. The mercury fell in the tube 0.25 of an inch during the thirty minutes the experiment lasted.

On testing with the spectroscope the lithium was found to have risen almost to the apex of the branch, only 0.25 of an inch being free from it, and the topmost leaf. The lithium therefore rose 13.35 inches in thirty minutes, or at the rate of 26.7 per hour. In the second-last leaf the lithium had risen 0.65 of an inch, making a total in stem and leaf of 14.0 inches in thirty minutes, or 28 inches per hour in a dull day, with the temperature under 14° Cent.

GENERAL CONCLUSION AND SUMMARY.

1. That under favourable circumstances a rate of ascent of 40 inches per hour can be obtained.

2. That, contrary to the generally received opinion, direct experiment has shown that the upward rapid current of water does not cease in the evening.

3. That checking the transpiration for a short time, by placing the branch in darkness, does not materially retard the rapid current of water.

4. That the removal of the cortical tissues does not impede the rapid current in the stem, which moves only through the wood (xylem) portion of the fibro-vascular bundles.

5. That a well-marked rapid flow of fluid will take place in a stem after the removal of the leaves.

6. That fluid will rapidly flow downwards as well as upwards in the wood (xylem) portion of the fibro-vascular bundles, as seen in a branch in which lithium solution was applied at the top.

7. That pressure of mercury does not exert any very marked influence on the rapidity of the flow, in the one experiment made with a pressure of 110.53 grammes of mercury.

RECAPITULATION OF EXPERIMENTS.

Experiment A 1. August 7, 1873. Transpiration. Cherry Laurel. Total amount transpired, 51.81 per cent. in five hours.

Experiment A 2. August 7, 1873. Transpiration. Elm. Total amount transpired, 65.61 per cent. in five hours.

Experiment A 3. August 7, 1873. Transpiration. Privet. Total amount transpired, 26.78 per cent. in five hours.

Experiment A 4. August 8. 1873. Transpiration. Cherry Laurel. Total amount transpired, 29.18 per cent. in six hours.

Experiment A 5. August 8, 1873. Ascent of fluid with lithium carbonate. Cherry Laurel. Four inches in ten minutes. 10.57 a.m. Temperature 20° Cent.

Experiment A 6. August 8, 1873. Transpiration. Privet. Total amount transpired, 29.18 per cent. in 6 hours.

Experiment A 7. August 8, 1873. Transpiration. Elm. Total amount transpired, 69.07 per cent. in six hours.

Experiment A 8. August 8, 1873. Ascent of fluid with lithium carbonate. Privet. One inch in ten minutes. 11.28 a.m. Temp. 20° Cent.

Experiment A 9. August 9, 1873. Transpiration. Privet. Total amount transpired, 28.07 per cent. in four hours.

Experiment A 10. August 9, 1873. Transpiration. Elm. Total amount transpired, 39.0 per cent in four hours.

Experiment A 11 August 9, 1873. Transpiration. Cherry Laurel. Total amount transpired, 3.27 per cent in two hours.

Experiment A 12. August 9, 1873. Ascent of fluid with lithium carbonate. Elm. 2.6 inches in ten minutes. 11.33 a m. Temp. 24.5° Cent.

Experiment A 13. August 9, 1873. Ascent of fluid with lithium carbonate. Cherry Laurel. No lithium found.

Experiment A 14. August 15, 1873. Ascent of fluid with lithium carbonate. Cherry Laurel. 2.2 inches in ten minutes. 11.33 a.m. Temperature 26.5° Cent.

Experiment A 15. August 15, 1873. Ascent of fluid with lithium carbonate. Cherry Laurel. 3.1 inches in ten minutes. 11.50 a.m. Temperature 26° Cent.

Experiment B 1. September 19, 1873. Ascent of fluid with lithium citrate. Cherry Laurel. In darkness for 17½ hours. Temperature 13° Cent.

Experiment B 2. September 19, 1873. Ascent of fluid with lithium citrate, and transpiration. Cherry Laurel. In darkness for 17½ hours. Temperature 13° Cent. Transpiration in 17½ hours, 0.63 per cent.

Experiment B 3. September 22, 1873. Ascent of fluid with lithium citrate, and transpiration. Cherry Laurel. In sun 30 minutes. Temperature 18° Cent. Transpired, 4.53 per cent.

Experiment B 4. September 22, 1873. Ascent of fluid with lithium citrate, and transpiration. Cherry Laurel. In darkness 30 minutes. Transpired 0.023 per cent.

Experiment B 5. September 22, 1873. Ascent of fluid with lithium citrate. Cherry Laurel. Branch cut in dark ; fluid absorbed in darkness, 30 minutes. 8.30 p.m. Temperature not noted. Ascended in stem 4.25 inches ; in stem and leaf, 5.45 inches, or 8.5 and 10.9 inches per hour.

Experiment B 6. September 22, 1873. Ascent of fluid with lithium

citrate. Cherry Laurel. Branch cut in dark. Fluid absorbed in darkness in twelve hours—in stem, 4.15 inches; in stem and leaf, 7.9 inches. Fluid transpired, 2.89 per cent.

Experiment B 7. September 22, 1873. Ascent of fluid with lithium citrate. Cherry Laurel. Branch cut in dark. Fluid absorbed in darkness in twelve hours—in stem, 4.35 inches; in stem and leaf, 7.85 inches. Transpired, 2.87 per cent.

Experiment B 8. September 23, 1873. Ascent of fluid with lithium citrate. Cherry Laurel. In sun, temperature, 23° Cent., for ten minutes. Ascended 2.75 inches in stem, 6.65 inches in stem and leaf; or 16.5 inches per hour in stem, and 39.9 inches per hour in stem and leaf.

Experiment B 9. September 23, 1873. Ascent of fluid with lithium citrate, and transpiration. Cherry Laurel. In darkness ten minutes. Temperature, 15° Cent. Transpired, 0.27 per cent.

Experiment B 10. September 23, 1873. Ascent of fluid with lithium citrate. Cherry Laurel. Twenty minutes in sun. Temperature, 28° Cent. Lithium rose in stem 24 inches per hour; in stem and leaf, 40.5 inches.

Experiment B 11. September 23, 1873. Ascent of fluid with lithium citrate. Cherry Laurel. Ten minutes in darkness.

Experiment B 12. September 25, 1873. Ascent of fluid with lithium citrate, and transpiration. Ten minutes' diffused light. Temperature, 14° Cent. Transpired 0.63 per cent.

Experiment B 13. September 25, 1873. Ascent of fluid with lithium citrate, and transpiration. Ten minutes in darkness. Temperature, 13° Cent Transpired. 0.18 per cent

Experiment B 14. September 25, 1873. Ascent of fluid with lithium citrate. Cherry Laurel. Thirty minutes in bright diffused light.

Experiment B 15. September 25, 1873. Ascent of fluid with lithium citrate. Cherry Laurel. Thirty minutes in bright diffused light, with cortical tissues removed.

Experiment B 16. September 27, 1873. Ascent of fluid with lithium citrate. Cherry Laurel. Thirty minutes in sun. Leaves removed.

Experiment B 17. September 27, 1873. Passage of fluid downwards with lithium citrate. Cherry Laurel. Thirty minutes in sun.

Experiment B 18. September 30, 1873. Ascent of fluid with lithium citrate. Cherry Laurel. Thirty minutes ; dull day.

Experiment B 19. September 30, 1873. Ascent of fluid with lithium citrate. Cherry Laurel. Thirty minutes. Dull day. The leaves removed.

Experiment B 20. September 30, 1873. Ascent of fluid with lithium citrate. Thirty minutes ; dull day ; under pressure of 110.53 grammes of mercury.

EXPLANATION OF PLATES.

Plate XVII.

No. 1. Graphic representation of the results of Experiment A 1 ; see page 345.

No. 2. Graphic representation of the results of Experiment A 2; see page 349.

Plate XVIII.

No. 1. Graphic representation of the results of Experiment A 4; see page 346.

No. 2. Graphic representation of the results of Experiment A 3; see page 347.

Plate XIX.

No. 1. Graphic representation of the results of Experiment A 6 on privet (see page 348); and A 7 on elm, see page 350.

No. 2. Graphic representation of the results of Experiment A 9 on privet, see page 348; and A 10 on elm, see page 350.

Plate XX.

Fig. 1. Experiment B 3; see page 357.

Fig. 2. Experiment B 4; see page 360.

Fig. 3. Experiment B 11; see page 360.

Fig. 4. Experiment B 12 ; see page 358.

Scale, .25 inch to one inch.

Plate XXI.

Fig. 1. Experiment B 6 ; see page 361.

Fig. 2. Experiment B 5; see page 361.

Fig. 3. Experiment B 7 ; see page 361.

Fig. 4. Experiment B 8; see page 357.

Scale, .25 inch to one inch.

Plate XXII.

Fig. 1. Experiment B 9; see page 360.
Fig. 2. Experiment B 13; see page 360.
Fig. 3. Experiment B 1; see page 359.
Fig. 4. Experiment B 2; see page 359.
Fig. 5. Experiment B 10; see page 358.
Fig. 6. Experiment B 16; see page 362.
Fig. 7. Experiment B 19; see page 362.
Scale, .25 inch to one inch.

Plate XXIII.

Fig. 1. Experiment B 15; see page 362.
Fig. 2. Experiment B 17; see page 363.
Scale, .25 inch to one inch.

Plate XXIV.

Fig. 1. Experiment B 14; see page 359.
Fig. 2. Experiment B 18; see page 359.
Fig. 3. Experiment B 20; see page 363.
Scale, .25 inch to one inch.

The shaded portions in the figures indicate the distance to which the lithium had passed.

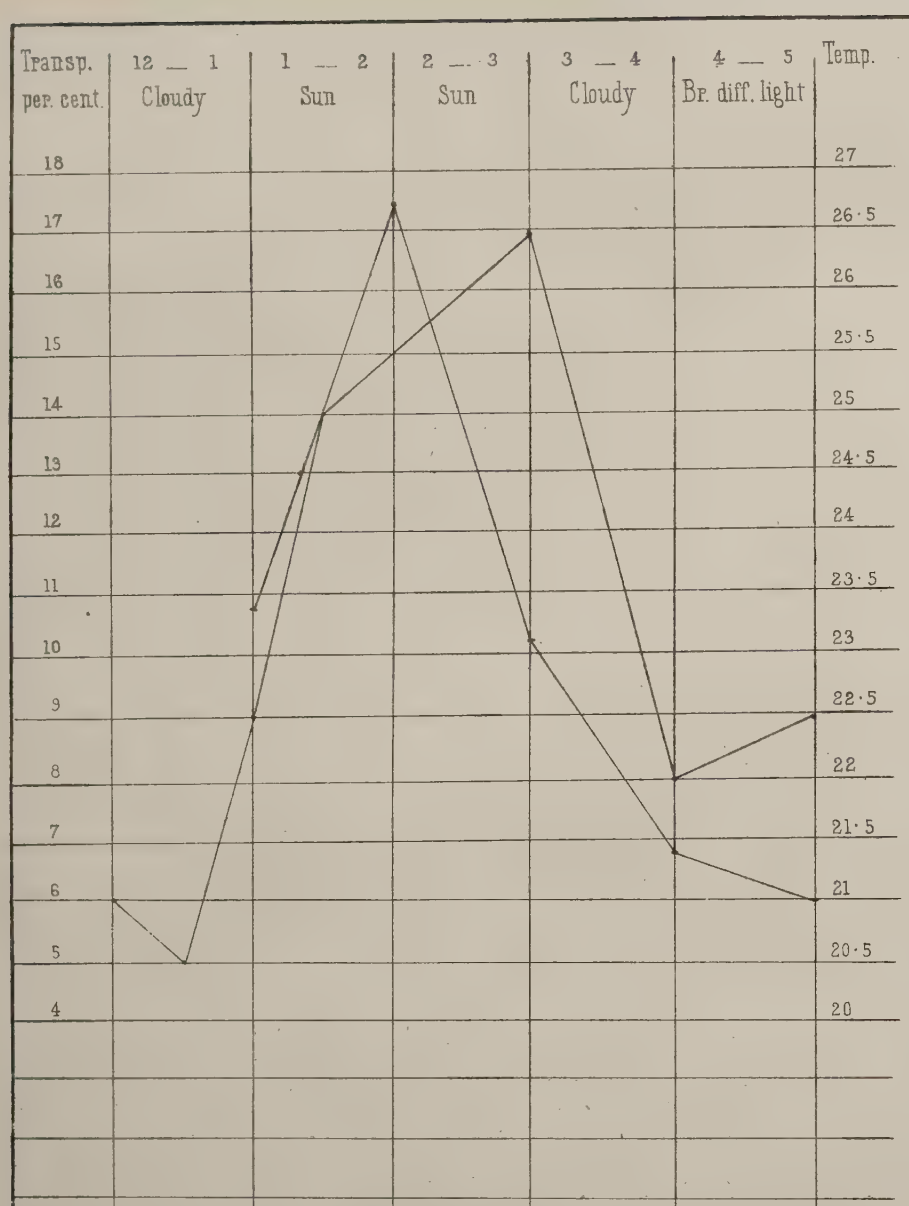


Fig. 1

A. 1. Cherry Laurel

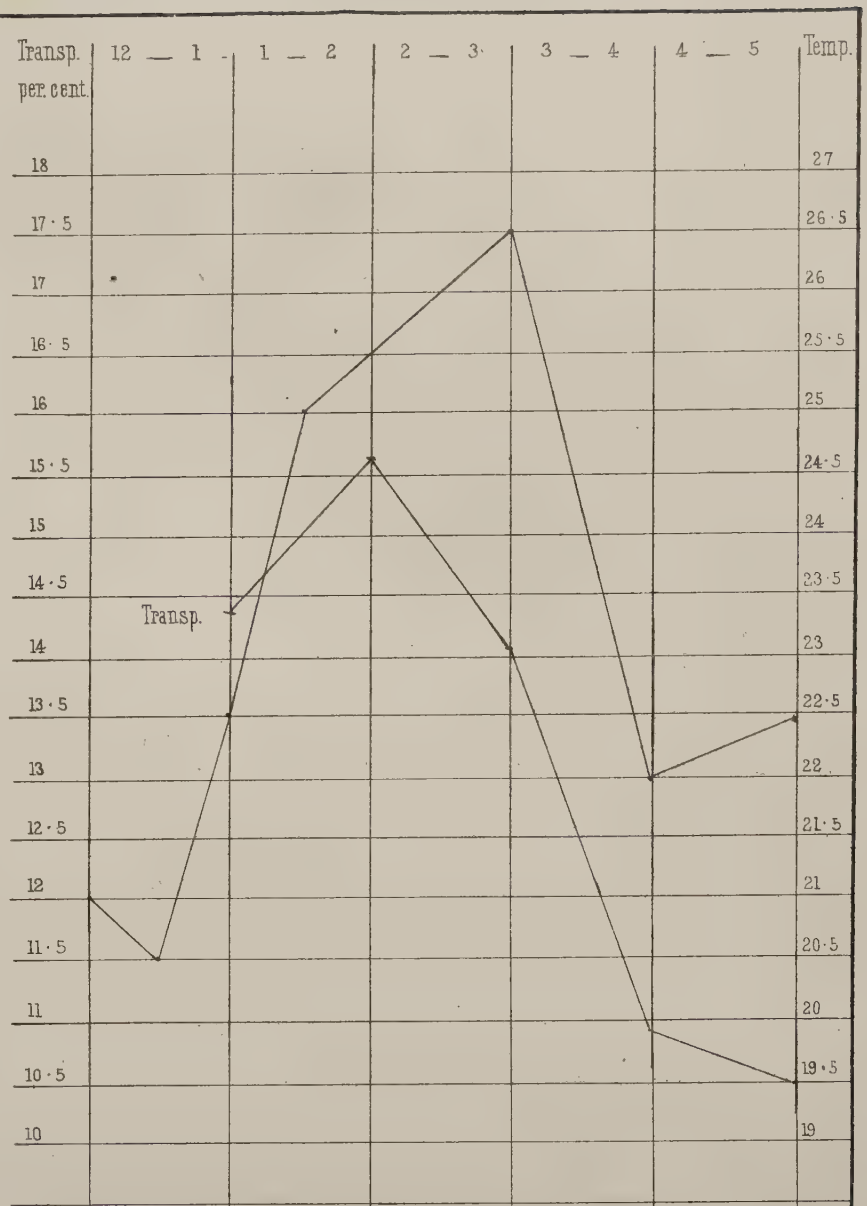


Fig. 2

A. 2. Elm

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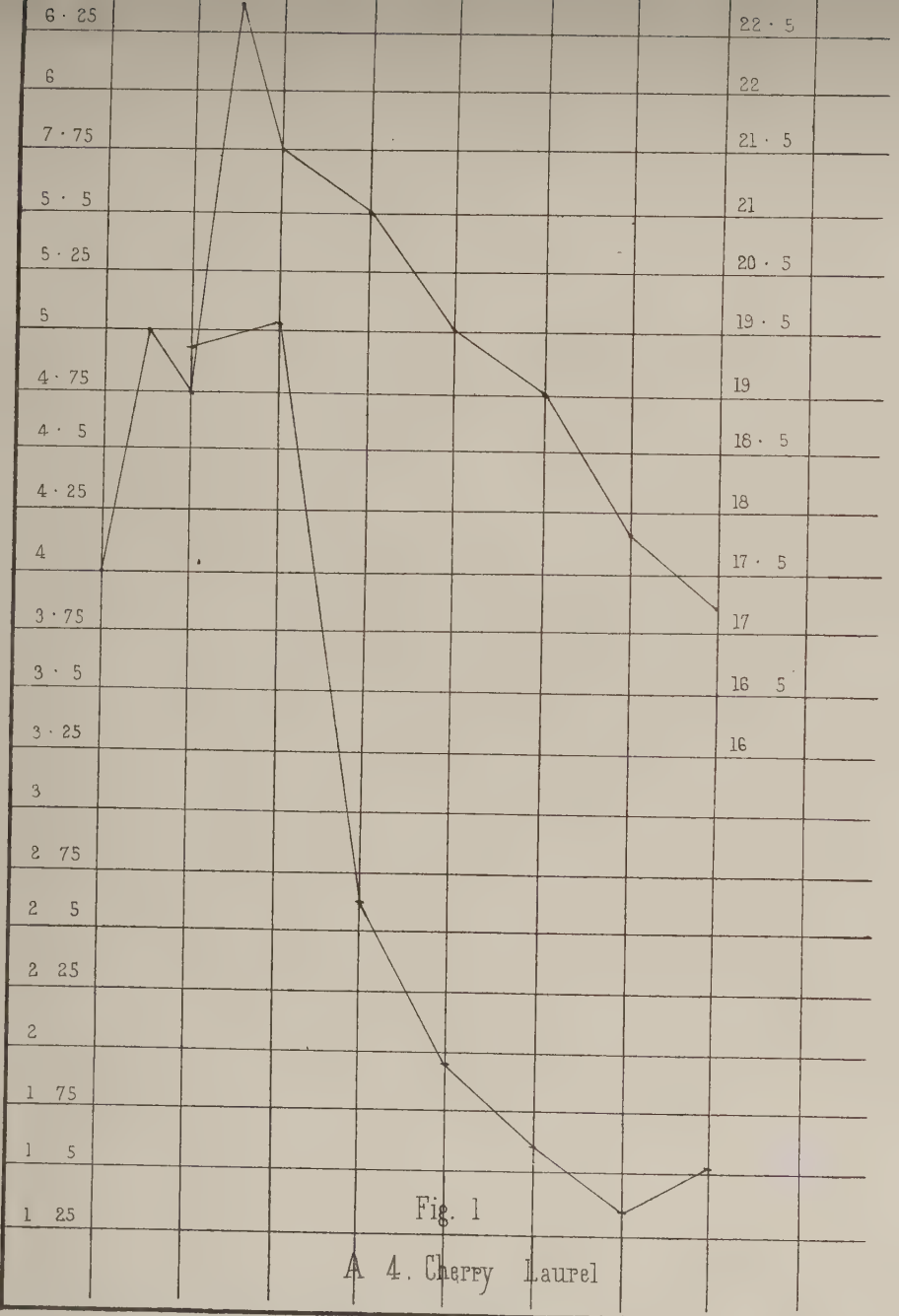

Fig. 1

A 4. Cherry Laurel

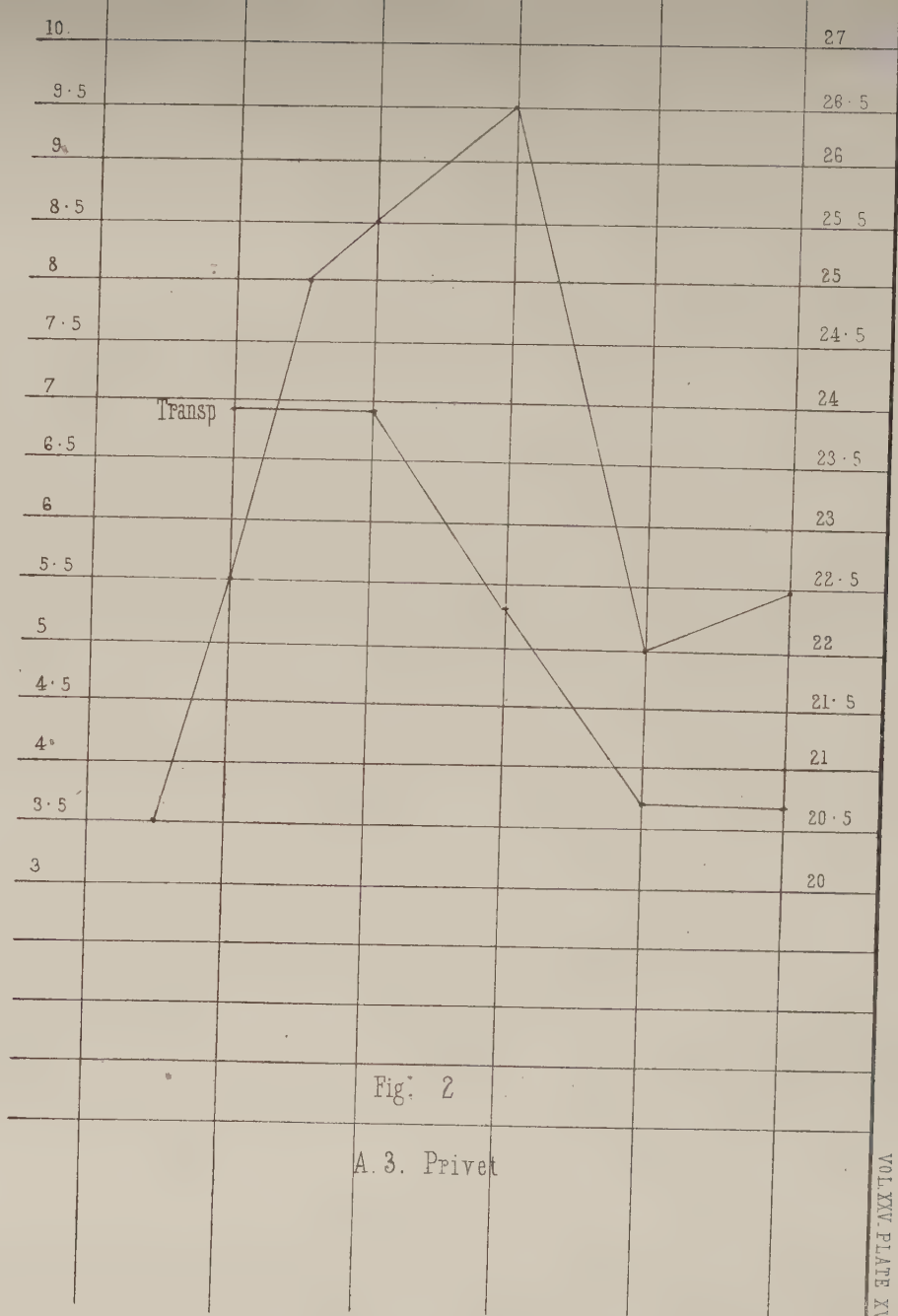

Fig. 2

A. 3. Privet

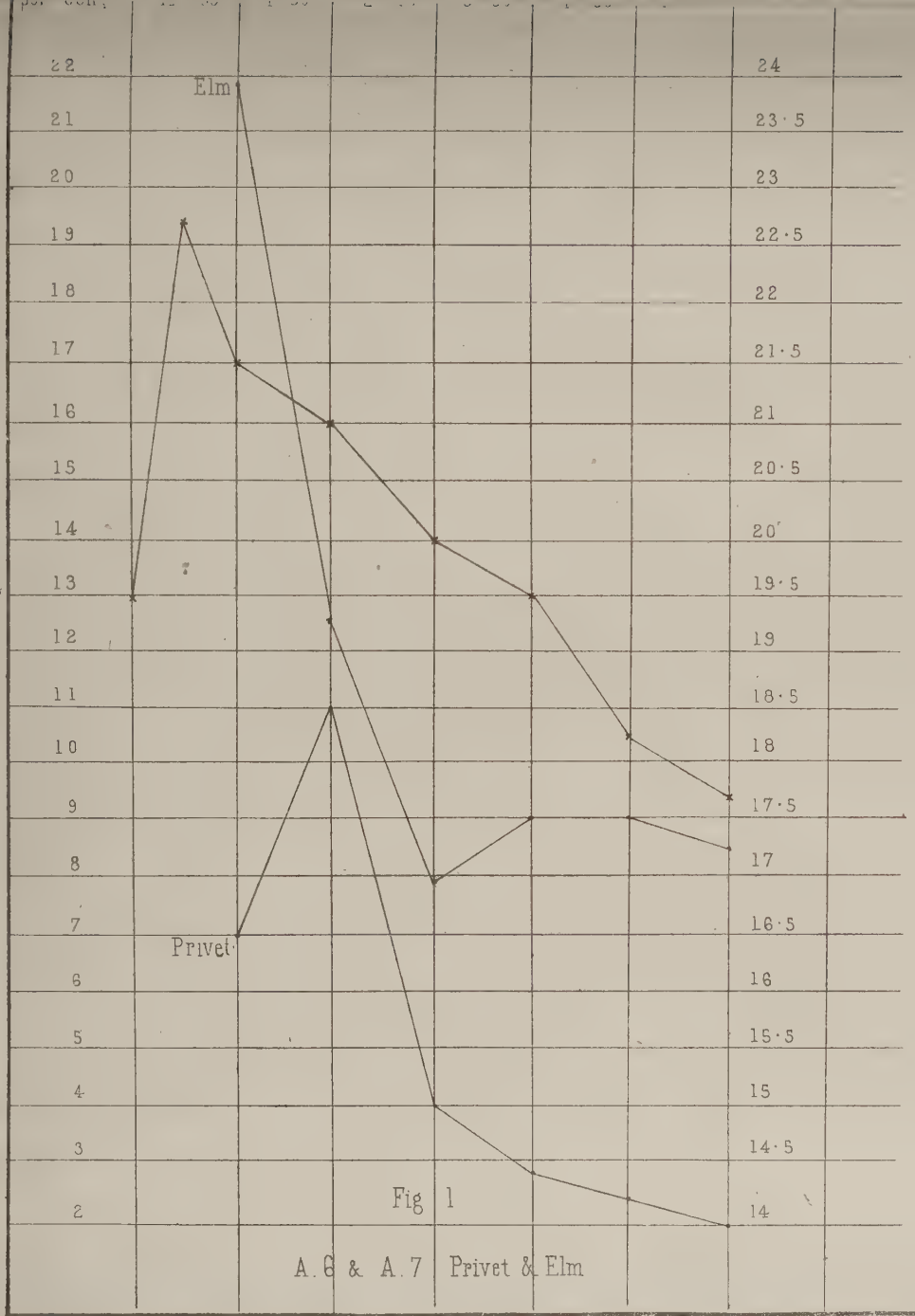

Fig 1

A.6 & A.7 Privet & Elm

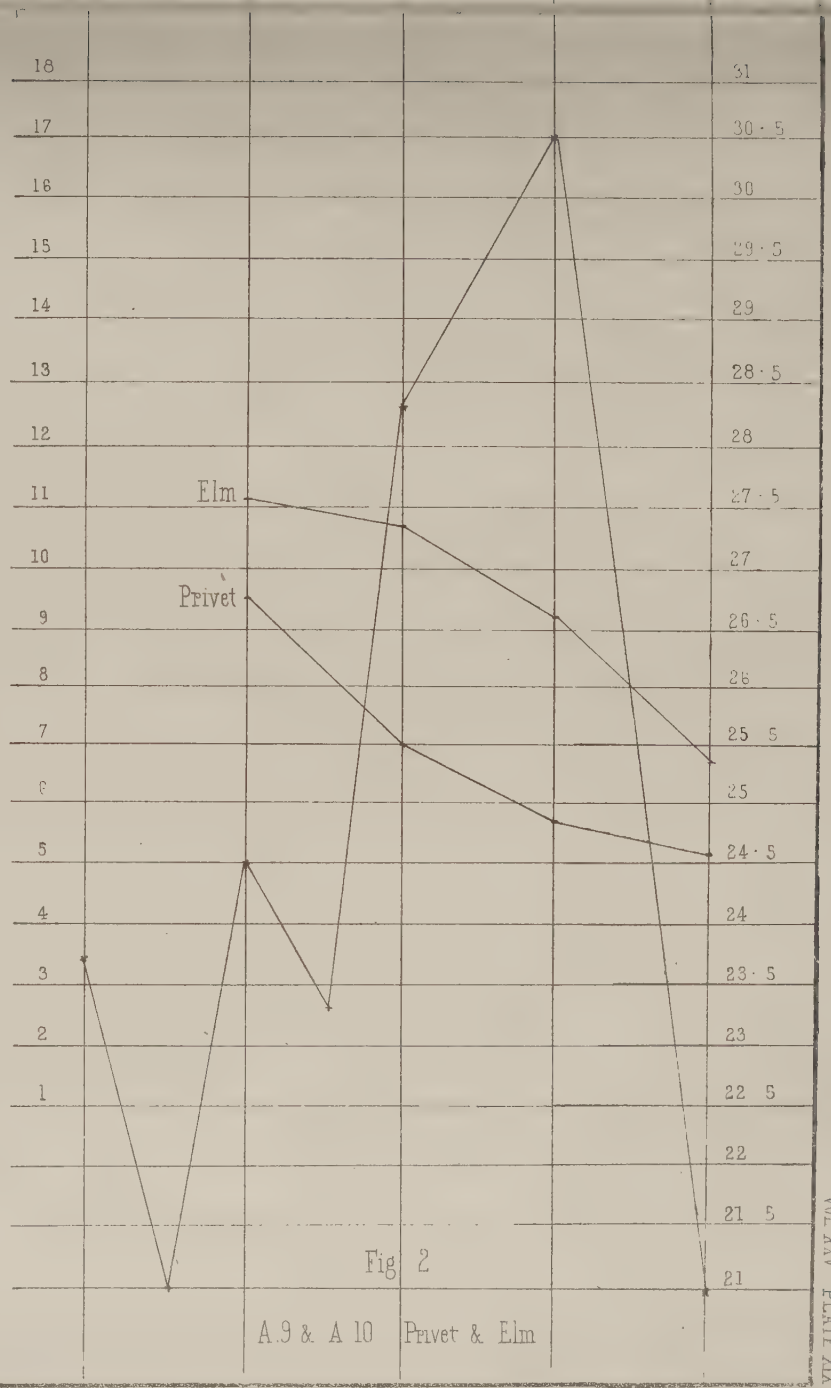

Fig 2

A.9 & A 10 Privet & Elm

Fig 1

B 3.

Fig 2

B 4

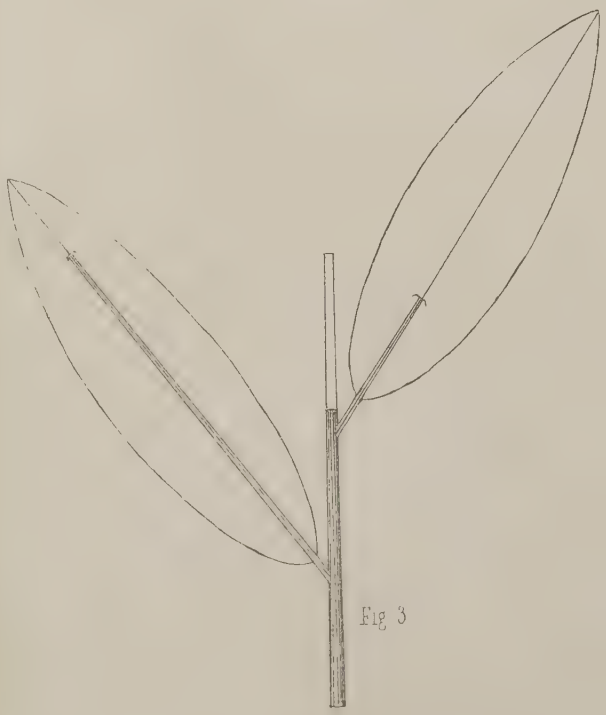

Fig 3

B 11

Fig 4

B 12

Fig 1

Fig 2

Fig 3

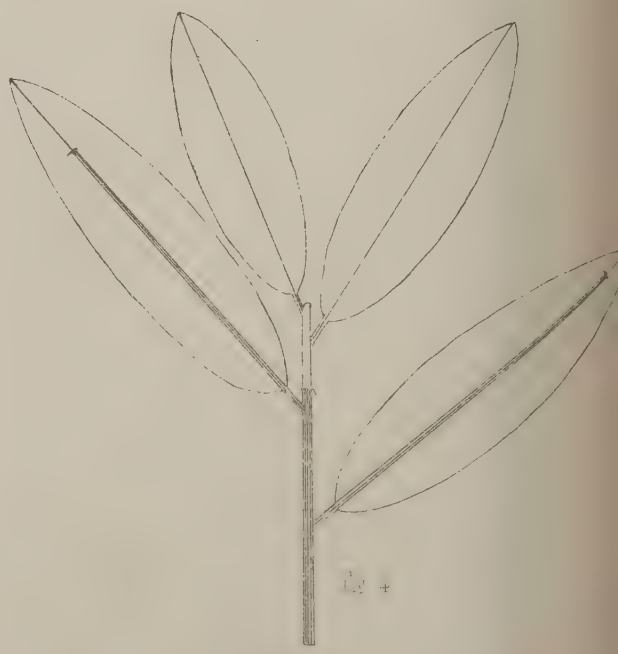

Fig 4

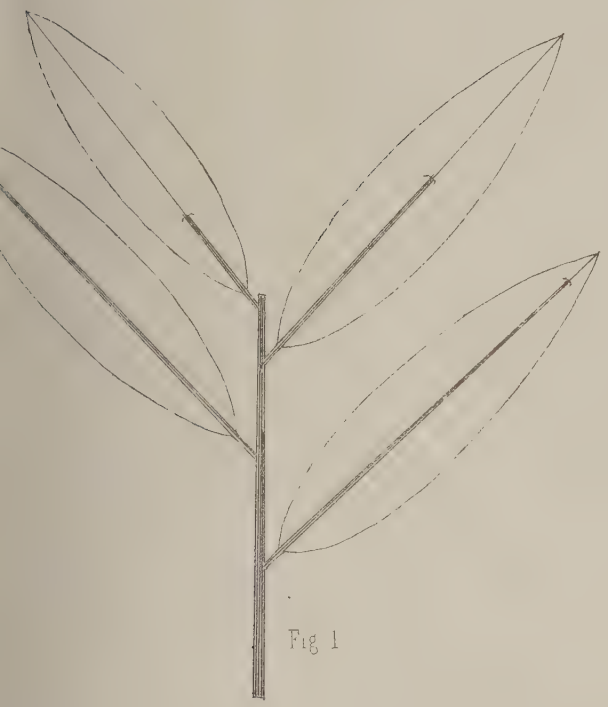

Fig. 1

B 4

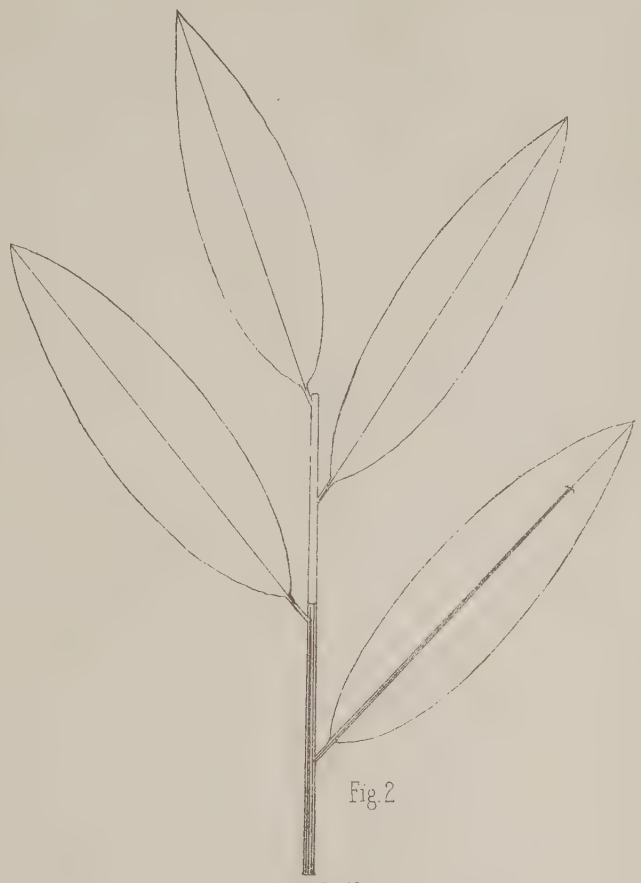

Fig. 2

B 13

Fig 3

B 1

Fig. 4

B 2

Fig. 5

B 10

Fig. 6

B 16

Fig. 7

B 19

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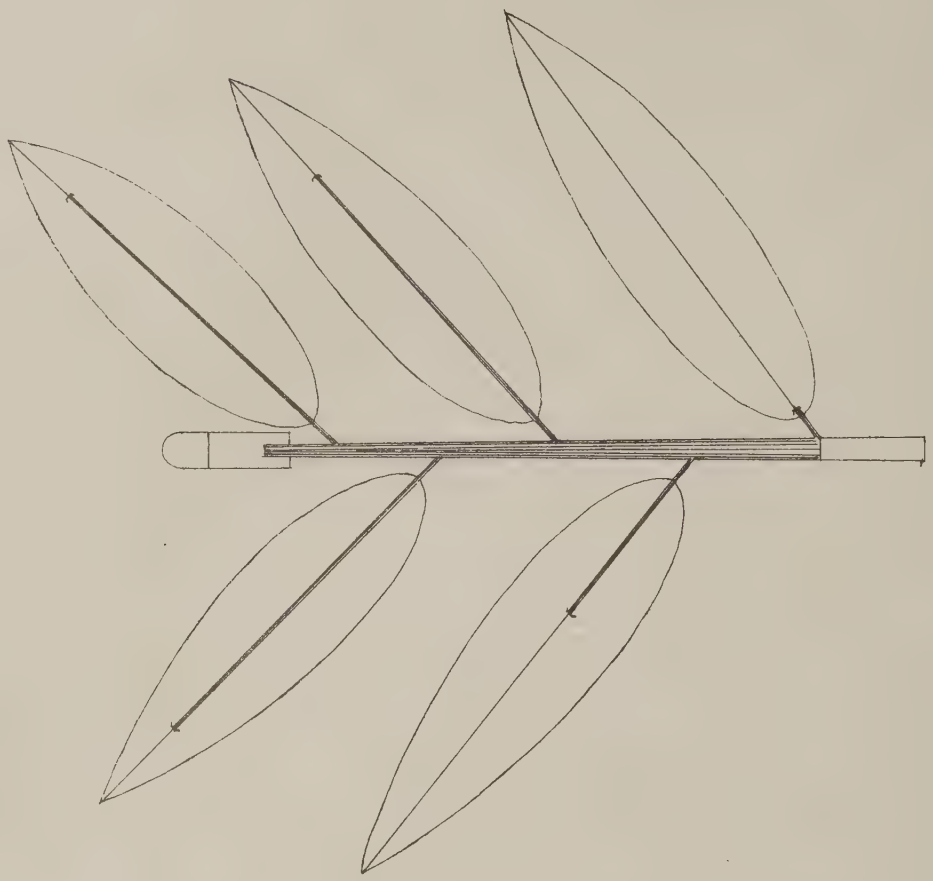


Fig 2

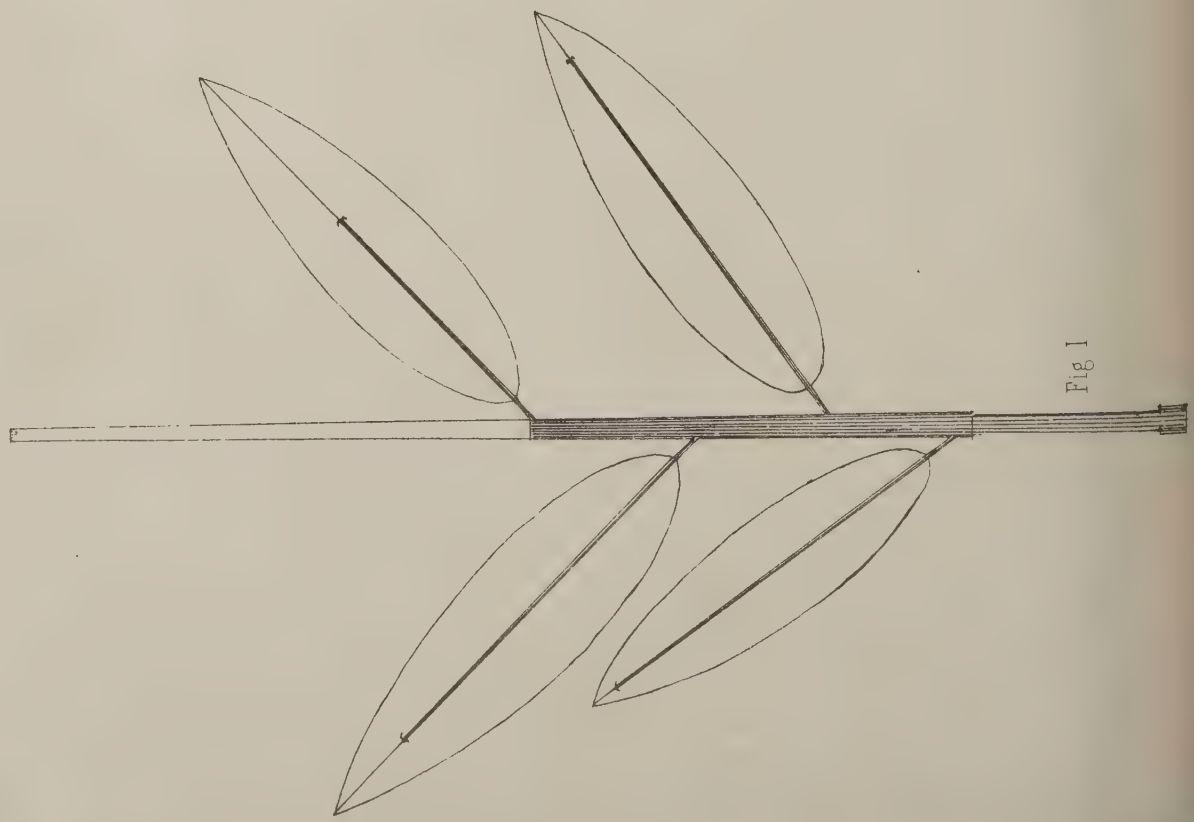


Fig 1

B 17

Fig. 1

Fig. 2

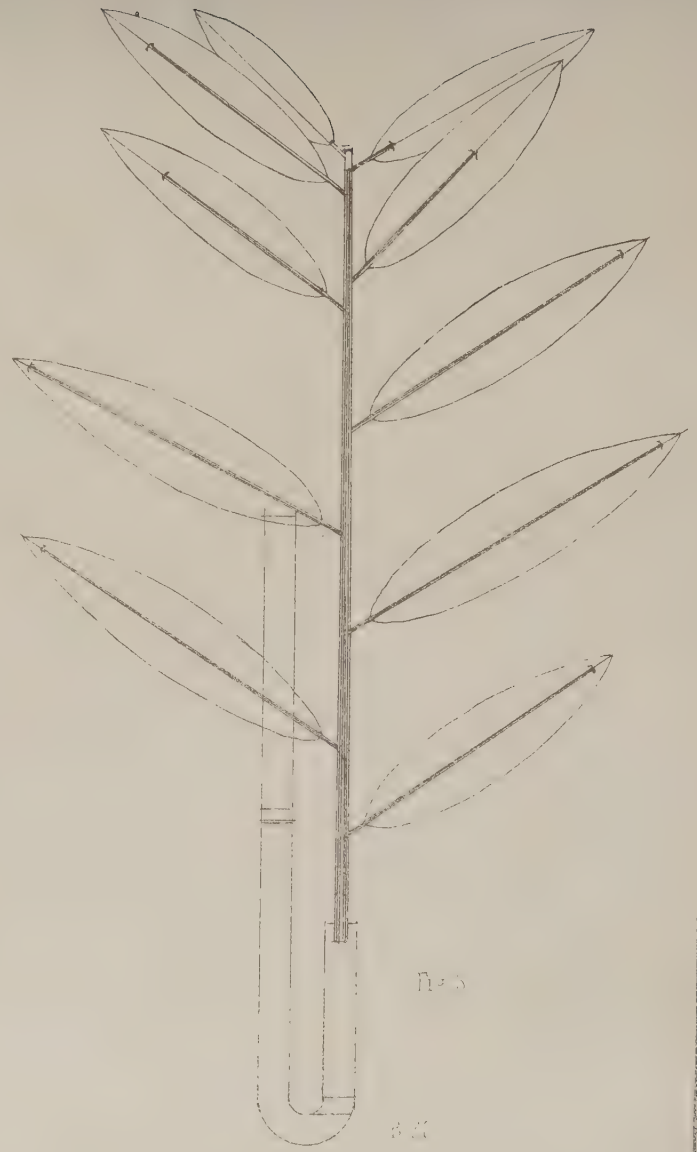


Fig. 3

X.—*Researches in Chemical Optics. By the* Rev. J. H. Jellett, B. D., Senior Fellow, Trinity College, Dublin.

Being the substance of Papers read June 25, 1860, January 26, 1863, and November 30, 1873.

1. No part of the scientific field appears to promise more important results to the investigator than the application of Optics to Chemistry. Already optical science has given to the chemist the solution of problems whose very investigation might have seemed to be beyond his reach. Not only has the spectroscope conferred on him a power of qualitative analysis, far exceeding (where it is applicable) that which he could derive from his own science, but it has also enabled him to apply analysis to cases in which chemistry proper has been hitherto, and probably would have remained, powerless. The analysis of a nebula or of a planetary atmosphere, effected to a considerable extent by the spectroscope, might seem to be beyond the powers of ordinary chemical methods.

The very striking character of these results has secured for this department of Chemical Optics an amount of scientific attention which leaves nothing to be desired. We may feel reasonably confident that in the department of spectral analysis whatever *can* be done will be done.

2. But there is another branch of Chemical Optics of which the same can hardly be said. The phenomena of rotatory polarization, considered in their relation to chemistry, have hardly received the attention which their importance deserves. If they are less striking than the phenomena of the spectroscope, they yet open a wide field for chemical research. And I hope to show that they resemble the phenomena of the spectroscope in giving a solution of a problem which appears to be difficult to ordinary chemistry.

3. The attention of physicists was first directed to this question by the remarkable discoveries made by Biot in the year 1815. That illustrious

philosopher, while conducting a series of experiments upon crystals of sulphate of lime, arrived almost accidentally at the remarkable and quite unexpected fact, that the effect produced on a ray of polarized light by transmission through a column of oil of turpentine is the same in kind with that which he had already observed in a ray transmitted along the axis of a quartz crystal.

The power of *rotating* the plane of polarization was, by the same observer, found to exist in other fluids, as, for example, in the essential oil of laurel, in the essential oil of lemon, and in the alcoholic solution of camphor. Biot observed further, that the effect produced was, in general, proportional to the length of the column of fluid through which the ray was transmitted. The list of substances possessed of the rotatory power has been greatly extended by subsequent observers. It has been shown that a large number, both of bodies naturally fluid, and of solutions of bodies naturally solid, are possessed of this power. And, by the application of the phenomena of rotatory polarization to the measurement of the quantity of sugar contained in a given syrup, these phenomena have received an important practical application. Much however still remains to be done, and without pretending to exhaust the subject, I shall state in the present memoir the results of a very long series of experiments, so far as they seem to throw any additional light upon this important branch of Optics.

4. In order to make an accurate examination of the rotatory powers of fluids, more especially of solutions, it was in the first place necessary to devise an instrument capable of giving measurements more exact than any which could be obtained by means of the polariscopes hitherto used. For the determination of this power is, in many cases, rendered very difficult by the sparing solubility of the substance examined, combined with the weakness of its rotatory power. Thus, for example, in the case of morphia, of which it is scarcely possible to form a cold alcoholic solution which will contain more than 1 gr. ·5 in one cubic inch, the greatest amount of rotation which can be obtained from a column of fluid, whose length is 8 inches, will not exceed 29′ for the mean ray. Now, if the instrument used be the saccharometer of Soleil, the amount of possible error can scarcely be estimated at less than one division on the scale, an error amounting for the mean ray to about 13′. It would, therefore, be impossible with such an instrument to determine, with any approach to

accuracy, the rotatory power of an alcoholic solution of morphia, inasmuch as the error might amount to nearly half of the quantity to be measured.

I.—*Description of the Instruments used.*

5. The rotatory power of a transparent fluid may be measured in one of two ways.

(1). By actual measurement of the angle through which the plane of polarization has revolved during the passage of a plane-polarized ray through a column of the fluid, the length of the column being supposed to be known.

(2). By the method of compensation, which consists in destroying the effect produced by the fluid under examination by transmission through a substance possessing an *opposite* rotatory power.

I have, in general, made use of the second method, as being, for reasons which will appear presently, much more convenient than direct measurement. But as I have also employed the first method, and as the principal part of the instrument here used is common to both methods, I shall commence by describing the mode of measuring directly the angle through which the plane of polarization has revolved, or, as it may be otherwise expressed, of *determining* the plane of polarization of the emergent light.

6. The instrument which has been ordinarily used for this purpose is the Nicol prism. In this instrument, as is well known, whatever be the nature of the incident light, the emergent ray is polarized in a certain definite plane depending solely on the construction of the prism. Theoretically then, if the incident ray be polarized in a plane perpendicular to this fixed plane, it will be extinguished by the prism ; and, conversely, if the light *be* extinguished, we know that the incident ray is polarized in a plane perpendicular to the fixed plane.

Hence, if it be required to determine the plane of polarization of a given ray, it is only necessary to transmit it through a Nicol prism, and then to turn the prism round its axis till the transmitted light vanishes. The plane of polarization of the incident light is then known.

Thus, for exemple, if it be required to measure the rotation of the plane of polarization caused by transmission through a fluid, it is only necessary to perform the experiment just described before and after transmission of the light

through the rotating fluid. The angle through which it has been necessary to turn the prism in passing from the first to the second experiment is the angle required.

Practically, however, the experiment does not admit of any great accuracy. It is not possible to obtain a beam of light whose component rays are mathematically parallel, and the same is of course true of the planes of polarization of these rays. Hence, it is evident that no position of the analyzing prism will completely extinguish the light. All that can be done is to place the prism in such a position that the transmitted light may be a *minimum*, and this it is difficult to do for two reasons.

In the first place, it is evident from the mathematical conditions of a *minimum* value that the variation in the intensity caused by a small change in the position of the prism will be very small. Hence a small error in the observation of the intensity will correspond to a large error in the position of the prism.

The second cause of inaccuracy, which is the more important of the two, is the fact that the observer is obliged to compare tints which are not simultaneously before the eye. In order to know that the *observed* intensity of the light, which is at any moment presented to the eye, is a minimum, it must be compared with the *recollected* intensity of the light which was seen the instant before. It is evident that the process must be very inferior in accuracy to the observation of tints *simultaneously* present. This difficulty is obviated in the analyzing prism which I have employed, and which is constructed as follows :—

Analyzer.

7. A rhombic prism of Iceland spar, whose longitudinal edges should have a length of about two inches, or a little more, is cut by two planes perpendicular to those edges, so as to form a right prism (Fig. 1). This prism is divided by a plane parallel to the edges, and making a small angle with the longer diagonal of the rhombic base. One of the two parts into which the prism is thus divided is then reversed, so as to place the base uppermost; the two parts are cemented together (as in Fig. 2),

Fig. 1.

Fig. 2.

with the surfaces of section in contact, and the ends of the prism thus formed are then ground and polished.

Now if two rays of light, parallel to the edges, be made to traverse the two parts of the prism respectively, it is evident from its construction, the angle of section being small, that the lines of separation of the extraordinary from the ordinary images in the two parts of the prism respectively will be in directions nearly opposite; and if the end at which the light is admitted be so chosen that these separations shall be *from* the plane of section, the separation of the extraordinary images from each other will be nearly double the separation of either from the corresponding ordinary image.

Suppose, now, that a cylindrical beam of plane polarized light is made to traverse the prism in a direction parallel to the edges, and so as to be equally divided by the plane of section. The emergent beam will consist of three separate parts, viz. :—1. A cylindrical beam whose section is OCO' (Fig. 3), formed by the union of the two ordinary beams. 2. Two semi-cylindrical extraordinary beams E, E'. If, then, the size of the incident beam be suitably determined, these latter may be completely separated from the ordinary beam ; and if the end of the prism be covered by a diaphragm pierced by a hole just large enough to allow this latter to pass, the extraordinary beams will, of course, be stopped. The instrument will then transmit a single beam of plane polarized light.

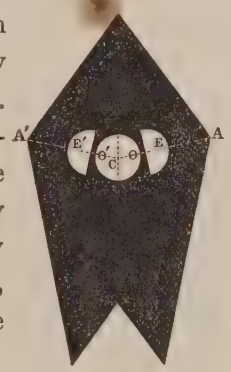


Fig. 3.

8. Now, it is easy to see that the planes of polarization of the two parts into which the beam is divided by the plane of section are inclined to each other at an angle, somewhat less than double the angle DCS. (Fig. 1.) Suppose then that, the plane of the paper being perpendicular to the beam, the traces of these planes of polarization are represented by AB', $A'B'$ (Fig. 4), and let CP, CP' be perpendicular to these lines respectively. Let Cp be the trace of the plane of polarization of a beam parallel to the edges which enters one end of the prism. Then so long as there is any difference between the angles PCp, $P'Cp$, the intensities of the two parts of the emergent beam will be dif-

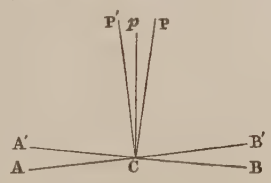


Fig. 4.

ferent, and conversely, if these intensities be equal, it is evident that the plane of polarization of the beam Cp will bisect the angle PCP'.

9. To apply this instrument to the determination of the plane of polarization of a given beam, it is only necessary to transmit through the prism a cylindrical beam composed of parallel rays, polarized in planes which are all parallel to one known plane, turning the prism round its axis until the intensities of the two semicircular images are equal. Having then removed this beam, introduce the given beam, and turn the prism till the equality of tints be re-established. The angle through which the prism has revolved, in passing from the first observation to the second, is the angle between the planes of polarization of the two beams. Thus for the direct measurement of the rotation caused by transmission through a fluid, it is only necessary to establish the equality of tints before and after transmission. The rotation of the prism is measured by attaching it to a graduated circle, which revolves with it.

10. The determination of the angle α ($= PCP'$), so as to secure for the prism the greatest amount of sensibility, will necessarily depend upon the nature of the incident beam. If the elementary rays of which this beam is composed be polarized in planes which are accurately parallel, it is desirable to use a prism cut at a very small angle and a powerful incident light. In fact, it may be shown that in such a case the sensibility of the instrument is always augmented by diminishing the angle α, and at the same time increasing the intensity of the incident light, so as to preserve the same intensity for the analyzed light. If the planes of polarization of the elementary rays be inclined to each other at small angles, the best value for α will be of the same order of magnitude as these angles.

11. To prove this, let ka_n be the intensity of one of the elementary rays which is incident upon the analyzing prism, where k is a constant depending on the intensity of the light made use of, and is the same for all the elementary rays. Let θ_n be the complement of the angle which the plane of polarization of this ray makes with one of the planes of analyzation. Then $\alpha - \theta_n$ will be the complement of the angle which the same plane of polarization makes with the other plane of analyzation. The intensities of these rays, when emerging from the prism, will be $ka_n \sin^2\theta_n$ and $ka_n \sin^2(\alpha - \theta)$.

Then, since the intensities of the emergent beams in the two images are equal, we have

$$\Sigma a_n \sin^2\theta_n = \Sigma a_n \sin^2(\alpha - \theta),$$

or

$$\Sigma a_n \sin(\alpha - 2\theta_n) = 0. \qquad (1)$$

Now if the prism be turned through a small angle $\delta\theta$, the difference between the intensities of the images will be

$$k\delta\theta\Sigma a_n \{\sin 2\theta + \sin 2(\alpha - \theta)\}$$

or

$$2k \sin \alpha\delta\theta\Sigma a_n \cos(\alpha - 2\theta_n).$$

The sensibility of the instrument is therefore proportionate to

$$k \sin \alpha\, \Sigma a_n \cos(\alpha - 2\theta_n).$$

Now, if the intensity of the incident light be so arranged that the intensity of the analyzed light shall be invariable, we shall have

$$k\Sigma a_n \sin^2\theta_n = k\Sigma a_n \sin^2(\alpha - \theta_n) = \tfrac{1}{2}k\Sigma a_n\{\sin^2\theta_n + \sin^2(\alpha - \theta_n)\}$$
$$= \tfrac{1}{2}\, k\Sigma a_n\{1 - \cos\alpha \cos(\alpha - 2\theta_n)\} = \text{const.}$$

Hence the measure of the sensibility of the instrument will be

$$S = \frac{\sin\alpha\Sigma a_n \cos(\alpha - 2\theta_n)}{\Sigma a_n - \cos\alpha\Sigma a_n \cos(\alpha - 2\theta_n)}.$$

Now since the elementary rays are polarized in planes making small angles with each other, we may assume

$$2\theta_n = \epsilon + \alpha_n$$

where ϵ is the same for all the rays, and α_n are small and independent of the angle at which the prism is cut.

Hence

$$\frac{1}{S} = \frac{\Sigma a_n}{\sin\alpha\{\cos(\alpha - \epsilon)\Sigma a_n \cos\alpha_n + \sin(\alpha - \epsilon)\Sigma a_n \sin\alpha_n\}} - \cot\alpha$$

or, assuming $A = \Sigma a_n$, $B = \Sigma a_n \cos\alpha_n$, $C = \Sigma a_n \sin\alpha_n$

$$\frac{1}{S} = \frac{A}{B \sin\alpha \cos(\alpha - \epsilon) + C \sin\alpha \sin(\alpha - \epsilon)} - \cot\alpha. \qquad (2)$$

Making the same substitutions in the equation

$$\Sigma a_n \sin(\alpha - 2\theta_n) = 0$$

we have

$$B \sin(\alpha - \epsilon) - C \cos(\alpha - \epsilon) = 0.$$

Hence, substituting for $\alpha - \epsilon$ in eq. (2)

$$\frac{1}{S} = \frac{A}{\sqrt{B^2 + C^2}} \operatorname{cosec} \alpha - \cot \alpha.$$

If the sensibility of the prism be a maximum, this expression considered as a function of α is a minimum, hence

$$A \cos \alpha = \sqrt{B^2 + C^2},$$

If all the rays be polarized in the same plane, $a_n = 0$, whence $C = 0$, $B = A$, and therefore

$$\cos \alpha = 1, \ \alpha = 0,$$

showing that the sensibility increases continually with the diminution of α. If a_n be a small quantity of the first order, it is evident that B will differ from A by a quantity of the second order, and that C will be of the first order. Hence, it is plain that $\cos \alpha$ will differ from unity by a quantity of the second order, or that α itself will be of the first order.

In experiments on the rotatory power of fluids, which form the subject of the present memoir, the planes of polarization are never coincident, but are dispersed through angles varying with the nature of the fluid under examination. For these experiments I have, in general, employed a prism, in which the value of α is about 1° 45′, the light made use of being that of a lamp.

Compensator.

12. For the direct measurement of the angle through which the plane of polarization of a ray has revolved, the analyzing prism already described is sufficient. But the method of direct measurement is not applicable tc the phenomenon of rotatory polarization unless the light employed be homogeneous. In fact, if the light be heterogeneous, the planes of polarization of

the elementary rays are so widely dispersed that we cannot speak, even approximately, of *the* plane of polarization of the emergent beam. And it is not easy to procure, without the aid of the sun, a beam of homogeneous light sufficiently intense to give a perceptible image after transmission through a prism by which it is so nearly extinguished.

The practical difficulty of obtaining an intense homogeneous light has caused physicists generally to prefer the method of *compensation*, in which the effect produced by transmission through one substance is not measured directly, but annulled by transmission through another substance, which causes the plane of polarization to revolve in a direction opposite to that of the rotation caused by the first. Provision being made for measuring the lengths of the columns of both substances, we have thus a method of comparing the rotatory powers of these substances, and therefore of expressing the rotatory power of any given substance in terms of that of some one substance which is taken as the standard of reference. Thus let R denote the rotatory power of the one substance, and l the length of the column through which the light passes, R' and l' having the same significations for the second substance. Then, if the lengths of these columns be such that the effect of the one is annulled by that of the other, we have (*infra*, p. 384)

$$Rl = R'l'$$

an equation containing but one unknown quantity R, as soon as the lengths l, l' have been ascertained. It only then remains to explain how these lengths are measured.

With regard to the first, l, there is no difficulty. In all instruments the fluid under examination is contained in a tube closed at both ends with plate glass, and whose length is accurately measured. The length l is then a constant depending on the construction of the instrument.

The mode of measuring l' depends upon the nature of the compensating substance. In the polariscope which I have used the compensating substance is a fluid contained in a vessel closed at the bottom with glass, and the length of the column of the fluid through which the light passes is regulated by a tube, also closed with glass, which is capable of moving in a direction coincident with its axis.

Polariscope.

13. The general plan of the instrument which I have used in these researches is represented by the accompanying diagram (Fig. 5).

Fig. 5.

The light is taken from a lamp burning paraffin or (better perhaps) some

one of the hydro-carbons extracted from petroleum.* The object being to procure a small intense light, the lamp is provided with a flat wick, which, when turned with its edge towards the polariscope, gives a light of considerable intensity. The light of this lamp is received upon a condenser, consisting of two large plano-convex lenses, the flame of the lamp being placed as nearly as possible in the principal focus of the lower lens. Above the condenser is a short tube, carrying at one extremity a small microscope-objective, and at the other extremity a diaphragm, pierced at its centre by a very small hole, which is situated in the focus of the objective, and also, when the instrument is adjusted, in the principal focus of the upper lens of the condenser. By this arrangement a beam of light is obtained, the rays of which are sensibly parallel to the common axis of the tubes. This beam is polarized by transmission through a Nicol prism, contained in the tube *DD*. The light now enters the compensator *EEEE*, constructed as above described, the length of the compensating column being regulated by the tube *F*, which is open at the upper end, and closed at the lower end with plate glass. This tube is attached to the transverse piece *VV*, which carries a vernier, whose divisions correspond to those of the scale *SS*, which is attached to the bar *ZZ*, carrying all the parts of the instrument. The transverse piece *VV* is capable of sliding along *ZZ*, this motion being produced by a chain attached at both ends to *ZZ*, and passing round a spindle with a milled head attached to *VV*. By these means a motion can be given to the tube *FF*, parallel to its axis, and by a very simple arrangement the zero of the vernier is made to coincide with the zero of the scale, when the lower extremity of the tube is in contact with the glass covering the lower aperture of the vessel *EEEE*. It is plain then that the number read off on the scale, which is graduated so as to be read to 0^{inch} ·001, will denote the length of the column of fluid interposed between the bottom of the vessel and the bottom of the tube. This is the compensating column spoken of in the description of the compensator.

The fluid under examination, through which the light next passes, is contained in a tube closed at both ends with glass. This tube is attached to a slide, *TT*, having a transverse motion across *ZZ*, thus enabling the observer

* The hydro-carbon which I have myself used, and which I have found very satisfactory, is "Lucigene," manufactured by Messrs. Dixon and Hughes, of Erne-street, Dublin.

to push the tube into or out of the line of vision. The slide is capable of holding at the same time two tubes, which can thus be brought successively into the line of light, an arrangement which greatly facilitates *comparative* observations.

GG is a tube containing an analysing prism constructed as above described, and *HH* a Ramsden's eye-piece.

To explain the mode of using the instrument, let it be supposed that the object is to determine the strength of a solution of cane sugar. In this case the compensating fluid may be French oil of turpentine. One of the tubes carried by the slide *TT* is filled with the given solution, and the other with a solution of given strength. The tubes having been placed in the slide, the first is pushed into the line of vision, and the slide *VV* moved until the intensities of the two images are equal. The reading of the scale is then noted. The tube containing the known solution is now pushed into the line of vision, and the same process repeated. Then if S, S' be the amounts of cane sugar contained in the same bulk of each solution respectively, and R, R' the readings of the scale in the two experiments, we have

$$S : S' :: R : R' ;$$

whence

$$S = \frac{R}{R'} S'.$$

Burette.

14. In many of the experiments which form the subject of the present memoir it is necessary to add to a given solution an accurately measured quantity of another fluid. For this purpose I have found the following arrangement to be very convenient as well as accurate.

ABCD (Fig. 6.) is a solid wooden frame, which may be firmly attached to a table; *TT* is a tube of boxwood set in the frame, and receiving a piston, *P*, worked by a screw, *S*; *GGGG* is a bent glass tube, connected at one extremity with the boxwood tube, and carrying at the other

extremity the graduated glass tube *EE*. The piston having been withdrawn from the tube *TT*, mercury is poured in so as to fill this tube, rising of course to the same height in the bent tube *GGGG*. The screw *S* is then turned so as to force the piston into the tube *TT*, and to cause the mercury to rise to a certain height, *H*, in the bent tube. If now the extremity of *EE′* be allowed to dip into a vessel containing the fluid which is to be added in a measured quantity, and the motion of the screw be reversed, the mercury will fall in the bent tube, and the required fluid will rise in *EE*. As soon as it has risen to the proper height, which may be very exactly measured by a cathetometer, the vessel containing this fluid is removed, and that containing the fluid to which the addition is to be made is brought under *EE*. Then by turning down the screw, the mercury is made to rise again in the bent tube, and therefore necessarily to force the fluid contained in *EE* into the vessel placed to receive it.

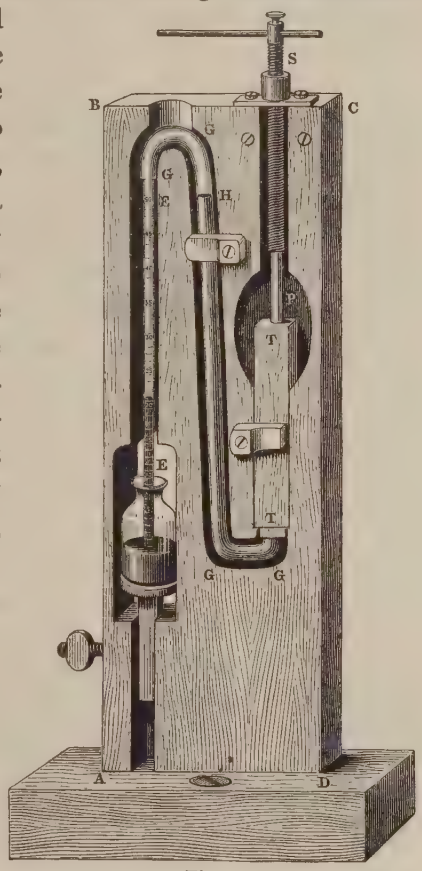


Fig. 6.

II.—*Definitions and Laws of the Rotatory Power—General Mode of Measurement.*

15. When a ray of polarized light is transmitted through a column of a substance possessing the rotatory power, the angle through which the plane of polarization is made to rotate is in general for the same substance proportional to the length of the column. Thus, let *R* be the angle of rotation, *l* the length

of the column, and C a constant depending upon the nature of the substance. Then

$$R = Cl.$$

If the rotatory substance be a liquid or a solid in its natural state, C represents the rotatory power. This definition may be thus expressed.

The rotatory power of a liquid or transparent solid is the actual rotation of the plane of polarization of a ray of a certain determinate colour, caused by transmission through a unit of length of the given substance.

If the rotating substance be a solid which is held in solution by a liquid, it is known that the actual rotation is proportional to the quantity of solid contained in a given bulk of the fluid, or, as it is commonly termed, the *strength* of the solution. Hence, if s be the quantity of the solid body contained in a certain bulk, b, of the solution, and C' the rotatory power of this body in solution,

$$R = \frac{C'ls}{b}.$$

Or, expressed in words,

The rotatory power of a solid in solution is the actual rotation of the plane of polarization of a determinate ray, caused by transmission through a unit of length of a solution containing in a unit of bulk a unit of weight of the given substance.

16. The method adopted in obtaining the rotatory powers of solid substances in solution was as follows :—

The solvent used was, in general, rectified or proof spirit. Occasionally I employed water as the solvent, and in some instances chloroform. But, in comparing the rotatory powers of different substances with each other, it is absolutely necessary to employ the same solvent, and for this reason, that the rotatory power of the same substance varies with the solvent used. Thus for example, it will be seen by a reference to the table,* that the rotatory power of narcotina when dissolved in chloroform greatly exceeds the rotatory power of the same substance when dissolved in spirit. Similarly, the rotatory power of salicine dissolved in spirit is much less than the rotatory

* *Vide* Table I. at the end of this Memoir.

power of salicine dissolved in water. I have therefore, in general, employed the same solvent, selecting for this purpose spirit, as being for the substances which I have examined a more general solvent than water, and much more convenient than chloroform. In constructing the table I have in every case been careful to state the solvent used, in order that the reader may be enabled to distinguish the experiments which are fairly comparable with each other from those which are not.

17. No fixed rule can be given for determining what degree of strength should be given to the solution which is the subject of the experiment. The cause of this uncertainty is to be found in the imperfection which necessarily attaches to the process of *compensation.* When a ray of plane-polarized white light passes through a substance possessed of the rotatory power, this power affects in very different degrees the planes of polarization of the elementary rays of which the white ray is composed. Thus, in quartz, the rotation of the plane in the case of the extreme violet is between two and three times as great as in the case of the extreme red. Now the theory of any compensating polariscope requires us to assume that, if the lengths of the columns be so adjusted that the compensation may be complete for any one ray, the same will be sensibly true for every ray.

Thus, if R_r, R_r' be the rotations (in opposite directions) impressed by the two media respectively on the plane of polarization of the red ray, and R_v, R_v' the same rotations for the violet ray, we assume that if $R_r = R_r'$, we have also (sensibly) $R_v = R_v'$. This is evidently equivalent to the assumption that the ratio of the rotatory powers of two substances is sensibly the same for all the elementary rays.

But this assumption is in many cases very far from the truth. Thus, for example, if the fixed tube be filled with American oil of turpentine, and the compensating vessel with French oil of turpentine, it will be found that, when the compensation is as perfect as it can be made, there is a marked difference of *colour* between the two images, the right hand image being red, and the left hand image blue. Now since the intensities of these images have been made the same, we may assume that the compensation is q. p. perfect for the mean ray. The plane of polarization of the mean ray has therefore been restored to its original position. But since the right hand image is *red*, thus

showing a deficiency of *blue* light, it is evident that if the incident light had been blue, the right-hand image would have been dark. Hence, according to the arrangement adopted in the polariscope, we should infer that the total result of the action of both media is to rotate the plane of polarization of the blue ray to the right. For the blue ray, therefore, the right-handed rotation caused by the American turpentine exceeds the left-handed rotation caused by the French. The reverse is evidently the case for the red ray. Here, as may be shown by similar reasoning, the rotation caused by the American turpentine is *less* than that caused by the French. These results may be expressed by saying that American oil of turpentine is more *dispersive* of the planes of polarization than the French oil.*

Conversely, if two substances differ sensibly in their powers of dispersing the planes of polarization, a beam of white light transmitted successively through two columns of these substances so proportioned as to compensate as nearly as possible, will, when analyzed, furnish two images differing sensibly in colour, the red appearing in the image of the same name as the *more* dispersive substance, i. e. in the right or left-hand image according as the right-handed or left-handed substance is the more dispersive.

If this difference of colour be considerable, it will render the observation exceedingly difficult, and that difficulty will obviously increase with the strength of the solution used. On the other hand, it must be remembered that the less the total reading is, the greater is the importance of a given error. On the whole, I have found the most convenient strength of the solution to be that which required as a compensating fluid (if the substance be left-handed) a column of American turpentine, whose length is from 1·5 to 2 inches. With a tube whose length is 9·6 inches this can only be attained in the case of substances, such as salicine, quinia, brucia, which are possessed of a considerable degree of solubility. With the less soluble substances, such as strychnia, cinchonia, narcotina, the solution may be made as strong as possible, taking care to keep sufficiently below the point of saturation to prevent the deposition of

* I have elsewhere shown (Proceedings, Royal Irish Academy, Vol. ix., p. 530) that, by mixing these fluids in a certain proportion, a fluid may be obtained which possesses the power of rotating in *opposite* directions the planes of polarization of rays at opposite ends of the spectrum.

crystals. The strength of the solution is best ascertained by drying completely a given bulk of the fluid, and weighing the residue. If the substance in question contain water of crystallization, I have always supplied a sufficient heat to expel it. The rotatory powers given in the table must therefore be understood to belong to the several substances in the anhydrous state.

18. Let l be the length of the tube containing the solution, s_1 the strength of the solution, measured by the number of grains contained in a cubic inch, and r_1 the rotatory power of the substance. Let also r be the reading of the scale, or the length of the column of the compensating fluid, and f the rotatory power of this fluid. Then we have

$$ls_1 r_1 = rf.$$

This equation would determine r_1, the required rotatory power, in terms, of that of the compensating fluid. But as the rotatory power of oil of turpentine, which was the compensating fluid generally used, varies much in different specimens, I have thought it better to select as the standard of comparison some substance whose rotatory power is more determinate. The substance chosen for this purpose is a solution of salicine in proof spirit. Let s_0 be the strength of this solution, r_0 the rotatory power, and r' the reading of the scale. Then

$$ls_0 r_0 = r'f.$$

Dividing these equations, one by the other, we find

$$\frac{r_1}{r_0} = \frac{rs_0}{r's_1}$$

If then we take as the unit of rotatory power the actual rotation of the plane of polarization of the mean ray caused by transmission through a column of this solution of salicine, the length of the column being one inch, and the solution containing one grain of salicine in each cubic inch, we shall have

$$r_0 = 1$$

and therefore

$$r_1 = \frac{rs_0}{r's_1}$$

The column of rotatory powers in the table (Table I.) contains the values of this fraction for the several substances which I have examined.

III.—*Saccharometry.*

19. One of the commonest, as well as (in a practical point of view) most important applications of the phenomena of rotatory polarization to chemistry, is *saccharometry*, or the determination of the quantity of sugar contained in a given syrup. The mode of obtaining this determination optically is founded on the known fact that, *ceteris paribus*, the rotatory power of a solution is proportional to the amount of the active substance dissolved in a given bulk of the solution. If, therefore a saccharine solution contain but one kind of sugar, the strength of the solution is at once obtained by the method stated in p. 384. Generally, however, more than one kind of sugar being present in the solution, the mode of investigation is not so simple. I shall here discuss the problem in the general form in which it has been considered by Dr. Apjohn, who has shown* that by a combination of the optical and chemical methods we can effect a complete analysis of a solution in which three kinds of sugar are present.

20. Let it be supposed that the species of sugar present in the given syrup are—1. Cane sugar or sucrose ; 2. Inverted sugar ; 3. Grape sugar. These are the species most commonly met with in practice.

First Observation.—The fixed tubes of the polariscope are filled, one with the syrup to be examined, the other with a solution of cane sugar, containing 100 grains in 3 cubic inches. Let the readings of the scale for these be l_1, l_2 respectively, the compensating fluid being French oil of turpentine.

Second Observation.—The two syrups used in the first observation are each mixed with one-tenth of its bulk of pure hydrochloric acid, and raised to a temperature of 150° Fahrenheit, this temperature being maintained for twenty minutes. The effect of this process is, as is well known, to convert the whole of the sucrose into inverted sugar, the grape sugar remaining unchanged. Let the fixed tubes be filled each with one of these inverted syrups, and let the readings of the scale be l'_1, l'_2 respectively, the compensating fluid being now a solution of cane sugar, containing 100 grains in one cubic inch.

* "Transactions of the Royal Irish Academy," Vol. XXIV., p. 587.

Let c, i, g be the quantities of cane sugar, inverted sugar, and grape sugar, respectively—and let C, I, G be the rotatory powers of these several sugars. Let also l be the length of the fixed tube, and F the rotatory power of French turpentine. Then, from the first observation,

$$l\,(Cc - Ii + Gg) = Fl_1 \tag{1}$$

$$100\ Cl = 3Fl_2 \tag{2}$$

Again, from the second observation,

$$l\,\{(c + i)\ I - Gg\} = 110\ Cl'_1 \tag{3}$$

$$Il = 3{\cdot}3 \times Cl'_2 \tag{4}$$

remembering that on account of the dilution of the syrups in the second observation, it has been necessary to multiply the reading l'_1, l'_2 by the coefficient 1·1.

Adding the first and third of these equations we have

$$l\,(C + I)\,c = Fl_1 + 110\ Cl'_1, \tag{5}$$

and assuming for the sake of brevity

$$I = KC,\ G = K'C,\ F = K''C, \tag{6}$$

we have from this equation

$$l\,(1 + K)c = K''l_1 + 110\,l'_1. \tag{7}$$

But from equations (2) and (4) of the preceding system

$$100l = 3K''l_2,\ 10Kl = 33l'_2. \tag{8}$$

These equations give the values of K'', K, and equation (7) then gives the value of c, the quantity of cane sugar contained in a cubic inch of the given syrup.

If now we substitute the values (6) for I, G in the equation (1) we have

$$K'g - Ki = \frac{K''l_1}{l} - c = \frac{KK''l_1 - 110\,l'_1}{(1 + K)l} \tag{9}$$

We have thus one equation between g and i as soon as K' is determined.

To determine this coefficient, it will be necessary to make a solution of grape sugar containing 100 grains in 3 cubic inches, and to introduce this solution into the fixed tube in the place of the given syrup in the first observation. If l_3 be the corresponding reading of the scale, we have

$$100Gl = 3Fl_3$$

or, dividing this equation by the second equation of the same system,

$$K' = \frac{0_3}{l_1}.$$

We are thus enabled by the optical method to determine c and to find one equation between g and i. The second equation between these quantities is obtained by means of Barreswil's copper solution, from which, as is well known, we can find the total amount of sugar contained in the given syrup.

21. It may be well to mention some precautions which are necessary to the success of this experiment.

(1). It has been shown that the amount of cane sugar in the solution can be found by the optical method alone, without the aid of the copper solution. The truth of this conclusion, however, depends on the identity of the temperature in the first and second observations. For the rotatory power of inverted sugar varies very rapidly with the variation of the temperature, while that of cane sugar continues nearly constant. If, therefore, the temperature be not the same, the value of I will be different in these two observations, and it will be no longer possible to eliminate g and i, as has been done, by simply adding two of the equations obtained from these observations. It will, therefore, be no longer possible to determine the amount of cane sugar by the optica method alone. Now, it may often happen that the determination of the cane sugar is the only point of practical importance in the investigation. It is, therefore desirable that the observations described in p. 388 should be made at precisely the same temperature.

(2). It is known that the rotatory power of a solution of grape sugar, when freshly made, is greater than that of a solution which has been allowed to stand for some time. This power, however, may be reduced to its *minimum* value

by heating the solution to 180° Fahr. and then allowing it to cool. It is therefore necessary to heat to the same temperature the given syrup containing the mixture of the three sugars, in order to reduce to a fixed value the rotatory power of the grape sugar which it contains.

22. It must be remembered that in equations (1), (2), (3), (4) the inverted sugar is reckoned as if its atomic weight were the same as that of cane sugar. For the unit of weight of the inverted sugar is, as is evident from the way in which equations (3), (4) are derived from equations (1), (2), not as in other cases a certain fixed weight, but the amount of inverted sugar which is derived from the unit of weight of cane sugar. Hence, in order to obtain the true weight of the inverted sugar, the value of i in equation (9) should be augmented in the ratio $171 : 198 = 19 : 22$. Now in the application of Barreswil's solution, all the sugars are naturally reckoned as having the atomic weight of grape sugar. Hence it is desirable to correct i for the difference of atomic weight in the equation (9) which will then become

$$K'g - \frac{19}{22} Ki = \frac{KK''l_1 - 110l'_1}{(1 + K)l}.$$

For a similar reason, the equation obtained from the copper solution will be

$$\frac{22}{19} c + i + g = \text{const.}$$

IV.—*On the Change produced in the Rotatory Powers of the Organic Alkaloids by the Addition of an Acid.*

23. It is known that the rotatory powers of solutions of several among the organic alkaloids are sensibly changed by the addition of an acid. Thus for example, the rotatory powers of solutions of quinia and cinchonia are increased, while those of strychnia, brucia, morphia, and codeia are diminished, and those of narcotina, and nicotylia are (if a sufficient quantity of acid be used) reversed.

The rotatory power of a salt of any one of these alkaloids may be determined by one of the following methods :—

(1.) If the salt can be obtained in a crystalline form we may operate upon it as a separate body by the method given in Section II.

(2.) Having made a solution of the base (of known strength) we may add by measurement a sufficient quantity of acid to convert the whole of the base into the required salt, and then determine the actual rotation produced by the solution thus acidulated. Let α be the atomic weight of the acid used, β that of the base, s the strength of the original solution, ρ the rotatory power of the salt, and r the actual rotation (measured by the reading of the scale) produced by the acidulated solution. Then if the salt under consideration be formed by the combination of m atoms of acid with n atoms of base, the strength of the acidulated solution, i.e., the amount of salt contained in a cubic inch, will be

$$\frac{m\alpha + n\beta}{n\beta} s,$$

we have then

$$r = \frac{m\alpha + n\beta}{n\beta} s\rho_1$$

whence

$$\rho_1 = \frac{n\beta}{m\alpha + n\beta} \frac{r}{s}.$$

This and the following results will be somewhat simplified by taking as the unit of salt, not as in other cases a certain fixed weight, but the quantity of salt which is derived from a unit of base. With this modification the preceding result will become

$$\rho_1 = \frac{r}{s}.$$

(3.) A third method, which is frequently convenient, depends upon the addition of a measured quantity of acid, insufficient to form the required salt. This method is as follows :—

Let it be supposed that the salt under consideration formed by the combination of m atoms of acid with n atoms of base, is the lowest or least acid salt

which the given acid and base are capable of forming. If then an amount of acid, insufficient to convert the whole of the base into the required salt, be added to the solution, the effect of such addition will be to convert a part of the base into salt, leaving the remainder in its free or uncombined state.

Let a be the amount of acid added to each cubic inch of solution, and b the amount of base originally present in the same fluid volume. Then it is evident that the amount of base which enters into combination with a is

$$\frac{n\beta}{m\alpha}a$$

Hence adopting as before for the unit of salt the amount of salt corresponding to a unit of base and denoting by r_1 ρ_1, the rotatory powers of the base and salt, and by r the actual rotation caused by the acidulated fluid, we have

$$r = \frac{n\beta a}{m\alpha}\rho_1 + \left(b - \frac{n\beta a}{m\alpha}\right)r_1,$$

an equation by which the rotatory power ρ_1 of the salt may be determined, when that of the base is known.

If the salt whose rotatory power is required be not the least acid combination which can be formed, this process requires a certain modification. Suppose that the salt immediately below (in the sense explained above) that under consideration is formed by the combination of m' atoms of acid with n' atoms of base, where

$$\frac{m'}{n'} < \frac{m}{n}.$$

Suppose also that the amount of acid added to the solution is more than sufficient to convert the whole of the base into the lower salt, but insufficient to convert it all into the higher salt. This condition is expressed algebraically thus

$$a > \frac{m'\alpha}{n'\beta}b < \frac{m\alpha}{n\beta}b.$$

Let x be the quantity of acid and y the quantity of base which are combined in the higher or more acid salt. Then $a - x$, $b - y$, are the quantities

of acid and base respectively combined in the lower salt. We have then for the determination of x and y the equations

$$\frac{\beta x}{\alpha y} = \frac{m}{n}, \quad \frac{\beta(a-x)}{\alpha(b-y)} = \frac{m'}{n'}.$$

Solving these equations for x, y, we find

$$x = \frac{m}{\beta} \cdot \frac{n'\beta a - m'\alpha b}{mn' - m'n},$$

$$y = \frac{n}{\alpha} \cdot \frac{n'\beta a - m'\alpha b}{mn' - m'n}.$$

Hence, preserving the same unit notation as before, the actual rotation due to the amount of the higher salt which is present in the solution is

$$\frac{n}{\alpha} \cdot \frac{n'\beta a - m'\alpha b}{mn' - m'n} \rho_2.$$

The amount of base which enters into composition in the lower salt being $b - y$, or

$$\frac{n}{\alpha} \cdot \frac{m\alpha b - n\beta a}{mn' - m'n},$$

the rotation due to the presence of this salt is

$$\frac{n'}{\alpha} \cdot \frac{m\alpha b - n\beta a}{mn' - m'n} \rho_1.$$

Hence, denoting as before by r the complete rotation caused by the acidulated mixture

$$r = \frac{(n'\beta a - m'\alpha b)\, n\rho_2 + (m\alpha b - n\beta a)\, n'\rho_1}{(mn' - m'n)\, \alpha}.$$

If, therefore, the rotatory power, ρ_1, of the lower salt be known, we can determine by this equation ρ_2 the rotatory power of the higher salt. The investigation must, therefore be commenced by adding to a solution of the base a quantity of acid insufficient to convert the whole into the least acid salt

which can be formed. If this salt be formed by the combination of m_0 atoms of acid with n_0 atoms of base, and if ρ_1 be its rotatory power, r the actual reading of the scale for the acidulated mixture, and a the amount of acid added to each cubic inch of solution, we have as in the first case,

$$r = \frac{n_0 \beta a}{m_0 \alpha} \rho_0 + \left(b - \frac{n_0 \beta a}{m_0 \alpha}\right) r_1 . \tag{5}$$

This equation determines ρ_0 the rotatory power of the least acid salt. Equation (4) gives then the rotatory power of the next, and so on for any number of salts which the given base and acid are capable of forming.

This method of course assumes as known the chemical constitution of the several salts. If this were not so, the several quantities

$$\frac{m}{n} \quad , \quad \frac{m'}{n'}$$

would be unknown, and the equation given would be insufficient. Here, however, a question arises of the highest interest in the application of optics to chemistry, namely—Does the optical method give us any information as to the number and constitution of the salts which are formed by the combination, in different proportions, of a given acid with a given base. I shall consider this question in the next section (V).

24. The second method may be conveniently employed when the base and acid are only capable of combining in one proportion. For in such a case it will be sufficient to add the acid in an amount very slightly in excess of that which is sufficient to form that compound. The same method may be applied to determine the rotatory power of the most acid salt which can be formed.

But in all cases in which the salt whose rotatory power is sought is not the most acid salt which the base and the acid are capable of forming, the third method is to be preferred. For in such a case any error either by excess or defect in the amount of acid added to the solution will necessarily cause a considerable error in the value obtained for the rotatory power by the second method.

Thus, for example, the second method may be advantageously applied to the salts formed by the combination of strychnia or brucia with nitric or hydrochloric acid, inasmuch as these bases and acids respectively form but a single com-

bination. But the same method cannot be as advantageously applied to the lower salts of quinia or cinchonia (whether these salts are to be denominated neutral or basic). For if the acid added to the solution be in excess, this solution will contain a portion of the higher (or more acid) salt; and if the acid be in defect the solution will contain a portion of uncombined base. In either case the method will give an inaccurate result.*

V.—*Application of the Optical Method to the determination of the several compounds which may be formed by combining the same base with the same acid.*

25. We have seen, p. 395, that if the least acid salt which can be formed by the combination of a given acid with a given base contain m atoms of acid, and n atoms of base, and if a quantity, a, of acid be added to each cubic inch of a solution of the base, the actual rotation caused by the acidulated liquid is given by the equation

$$r = \frac{n\beta a}{ma}\rho_1 + \left(b_1 - \frac{n\beta a}{ma}\right) r_1 \qquad (1)$$

the amount of acid which is added being insufficient to convert the whole of the base into salt.

Now it is evident that the term $b_1\, r_1$ denotes the actual rotation caused by the unacidulated solution. Hence, denoting this rotation by r', we have

$$r - r' = ka$$

where

$$k = \frac{n\beta}{ma}(\rho_1 - r_1).$$

It is evident, therefore, that as long as the quantity of acid added to the solution is insufficient to convert the whole of the base into the least acid salt which can be formed, the fraction

$$\frac{r - r'}{a}$$

* Vid. Note on Sec. V.

will preserve a constant value. We shall now show that in general as soon as the first definite combination is passed over, this fraction will commence to vary.

Retaining the same notation for the base and the first salt, let ρ_2 be the rotatory power of the next salt, if there be one. Let it be supposed that this salt is formed by the addition of m' atoms of acid to n' atoms of base. Let it be supposed that the quantity of acid, a, is more than sufficient to convert the whole of the base, b, into the first salt, but insufficient to convert the whole into the second salt. None of the base will then remain in the free state. A part will combine with the acid to form the first salt, the remainder combining with the remainder of the acid to form the second salt. Let x, y, be the quantities of acid which enter into the composition of these two salts respectively. We have then, for the determination of x and y, the equations

$$x + y = a, \quad \frac{n\beta x}{ma} + \frac{n'\beta y}{m'a} = b$$

whence, putting $\quad n = \mu m, n' = \mu' m', \beta = \iota a,$

$$x = \frac{b - \iota\mu' a}{(\mu - \mu')\iota}, \quad y = \frac{\iota\mu a - b}{(\mu - \mu')\iota}$$

the corresponding quantities of base which are combined with these portions of acid being

$$\frac{n\beta x}{ma} = \frac{\mu(b - \iota\mu' a)}{\mu - \mu'}, \quad \frac{n'\beta y}{m'a} = \frac{\mu'(\iota\mu a - b)}{\mu - \mu'}.$$

The rotation caused by transmission through the fluid is then

$$\frac{(b - \iota\mu' a)\mu\rho_1 + (\iota\mu a - b)\mu'\rho_2}{\mu - \mu'}.$$

Denoting this, as before, by r, and the rotation caused by the unacidulated solution by r' $(= br_1)$ we have

$$r - r' = \frac{b\{\mu\rho_1 - \mu'\rho_2 - (\mu - \mu')r_1\} + \iota\mu\mu' a\,(\rho_2 - \rho_1)}{\mu - \mu}. \qquad (2)$$

It is evident from the form of this expression that the value of the fraction

$$\frac{r - r'}{a}$$

will not remain constant when a varies, unless the coefficient of b vanish. This would give

$$\mu\rho_1 = \mu'\rho_2 + (\mu - \mu')r_1. \tag{3}$$

26. We infer therefore as follows:—

(1.) If we perform a series of experiments in which the quantities of acid added are

$$a_1, a_2, \qquad a_n, a_{n+1} \ldots$$

tabulating for each experiment the value of the fraction

$$\frac{r_n - r'}{a_n}$$

(which we shall denote generally by A_n), and if we find that the value of this fraction having remained constant from a_1 to a_n, changes sensibly in passing from a_n to a_{n+1}, we infer at once that a definite combination has been passed over in this interval. In other words, the amount of acid requisite to form the least acid combination of the given acid with the given base lies between a_n and a_{n+1}. Now we know that, in the formation of a salt, the number expressing the ratio of the number of atoms of the acid to the number of atoms of the base is either a very small integer or a very simple fraction. If, therefore, we write down the value of the two fractions which express this ratio in the two experiments respectively,

$$\frac{\beta a_n}{ab}, \quad \frac{\beta a_{n+1}}{ab}$$

it will be found that a very simple integer, or a very simple fraction lies between them, and, by taking a_n, a_{n+1} sufficiently near in value, this integer or fraction may be determined with perfect precision.

(2.) As soon as the highest or most acid salt has been reached, the observed rotation (corrected as usual for dilution) will show no further change, or will

vary continuously and very slowly, this variation being due to the change in the solvent caused by the continual addition of acid.

(3.) So long as the value of A_m remains unchanged we infer either that no definite combination has been passed over, or that if there be such a combination, a certain special relation exists between its rotatory power and the power of the compounds which precede and follow it. This relation is expressed by the equation (3).

Thus, for example, let the given base and acid be capable of forming two salts, namely, a neutral and an acid salt, or a basic and a neutral salt. In these cases we have respectively,

$$\mu = 1,\ \mu' = \tfrac{1}{2},\ \text{and}\ \mu = 2,\ \mu' = 1.$$

In both these cases eq. (3) becomes

$$2\rho_1 = \rho_1 + r_1.$$

If this condition be satisfied, the formation of the lower or less acid salt in each case would not be indicated by the optical method.

If $\rho_2 = \rho_1$ it is easy to see that the formation of the higher salt would not be indicated. In all other cases the optical method would indicate the formation of both salts.

27. In general, if the quantity of acid added to the solution be less than that which is required for the formation of the nth salt, but exceed the amount required for the formation of the $(n-1)^{th}$ salt, and if this excess be denoted by a, the actual rotation caused by the acidulated fluid will be expressed by an equation of the form

$$r = Aa\rho_n + (B - Aa)\rho_{n-1},$$

A, B being constants. Hence it is evident that if distances be measured along the axis of abscissæ to represent the quantities of acid added to a given bulk of the solution, and ordinates be raised proportional to the corresponding rotations (corrected for dilution), the locus of the extremities of these ordinates will be a right line, so long as the amount of acid so represented is more than enough to convert the whole of the base into the $(n-1)^{th}$ salt, but not

enough to convert the whole of the base into the nth salt. Beyond these limits on either side the locus is still a right line, but not in general the same line.

Hence if a series of experiments be made, commencing with the addition of a very small quantity of acid, the graphic representation (made as above described) of these experiments will be a series of right lines.

The first of these lines ought to cut the axis of ordinates at a distance from the origin corresponding to the rotation caused by the unacidulated fluid. If, therefore, it be found that this is sensibly untrue, we infer that a definite combination has been passed over either before the first experiment or between the first and the second. But this may always be obviated by making the first and second additions of acid in quantities sufficiently small. It is in general necessary that the successive additions of acid should be so near to each other in quantity, that at least three experiments should correspond to each definite compound. If this precaution be observed, the locus of the extremities of the ordinates will be a series of right lines, each line corresponding to a distinct compound. When the highest or most acid compound has been attained, the acidulation, if extended further, will produce no chemical change. The rotation caused by the fluid ought thenceforth to be constant, and therefore the locus of the extremities of the ordinates ought to be a line parallel to the axis of the abscissæ. This line may, however, be inclined to the axis at a small angle. For although no chemical change can be produced in the dissolved substance, yet the solvent itself is necessarily changed by acidulation continued after the formation of the highest salt. This change, as we have seen before, (p. 384) may affect the rotation, and therefore the locus may not be accurately parallel to the axis. But as the quantity of acid is in general very small as compared with the quantity of the solvent, this change, and therefore the effect produced on the rotation, is necessarily slow. Hence the angle of inclination is small. The graphic representation of the effect produced by the continued addition of acid, after the formation of the highest salt has been completed, differs also from that which was obtained while the several compounds are in process of formation in the *continuity* of the locus which is thus constructed. This locus will thenceforward show no *breaks* such as those which marked the completion of each definite compound. It will be a con-

tinuous line, in general nearly or completely straight, and nearly or completely parallel to the base line.

28. The number of definite compounds formed by the base and acid is thus in general indicated by the number of *breaks* in the locus, constructed as above described. But further it is easily seen that the *nature* of these compounds, or in other words the relative numbers of atoms of base and acid which enter into their composition, is indicated by the *position* of those breaks. Thus, referring to the equation

$$r = \frac{n\beta a}{ma}\rho_1 + \left(b_1 - \frac{n\beta a}{ma}\right)r_1$$

it is evident that the first break in the locus will occur when

$$b_1 = \frac{n\beta a}{ma},$$

whence

$$\frac{n}{m} = \frac{b_1 a}{\beta a}.$$

In the right-hand member of this equation, b_1 is known from the strength of the solution employed, and a from the point in the base line above which the break in the locus occurs. We have then from the equation the value of the ratio $n \div m$, which defines the salt.

This composition will be more distinctly indicated if we make a slight change in the quantities which are graphically represented.

Let the number of atoms of base in a unit of solution be represented (arbitrarily) by 100, and let x denote the number of atoms of acid added to this unit. Let y denote the change in the actual rotation caused by the addition of the acid, divided by the quantity of base present in a unit of solution. Then

$$x = \frac{100\beta a}{ab_1}, \qquad y = \frac{r - b_1 r_1}{b_1}.$$

Hence and from (1)

$$x = \frac{100my}{(\rho_1 - r_1)n}.$$

Let y and x be measured along the axes of ordinates and abscissæ respectively. The first of the lines of which the locus consists will then pass through the

origin, and there will be in general a break at every point defined by any of the equations

$$\frac{n\beta a}{ma} = b_1, \quad \frac{n'\beta a}{m'a} = b_1 \text{ \&c.},$$

or,

$$\frac{n}{m} = \frac{x}{100}, \quad \frac{n'}{m'} = \frac{x}{100}.$$

It is then only necessary to measure the value of x corresponding to each break in the locus in order to find the values of the several ratios $n \div m$, $n' \div m'$, &c., defining the successive salts.

29. As an example of this method, we may consider the combination of quinia with nitric acid. The experiments are conducted as follows :—

A solution of quinia is made in proof spirit containing five grains in each cubic inch. Successive portions of this solution are acidulated with nitric acid diluted with proof spirit, the first addition of acid being in a very small quantity. Assuming 100 to represent the number of atoms of quinia in each cubic inch of solution, the first column in the following table gives the number of atoms of HNO_3 in each acidulated mixture. The second column gives the successive differences in the reading of the scale for the un-acidulated and the acidulated fluids, the latter reading being corrected for the dilution caused by the addition of the acid. In the third column is given the ratio of the numbers corresponding to each other in the first and second columns.

I.	II.	III.
20	149	7·4
35	277	7·9
61	459	7·5
83	629	7·6
103	772	7·6
137	944	6·9
171	1082	6·3
203	1184	5·8
258	1169	4·5

30. It is evident from inspection of the table that the numbers in the third column are sensibly the same, while the number of atoms of acid in the solution is increased from zero to 103, and that after this the number in the third column begins to vary sensibly. We infer, therefore, that the completion of a definite salt is reached at or about the degree of acidulation indicated by the number 103 in the first column. This is evidently the neutral or the basic salt, according to the value which we choose to assign to the atomic weight of quinia, and corresponds to 100 in the first column. It is evident also that no indication of any salt lower than this is given by the table.

If now quinia were capable of forming but *one* combination with nitric acid, the numbers in the second column should remain nearly constant, the only cause of variation being the slow change of the solvent by the addition of acid which would necessarily remain free in the solution. It is evident that this is not the case. The numbers in the second column, although not varying as rapidly as before, do nevertheless show a very marked change. Moreover, it will be seen that this variation terminates when the number of atoms of acid reaches 200, being double the number of the atoms of the base, or equal to that number, according to the atomic weight which we assign to quinia. After this number has been reached, the second column, as will be seen by comparing the last two numbers, shows only a very slight variation, and that in a direction opposite to that which appeared before. This variation, which is indeed very little greater than that which might arise from the errors of experiment, is probably due to the change in the solvent.

On the whole then, it appears from this investigation that quinia is capable of forming two combinations with nitric acid, represented respectively by

$$Qu, HNO_3, \quad Qu, 2HNO_3$$

or

$$2Qu, HNO_3, \quad Qu, HNO_3,$$

according to the atomic weight which we assign to quinia.

It appears also that no indication of the formation of a salt higher than the second of these is presented by the optical method.

31. It remains to consider the phenomena presented when the amount of acid added to the solution exceeds 100 atoms, and falls short of 200.

But, on account of the importance of the question, I have thought it better to examine separately and more minutely the combination formed by adding acid to the lower salt. This was done as follows :—

A solution of the ordinary nitrate of quinia in proof spirit was made, containing five grains in the cubic inch. To successive portions of this is added, in gradually increasing proportion, nitric acid diluted with proof spirit, the dilute acid containing 1·5 per cent. of HNO_3. The reading of the scale for each acidulated solution is then observed, and the difference between the result, corrected for dilution, and the reading for the unacidulated mixture is tabulated. The results are contained in the following table constructed as before :—

n	$r - r_0$	$\frac{r - r_0}{n}$	Temp.
302	135	·447	58
467	187	·398	60
623	227	·364	60
760	284	·373	60·5
911	318	·349	60·5
1049	336	·320	61

Number of atoms of Qu, HNO_3 in each solution—1000.

32. It is obvious from inspection of this table that the numbers in the third column do not remain constant, but diminish perceptibly with the increase of the acid. As the temperature is nearly constant, and the solvent remains unchanged, the violation of the law stated in p. 396, appears to indicate that the whole of the nitric acid does not enter into combination with the lower salt, but remains free, or, as I should prefer to say, combined with the solvent. It seems that we must look on the solvent as in a certain respect partaking of the properties of a base. It is known* that when an acid is presented to a mixture of two bases, no insoluble compound being formed, neither base takes the whole of the acid, but that this acid is divided between the two. The investigation, of which I have given the results above, appears to show that the

* Vid. Sec. VI.

same phenomenon may take place when a single base is present, the partition in this case being between this base and the solvent used. In the present case, the solvent being proof spirit, we are at liberty to suppose that the water, or the alcohol, or both, possess the properties of a base.

In a subsequent case I shall show that this property is possessed by water.

33. The general class of questions to which this belongs will be more fully considered in the following section : for the present it is enough to say that it seems a more probable account of the phenomenon to speak of the acid which does not combine with the quinia as taken by the solvent than to describe it as absolutely free, although in chemical presence of a substance which is capable of combining with it.

It is right to observe that these experiments were attended by much difficulty in consequence of the very imperfect *compensation.* The power of *dispersing* the planes of polarization of the elementary rays possessed by the salts of quinia is very great, even as compared with that of American oil of turpentine. We are, therefore, met by the difficulty stated pp. 384–5, the results being necessarily liable to a corresponding error. Probably the interruption to the general law of diminution of the figures in the third column, occurring in the fourth experiment, is due to this cause. But there can be no doubt of the general fact that these numbers form a diminishing series. The changeo f temperature is quite too small to account for this variation.

34. As another instance of the application of this method, we may consider the effect produced by the addition of hydrochloric acid to an aqueous solution of the ordinary hydrochlorate of quinia.

It has been inferred from the researches of Andrews that an acid salt is decomposed by solution in water. Hence, if to an aqueous solution of a neutral salt we add a fresh portion of its own acid, it is evident that the acid salt will not be formed, and conversely, if we have evidence that a chemical combination does take place between the acid and the dissolved salt, we infer that, if the original inference be true, this salt is not a neutral salt.

If now, applying the method of p. 404, we add to separate portions of an aqueous solution of hydrochlorate of quinia, continually-increasing quantities

of acid, tabulating as before for each acidulated solution the value of the fraction

$$\frac{r - r_0}{n},$$

we shall be able to determine by the nature of the series formed by these values whether or not chemical combination has taken place. The annexed table is constructed as in p. 404.

No. of atoms of acid.	Change in rotation.	$\frac{r - r_0}{n}$.	Temp.
454	371	·817	59°
713	580	·814	60°·5
932	742	·796	61
1179	774	·656	61·5
1332	742		61·5

35. It appears from inspection of this table that the fraction is nearly constant for the first three experiments. The value which corresponds to the fourth experiment shows a sensible variation, and after this the numbers in the second column instead of increasing as before, begin to diminish slowly.

Hence, if the locus were constructed as in p. 401, this locus would show a *break* for a value of the abscissa intermediate between values (3) and (4). After this the locus would become nearly parallel to the base line, being inclined to that line at a very small angle. We, infer, therefore, as before, that the acid does combine with the hydrochlorate of quinia, and that this combination becomes complete for a value of the number of atoms of acid added to the solution intermediate between 93 and 118. It is plain, therefore, that the salt formed by the addition of an atom of hydrochloric acid to an atom of the ordinary hydrochlorate of quinia is not decomposed (at least completely) by solution in water. If then we are to accept the above-mentioned theorem, we cannot look upon this salt as acid. We must regard it as neutral, and therefore consider the ordinary hydrochlorate to be a basic salt.

36. Schorlemmer states (Chemistry of Carb. Comp., p. 478), that the salt *Qu*, 2*HCl*, which he terms the normal salt, is decomposed by solution in water into free acid, and the basic salt *Qu*, *HCl*. He has not given the method by which he arrives at this conclusion, but if it be meant that the salt is *completely* decomposed, the considerations just adduced seem to show that the conclusion is inadmissible. If the salt were completely decomposed by solution it is evident that it could not be formed. The acid added would then have no effect but in changing the solvent, and the rotatory power of the acidulated solution would vary, either not at all, or slowly and continuously. This, as we have seen, is not the case. The ratio of the change in the rotation to the quantity of acid added remains nearly the same until the amount of acid is sufficient to form the salt *Qu*, 2*HCl*, and then undergoes an abrupt change indicating the completion at or near the point of a definite chemical combination. This combination is of course *Qu*, 2*HCl*.

It appears, however, from inspection of the foregoing table that the numbers in the third column are not even from the beginning absolutely constant, but diminish slowly with the increase of the acid. We infer then, as in the case of the combination of quinia with nitric acid, that the whole of the acid added to the solution does not combine with the lower salt, and therefore that if the higher salt were dissolved in water it would be partially decomposed. Here as in the case of the higher combination of quinia with nitric acid, the solvent appears to act in a manner analogous in this respect to the action of a base, determining a division of the acid between itself and the lower salt, which is to be regarded as the second base.

VI.—*Problem of Chemical Equilibrium.*

37. The problem which I propose to consider in the present section has been long regarded as one of the obscure questions of chemistry, and to which indeed chemical science furnishes no sufficient answer. Although the nature of the problem renders it in the highest degree probable that it admits of an exact mathematical solution, no such solution has ever been given, certain general principles only having been pointed out to which the solution must necessarily conform. The problem, as ordinarily enunciated, is as follows:—

When an acid is brought into chemical presence (as in a solution) of two bases, the quantity of acid being insufficient to convert the whole of both bases into the lowest salts which it is capable of forming, what are the combinations actually formed? If the acid be added gradually, is the combination with one base completed before the combination with the other base is begun, or is there from the beginning a division between the two bases? If there be a division, by what law is it governed?

38. Chemical science has given but a partial answer to these questions. It is in general agreed that there *is* a division of the acid between the two bases to which it is presented, and this principle of division enables us, as is well known, to solve the problem completely for the case in which either of the compounds formed is insoluble, and is, therefore, immediately upon its formation, removed from the sphere of chemical activity. But the other and more important case, in which all the four compounds continue in chemical presence of each other, remains still unsolved. Chemists have indeed agreed upon one principle by which the solution of the problem is largely influenced, namely the effect of *mass*. It is agreed that when we augment the relative quantity of either bases, we augment also the proportion of acid which enters into combination with that base. But the actual law according to which the division of the acid between the two bases is effected, has not yet been assigned. I hope to supply this deficiency in the present section.

39. The ratio in which an acid divides itself between two bases depends, as has been said, upon the quantities of the bases which are in chemical presence of each other. But it does not appear to have been so clearly seen that this ratio must depend also upon the quantity of the acid. It is, however, easily shown that it must be so. Let it be supposed that the acid is added in successive portions. Then, unless the division which ensues be in exact proportion to the masses, which is rarely the case, one base will take in proportion to its mass a larger quantity of acid than the other. A quantity of this base larger in proportion will, therefore, be converted into salt, and consequently the quantity of this base remaining free, will be less in proportion to the free portion of the other base than before the addition of the acid. The second addition of acid will, therefore, be made under altered circumstances. The mass of the stronger base will have been relatively diminished, and this base will therefore take a smaller part of the acid than before. The ratio of division will consequently have different values in the first and second additions of acid. Hence it is evident, as has been said, that this ratio depends upon the quantity of acid used in the experiment.

If then we denote by b, b' the total quantities of each base present in a given bulk of solution, and by a the quantity of acid so present, the ratio of division will be expressed by an equation of the form

$$\frac{x}{x'} = F(b, b', a), \tag{1}$$

where x, x', are the quantities of acid taken by each base respectively. The solution of the problem thus enunciated would therefore require the determination of the form of the function F. The method to be hereafter explained does enable us to effect this determination, but I have found it in all respects more satisfactory to adopt a different statement of the whole question. I propose, therefore, to consider the problem as one of *chemical equilibrium*, proceeding now to define the sense in which these terms are used.

Chemical Equilibrium.

40. *Two or more substances may be said to be in chemical equilibrium, if they can be brought into chemical presence of each other (as in a solution) without the formation of any new compound, or change in the amount of any of the former compounds which have been thus brought together.*

It must be observed that this definition does not necessarily imply absolute rest. Chemical equilibrium, as here defined, may result either from the absolute unchangeableness of each molecule of the several compounds which are present, or from a continual process of decomposition and recomposition, these effects being in amount such as exactly to balance each other.

Thus, for example, soda and sulphate of soda, if present in the same solution, are necessarily in chemical equilibrium, the amount of each of these substances being invariable. But it is not implied by this statement that the combination of soda and sulphuric acid by which the sulphate is formed continues throughout between the same molecules. The same effect will be produced by a series of decompositions and recompositions, each molecule of sulphate being in its turn decomposed, the acid uniting with a molecule of the free base, and the base which had been in combination with the molecule of acid becoming free. In the language of mechanics, chemical equilibrium may be either *statical*, denoting the absolute unchangeability of each molecule, or *dynamical*, denoting merely the equivalence of opposite effects. I shall adduce further on some considerations which seem to show that the latter account of the phenomenon is more probable than the other, but for the present it is sufficient to observe that either account is consistent with the definition of chemical equilibrium, which I have here given.

41. Now, let it be supposed that to a solution containing two bases, we add a quantity of acid insufficient to convert both bases into the least acid salts which they are respectively capable of forming. Suppose also that neither of these salts is insoluble. We shall then in general have in the solution four substances in chemical presence of each other ; namely, two salts formed by the combination of the acid with portions of each base, and the remaining

portions of these bases which necessarily continue uncombined. After a certain time, which is probably in general very short, the quantities of these several substances will become permanent. This is expressed, in the language of the foregoing definition, by saying that chemical equilibrium exists between the four substances which are present in the solution. The same would be true if these four substances had been in the same quantities originally brought together in the solution. The problem which I propose to consider is; what relation or relations must subsist between the quantities of these several substances, in order that chemical equilibrium may be preserved when they are brought into chemical presence of each other.

42. Before proceeding to explain the special mode of investigation which I have adopted in the present case, I may observe that we are enabled to solve without any reference to this special method the first question which naturally presents itself in our inquiry.

What is the *number* of the equations of equilibrium? When an acid is added to a mixture of solutions of two bases, we have in general, as has been said, four substances present. Are these substances connected by one, two, or three equations? To answer this question, it is only necessary to observe that these four quantities are necessarily functions of *three* independent variables—namely, the original quantity of each base, and the original quantity of acid; the first two of these, and, within certain limits, the third, being perfectly arbitrary. It is evident, therefore, that the four quantities which are in chemical presence of each other, are connected by a single equation. In the language of mechanics, there is but one equation of equilibrium. This single equation is necessarily of the form

$$U = 0 \tag{2}$$

where

$$U = F(B_1, B_2, S_1, S_2),$$

B_1, B_2, being the quantities of uncombined base, and S_1, S_2, the quantities of salt. The object of the present investigation is to determine the form of the equation

$$U = 0.$$

43. The substances selected for experiment were quinia, brucia, and hydro-

chloric acid, the latter being used in the form of a very dilute solution of the gas in proof spirit, which was also the solvent employed for each of the two bases. It is known (Table I.) that a solution of either of these bases causes a powerful left-handed solution of the plane of polarization of a transmitted ray. It is also known (Table III.) that the rotatory power of a salt of quinia *exceeds* the rotatory power of the base, while that of a salt of brucia is less than that of the base. Hence it is evident that, in a solution containing the four above-mentioned substances, if by any means a portion of the acid which is combined with the quinia be transferred to the brucia, the rotatory power of the mixture will be diminished in two ways; namely—1. By the substitution of a certain quantity of free quinia for a corresponding quantity of a salt of quinia; and 2. By the substitution of a certain quantity of a salt of brucia for a corresponding quantity of free brucia. The subsequent part of the investigation is largely dependent upon this principle.

Criterion of Equilibrium.

44. If two fluids, whose rotatory powers are represented by r_1, r_2, respectively, be mixed in the proportion $m : n$, and if there be neither dilatation nor contraction, it is evident that the actual rotations which would be produced by the quantities of each fluid which are present in the mixture, if these fluids acted independently of each other, would be

$$\frac{mr_1}{m+n} \text{ and } \frac{nr_2}{m+n}$$

respectively. The complete rotation, r, is therefore the sum of these, or

$$r = \frac{mr_1 + nr_2}{m+n}. \tag{3}$$

The condition of independent action will be fulfilled by two solutions, if the solvents be the same, and if, moreover, the substances dissolved be such, that when brought into chemical presence of each other, they are in chemical equilibrium. On the other hand, if the substances be not in chemical

equilibrium, the actual rotation produced by the mixture will in general have a value sensibly different from that given by the foregoing equation.

Thus, for example, let it be supposed that each of the two solutions contains the four above-mentioned substances B_1, B_2, S_1, S_2, and let it be supposed further that these substances cannot be brought into chemical presence of each other without a rupture of equilibrium. This rupture of equilibrium can take place only in one of two ways—namely, either by a partial decomposition of the hydrochlorate of quinia, the liberated acid combining with a part of the free brucia, or by a partial decomposition of the hydrochlorate of brucia, the liberated acid combining with part of the free quinia. It is evident that the actual rotation will be less or greater than that indicated by the equation (3) according as equilibrium is broken in the former or in the latter of these two ways. On the other hand, if the actual rotation be equal to that indicated by eq. (3), it is evident that there has been *no* rupture of equilibrium, and therefore that the substances contained in the two solutions are present in such quantities as to satisfy the condition of equilibrium. It is, therefore, only necessary to measure separately the rotation caused by each solution, and having added these solutions in any ratio, to measure the rotation caused by the mixture. The agreement of this latter value with that given by eq. (3) is the *experimental criterion of equilibrium.*

I now proceed to the reduction of the general equation (2).

First Experiment.

45. Two solutions (in proof spirit) of quinia and brucia respectively, are mixed together, and a quantity of dilute hydrochloric acid, considerably less than that which is required to convert the whole of both bases into salts, is added to the mixture. One of the tubes of the polariscope is filled with the acidulated mixture, the other being filled with the same mixture diluted with the same spirit in the proportion $1 : m$. The readings of the scale being taken for the two solutions are found to be, within the limits of error of observation, as $m : 1$.

Now, if there were any rupture of equilibrium, it is evident (Art. 44) that the second reading would not be less than the first in the exact proportion of the dilution.

We infer then that there has been no rupture of equilibrium, or in other words that

Equilibrium is not troubled by dilution.

Hence, it is evident that equilibrium is preserved so long as the ratios of the quantities B_1, S_1, B_2, S_2, retain the proper values. We have then

$$U = F\left(\frac{B_1}{S_1}, \frac{B_2}{S_2}, \frac{B_1}{B_2}\right), \tag{4}$$

all the other ratios being capable of being expressed in terms of these three.

Second Experiment.

46. A solution of quinia is made containing in each cubic inch a known quantity of anhydrous quinia, the solvent being proof spirit.

A solution of brucia is made in the same solvent containing in each cubic inch a known quantity of anhydrous brucia.

These solutions are mixed in a known proportion.

To a measured quantity of this mixture is added a measured quantity of dilute hydrochloric acid of known strength.

The tubes of the polariscope are now filled, one with the unacidulated mixture, the other with the acidulated mixture, and the reading of the scale is carefully taken for both.

Let r_1, r_2, be the rotatory powers of the two bases, and ρ_1, ρ_2, the rotatory powers of the salts, the unit of mass for the salt being, as before, the quantity of salt which is derived from a unit of base. Let b_1, b_2, a be the quantities of each base and of HCl, which are present in a unit of bulk of the acidulated mixture. Let also β_1, β_2, α be the atomic weights of quinia, brucia, and hydrochloric acid respectively. Then if we denote by x the quantity of acid

which enters into combination with the quinia, the quantities of combined quinia and free quinia respectively will be for a unit of mixture

$$\frac{\beta_1 x}{a} \text{ and } b_1 - \frac{\beta_1 x}{a}.$$

Similarly the quantities of combined and free brucia respectively will be

$$\frac{\beta_2(a-x)}{a} \text{ and } b_2 - \frac{\beta_2(a-x)}{a}.$$

Hence denoting by r the rotation caused by the acidulated mixture, we have

$$r = \frac{\beta_1 x}{a}\rho_1 + \left(b_1 - \frac{\beta_1 x}{a}\right) r_1 + \frac{\beta_2(a-x)}{a}\rho_2 + \left\{b_2 - \frac{\beta_2(a-x)}{a}\right\} r_2.$$

Solving this equation for x, we find

$$x = Aa + B(r - b_1 r_1 - b_2 r_2), \tag{5}$$

where

$$A = \frac{\beta_2(r_2 - \rho_2)}{\beta_2(r_2 - \rho_2) + \beta_1(\rho_1 - r_1)}$$
$$B = \frac{a}{\beta_2(r_2 - \rho_2) + \beta_1(\rho_1 - r_1)}. \tag{6}$$

Equation (5) gives the value of x, the quantity of acid combined with the quinia, as soon as the constants A and B have been determined. This equation may be written in a form more easily applied to experiment, thus—

It is evident that the rotation caused by the unacidulated mixture is $b_1 r_1 + b_2 r_2$. Hence, denoting this rotation by r_0, we have

$$x = Aa + B(r - r_0). \tag{7}$$

The value of x being found from this equation, we have at once the quan-

tities of each of the four substances which are present in the solution, these quantities being

$$B_1 = b_1 - \frac{\beta_1 x}{a}, \qquad S = \left(\frac{\beta_1}{a} + 1\right)x.$$
$$B_2 = b_2 - \frac{\beta_2(a-x)}{a}, \qquad S_2 = \left(\frac{\beta_2}{a} + 1\right)(a-x). \tag{8}$$

Determination of the Constants A and B.

47. We have seen (p. 396) that when a quantity of acid (insufficient to convert the whole of the base into the lowest salt) is added to a solution of an alkaloid, the change in the rotation is given by the equation

$$\frac{r-r'}{a} = \frac{n\beta}{ma}(\rho_1 - r_1).$$

In the cases of the hydrochlorates of quinia and brucia $m = n$. If then we denote by D_1, D_2 the values of the quotient

$$\frac{r-r'}{a}$$

or quinia and brucia respectively, it is evident that

$$A = \frac{D_2}{D_2 + D_2}, \quad B = \frac{1}{D_1 + D_2}. \tag{9}$$

These constants are, therefore, determined experimentally by filling one of the tubes of the polariscope with a solution of quinia, and the other with the same solution acidulated with a quantity of hydrochloric acid, not sufficient to convert the whole into the hydrochlorate. The readings of the scale being taken for both solutions, we obtain, by substraction, the value of D_1, and by the same method the value of D_2. The constants A, B, are then determined by the equations (9).*

* Vid. Table IV.

Third Experiment.

48. We have seen in the preceding experiment how to determine a system of values of the four quantities B_1, S_1, B_2, S_2, which satisfy the condition of equilibrium. In the present experiment I commence by adding to a solution of quinia such a quantity of dilute hydrochloric acid, that the ratio of the quantity of base remaining free to the quantity of salt may be the same as that of B_1 to S_1 in the foregoing experiment. Let b'_1 be the quantity of base originally present in a cubic inch of the solution, and x' the quantity of acid added. Then it is easily seen that x' is determined by the equation

$$\frac{x'}{b'_1} = \frac{x}{b_1},$$

x, b_1, having the same meaning as in the foregoing experiment.

Similarly, a solution of brucia is so acidulated that the ratio of the free base to the salt shall be the same as that of B_2 to S_2.

The tubes of the polariscope being filled with these solutions respectively, the reading of the scale is taken for each. Let these readings be r_1 and r_2.

The solutions are then mixed in the proportion $m : n$, and the tube of the polariscope having been filled with the mixture, the reading of the scale is again taken. Let this reading be r. Then it is found that whatever be the ratio $m : n$, the value of r is, within errors of experiment, given by the equation

$$r = \frac{mr_1 + nr_2}{m+n}. \tag{10}$$

We infer then, according to the principle stated in p. 412, that there is no rupture of equilibrium when these solutions are added together.

Now it is evident, from the manner in which this mixture has been made, that while the values of the ratios $B_1 : S_1$, $B_2 : S_2$ remain the same, the value of

the ratio $B_1 : B_2$ varies with that of $m : n$. Hence it appears that if the values of $B_1 : S_1$ and $B_2 : S_2$ be preserved, the value of $B_1 : B_2$ may be varied *ad lib.* without a rupture of equilibrium. We infer, therefore that U (p. 414) depends only on the first two of these ratios. Hence

$$U = F\left(\frac{B_1}{S_1}, \frac{B_2}{S_2}\right).$$

Details of the Third Experiment.

	(1). Sol. of Quinia.	(2). Sol. of Brucia.
Strength of solution, ...	5·01 grs.	7·45 grs.
Quantity of solution, ...	·8825 cub. inch.	·9255 cub. inch.
Strength of dilute *HCl*, ...	·01 per cent.	·01 per cent.
Quantity of dilute *HCl*, ...	21·57 grs.	16·46 grs.
Do. per cubic inch, ...	23·89	17·79 ,,
Reading of scale, ...	1978	1180
Temperature, ...	55	55

Mixture of (1) *and* (2).

	Exp. 1.	Exp. 2.
Quantity of acidulated solution of Quinia, . . .	·5069	·2483
,, ,, ,, Brucia, . . .	·2548	5023
Value of ratio $m : n$, ...	2·041	·507
Reading of scale observed, ...	1724	1463
Do. calculated from eq. (10), ...	1711	1449
Error, ...	13	14
Temperature, ...	53	53

It will be observed here that the value of the ratio $m : n$ is four times as great in the first experiment as in the second.

49. To understand the significance of this experiment, let it be supposed that when the acidulated solutions are mixed, a certain quantity, x, of the acid present in a cubic inch of the mixture passes from one base to the other, e. g. from the brucia to the quinia. The rotation due to the quinia is then augmented by

$$\frac{\beta_1 x}{a}(\rho_1 - r_1),$$

inasmuch as a portion of this base represented by

$$\frac{\beta_1 x}{a}$$

passes from the free to the combined state.

Similarly the rotation due to the brucia is augmented by

$$\frac{\beta_2 x}{a}(r_2 - \rho_2).$$

Hence the whole augmentation of the reading of the scale will be

$$\frac{x}{a}\left\{\beta_1(\rho_1 - r_1) + \beta_2(r_2 - \rho_2)\right\}$$

or, adopting the notation of p. 416,

$$(D_1 + D_2)\,x.$$

This quantity will be the difference, δ, between the observed and calculated values of the reading of the scale. If x be reckoned in grains of dilute acid,

$$D_1 = 11{\cdot}58 \qquad D_2 = 16{\cdot}81.$$

Hence

$$\delta = 28{\cdot}39 \times x.$$

It appears, therefore, that the transference of one grain of the dilute acid from one base to the other would cause an error of 28 in the reading of the scale. Even then if the error noted in the foregoing table were wholly due to this cause, it would indicate a transference of less than one-half of a grain, in other words, a transference of less than one-fiftieth of the acid actually present in a cubic inch of the mixture. There is, however, another cause of error which will slightly raise the observed above the calculated value—namely, a

certain amount of evaporation, which in an alcoholic solution cannot be wholly avoided, attending the process of mixing the two acidulated solutions.

Fourth Experiment.

50. A solution is made containing brucia and quinia in the same proportion as the free brucia to the free quinia in Experiment 2.

A second solution is made containing brucia and quinia in the same proportion as the combined brucia to the combined quinia in the same experiment.

Thus if b_1, b_2 be the quantities of quinia and brucia, respectively, present in a given bulk of the first solution,

$$\frac{b_1}{b_2} = \frac{B_1}{B_2} \text{ (p. 416).}$$

Similarly if b'_1, b'_2 denote the quantities of quinia and brucia present in a given bulk of the second solution

$$\frac{b'_1}{b'_2} = \frac{\beta_1}{\beta_2} \frac{x}{a-x}.$$

A quantity of hydrochloric acid, just sufficient to convert both bases into salts, is now added to the second solution. This quantity is evidently given by the equation

$$a = \alpha\left(\frac{b'_1}{\beta_1} + \frac{b'_2}{\beta_2}\right).$$

The tubes of the polariscope having been now filled with these solutions respectively, the reading of the scale is taken for each.

The solutions are then mixed in the proportion $m : n$, and one of the tubes having been filled with the mixture, the reading of the scale is again taken.

Then if these readings be denoted severally by r_1, r_2, r, we find that

$$r = \frac{mr_1 + nr_2}{m+n}$$

within the errors of observation. We infer, therefore, as in p. 412, that in the mixture of the solutions (1) and (2) there is no rupture of equilibrium.

Now it is evident from the way in which the mixture is made that the values of the ratios $B_1 : B_2$ and $S_1 : S_2$ are preserved, while the ratio $B_1 : S_1$ or $B_2 : S_2$ varies with the ratio $m : n$. We infer, therefore as in p. 418, that

$$U = f\left(\frac{B_1}{B_2}, \frac{S_1}{S_2}\right).$$

Assume

$$x = \frac{B_1}{S_1}, \quad y = \frac{B_1}{B_2}, \quad z = \frac{B_1 S_2}{B_2 S_1}.$$

These variables are evidently independent. Introducing now these values into the expressions for U (pp. 418, 421), we have

$$U = F\left(x, \frac{z}{x}\right) = f\left(y, \frac{z}{y}\right).$$

But inasmuch as x and y are independent of each other, this condition can be satisfied only by the disappearance of x and y from these functions. We have, therefore,

$$U = F(z) = F\left(\frac{B_1 S_2}{B_2 S_1}\right).$$

Equating this to zero, we obtain finally as the equation of equilibrium

$$\frac{B_1 S_2}{B_2 S_1} = \text{const.}$$

or,

$$B_1 S_2 = K B_2 S_1. \qquad (11)$$

This equation may be written in another form which will be found useful when we come to consider the question from the theoretic point of view. Let a_1, a_2 be the quantities of acid which enter into the composition of S_1, S_2. Then

$$S_1 = \left(\frac{\beta_1}{\alpha} + 1\right) a_1, \quad S_2 = \left(\frac{\beta_2}{\alpha} + 1\right) a_2.$$

Substituting these values in (11), we have

$$B_1 a_2 = K' B_2 a_1 \qquad (12)$$

where

$$K' = K \frac{\beta_1 + \alpha}{\beta_2 + \alpha}.$$

Details of Fourth Experiment.

	1st Solution.	2nd Solution.
Strength of solution of quinia,	$4^{gr}\cdot93$	$4^{gr}\cdot93$
Quantity of solution in mixture,	·9245	1·4259
Quantity of quinia in 1 cubic inch,	2·10	3·11
Strength of solution of brucia,	$7^{gr}\cdot22$	$7^{gr}\cdot22$
Quantity of solution in mixture,	1·2448	·8345
Quantity of brucia in 1 cubic inch,	$4^{gr}\cdot14$	$2^{gr}\cdot67$
Quantity of HCl added to 1 cubic inch,	–	$0^{gr}\cdot5973$
Value of $B_1 \div B_2$,	·507	–
Value of $a_1 \div a_2$,	–	1·416
Reading of scale,	1583	1534
Temperature,	59°	59°

Mixtures of (1) *and* (2).

	A.	*B.*
Value of $m \div n$,	·409	2·520
Reading of scale observed,	1552	1568
Do. calculated from eq. (10),	1548	1571
Error,	– 4	+ 3
Temperature,	60°.5	57°.5

51. The form of the equation (11), p. 421, having been once obtained, it is not necessary to repeat for other cases the lengthened process by which the law contained in that equation was originally arrived at. It will be sufficient to calculate by means of eq. (7), p. 415, the values of the quantities of acid taken respectively by the two bases which are present in the solution, for different values of a, and then ascertain whether the expression

$$\frac{B_2 S_1}{B_1 S_2}$$

is constant.

It must be observed, however, that from the form of this expression any error arising from experiment will necessarily be greatly magnified.

Let a, as before, represent the quantity of acid added to a cubic inch of the mixture, and x the quantity which combines with one of the bases.

Then $a - x$ will represent the amount taken by the second base. We shall have consequently

$$S_1 = \left(\frac{\beta_1}{a} + 1\right) x, \quad S_2 = \left(\frac{\beta_2}{a} + 1\right)\left(a - x\right)$$

$$B_1 = b_1 - \frac{\beta_1 x}{a}, \quad B_2 = b_2 - \frac{\beta_2}{a}\left(a - x\right).$$

Hence if we assume

$$c_1 = \frac{b_1 a}{\beta_1}, \quad c_2 = \frac{b_2 a}{\beta_2}$$

it is evident that

$$\frac{B_2 S_1}{B_1 S_2} \propto \frac{x\,(c_2 - a + x)}{(a - x)\,(c_1 - x)}.$$

Denoting the latter fraction by V, let δx be the error in x arising from an error of observation, and δV the corresponding error in V. Then

$$\frac{\delta V}{V} = \delta x \left(\frac{1}{x} + \frac{1}{a - x} + \frac{1}{c_1 - x} + \frac{1}{c_2 - a + x}\right). \tag{13}$$

These terms having all the same sign, the relative error in V will be much greater than the relative error in x, and may become very considerable. It is to be observed further, that if the amount of acid added to the solution be small, the terms

$$\frac{1}{x}, \quad \frac{1}{a - x},$$

will both be large, and that if the amount of acid be nearly enough to convert both bases into salts, the terms

$$\frac{1}{c_1 - x}, \quad \frac{1}{c_2 - a + x}$$

will be large. Hence in making experiments with the purpose of verifying the law of p. 421, it will be necessary that the quantity of acid added to the solution should differ considerably from these extreme values.

52. As an example of this method, I may give the result of the verification for the two bases, quinia and codeia, the acid being, as before, hydrochloric.

The experiments are made as described in p. 414, and the results are given in the following table. I shall first make a slight change in the form of the expression for V.

Denoting by m_1, m_2 the numbers of atoms of the two bases, respectively, contained in a cubic inch of mixture, and by n_1, n_2 the number of atoms of acid combining respectively with these bases, we may write the value of V in the form

$$V = \frac{n_1(m_2 - n_2)}{n_2(m_1 - n_1)}. \tag{14}$$

The values of these several quantities for four experiments are given in the table—

m_1.	m_2.	n $(= n_1 + n_2)$.	n_1.	n_2.	V.	Temp.
100	104	70·7	42·7	28	1·91	60·5
,,	,,	91·9	55	36·9	2·08	62
,,	,,	112·4	66	46·4	2·10	62
,,	,,	130·3	73	57·3	2·02	62
Mean of the values of V, . . .					2·03	

53. In this table, the object being to determine the value of V, I have taken no notice of experiments in which the quantity of acid added is either very small or nearly sufficient to saturate both bases. But having obtained this value, we may further test the truth of the equation (11), p. 421, by calculating conversely from this equation the reading of the scale for the acidulated mixture, and comparing the result so obtained with the reading actually observed. This is the only exact method of testing the accuracy of the

equation. For, as we have seen, when the amount of acid added to the solution is either small or nearly sufficient to neutralize both bases, the errors of observation are so multiplied in the value of V, that it is difficult to determine for such values of the quantity of acid, whether V be, or be not, constant. But if the reading of the scale for the acidulated mixture be calculated on the hypothesis of the constancy of V, we can then compare the result with the reading actually observed, and thus determine whether the difference be beyond the error of experiment. This is done as follows:—

If we represent by x, as before, the quantity of acid which, in a cubic inch of solution, combines with one base, and therefore by $a - x$ the quantity similarly combining with the second base, it is easily seen that the equation

$$\frac{B_2 S_1}{B_1 S_2} = \text{const.}$$

is equivalent to

$$x\left(\frac{b_2 a}{\beta_2} - a + x\right) = K\left(a - x\right)\left(\frac{b_1 a}{\beta_1} - x\right),$$

or, putting

$$\frac{b_1 a}{\beta_1} = P_1, \qquad \frac{b_2 a}{\beta_2} = P_2,$$

and arranging

$$x^2 - \left(\frac{KP_1 + P_2}{K-1} + a\right)x + \frac{KP_1\, a}{K-1} = 0. \tag{15}$$

One of the roots of this equation, being either negative or greater than a, is evidently foreign to the question.

Assuming

$$Q_1 = \frac{KP_1}{K-1}, \qquad Q_2 = \frac{P_2}{K-1}, \qquad \sin^2\theta = \frac{4Q_1 a}{(Q_1 + Q_2 + a)^2},$$

it is easily seen that

$$x = (Q_1 + Q_2 + a)\sin^2\tfrac{1}{2}\theta \tag{16}$$

a form suited to logarithmic computation.

Then since (p. 415)

$$x = Aa + B\,(r - r_0),$$

the reading r for the acidulated solution is given by the equation

$$r = r_0 + \frac{x - Aa}{B}. \tag{17}$$

This value must be compared with the reading actually observed. The results are exhibited in the following table:—

m_1.	m_2.	n.	r calc.	r obs.	Difference.	Temp.
100	104	32	1793	1804	− 11	60°
,,	,,	72	1855	1852	+ 3	62°
,,	,,	93	1887	1888	− 1	62°
,,	,,	112	1915	1914	+ 1	62°
,,	,,	132	1925	1931	− 6	62°
,,	,,	154	1955	1947	+ 8	62°
,,	,,	164	1954	1946	+ 8	61°.5
,,	,,	187	1980	1956	+ 24	62°

All these differences are within the range of errors of observation with the exception of the last, which is somewhat in excess. This variation probably resulted from some accidental inaccuracy in the experiment.

54. The value of the function V, as calculated above, may advantageously be corrected by another method, which I shall now proceed to explain.

Having obtained, as in p. 415, a system of values of x and $a - x$, the quantities of acid combining respectively with the two bases, let two acidulated solutions be prepared as in p. 417. These solutions having been mixed in the proportion $m : n$, and the tube of the polariscope filled with the mixture, let the reading of the scale be r, and put

$$r' = \frac{mr_1 + nr_2}{m + n},$$

Then we have seen that, if the value of x were correctly assigned, and the acidulations, consequently, correctly made, we should have

$$r' = r.$$

Let δx be the error in the value of x. The quantity of acid represented by δx (positive or negative) will necessarily be transferred from combination with the second base to combination with the first base. Then, adopting the notation of p. 415, it is easy to see that the change in the reading of the scale will be

$$\frac{\{\beta_1 (\rho_1 - r_1) - \beta_2 (\rho_2 - r_2)\} \delta x}{a} = \frac{\delta x}{B}.$$

Hence, putting δr for $r - r'$, we have

$$\delta r = \frac{\delta x}{B}, \quad \text{or } \delta x = B \delta r,$$

an equation which gives the correction to be made in the value of x. The corresponding correction in V is then given by eq. (13), p. 423.

The following Table gives the values of this function for combinations of the three bases, quinia, brucia, and codeia :—

Bases combined.	V.
Quinia and codeia, ...	2·03
Codeia and brucia, ...	1·58
Brucia and quinia, ...	·32

I shall return to the consideration of these results when examining the question from the theoretic point of view.

55. The method here given is evidently applicable to a combination of two bases, one of which possesses the rotatory power, the other being devoid of it. To apply the equation (7), p. 415, to such a case, it is only necessary to make $\rho_2 = 0$, $r_2 = 0$. Theoretically, therefore, we should be able to estimate the relative combining powers of the inorganic bases by comparing them with some

one organic base, as for example quinia or brucia. It is, however, practically impossible to apply this method. For the combining power of the inorganic bases, as potassa, soda, ammonia, is so great as compared with that of any of the organic bases, that if an acid be added to a solution containing an organic and an inorganic base, there does not appear to be any division of the acid between the two bases, the inorganic base taking sensibly the whole of the acid until enough has been added to convert the whole of this base into a salt.

Thus, for example, if acetic acid be added to a solution containing potassa and strychnia, it is found that the rotatory power of the mixture is not sensibly altered (except by the dilution caused by the addition of the acid), until enough of the acid has been added to convert the whole of the potassa into a salt. Hence it is evident that no sensible quantity of the acid has combined with the strychnia. For, had any sensible quantity of this base been converted into an acetate, the rotatory power of the solution would have been sensibly diminished.

These results are quite in accordance with the law contained in eq. (11), p. 421. For if, in the equation

$$B_1S_2 = KB_2S_1,$$

we suppose the constant K to be very small, it is plain that one of the quantities B_1, S_2 must also be very small. Now, unless the first base, b_1, was originally present in very small quantity,

$$B_1\left(= b_1 - \frac{\beta_1 x}{a}\right)$$

cannot become small until this base has been almost wholly converted into salt. But even if the whole of the acid added to the solution were supposed to combine with the first base, the conversion of this base into salt would not be complete until the quantity, a, of acid had attained the value given by the equation

$$a = \frac{ab_1}{\beta_1}.$$

Until this value is nearly attained, the other factor, S_2, of the product, $B_1 S_2$, is necessarily very small, indicating that a very small part of the acid has entered into combination with the second base.

Hence, if we present an acid to a solution containing two bases whose combining powers are very unequal, the stronger base will take, sensibly, the whole of the acid, until enough has been added to saturate almost the whole of this base.

After this, the second base will take, sensibly, the whole of any further addition of acid until this base also is nearly saturated. The point of actual saturation is reached at the same time for both bases. For it is evident that in eq. (11), B_1 and B_2 must vanish together.

Theoretical Considerations.

Nature of Chemical Combination.

56. I have already alluded (p. 410), to two wholly different theories of the nature of chemical combination. These may be denominated with sufficient accuracy the *statical* and the *dynamical* theories.

According to the statical theory, two substances when chemically combined are in a state of relative rest. Whatever be the physical nature of the union existing between two molecules which are said to be chemically combined, this union is supposed to exist continually between the same molecules. Each molecule of the one substance is supposed to attach itself to one determinate molecule of the other substance, and this union continues until the compound is decomposed.

According to the dynamical theory, the molecules of which the compound consists are in a state of constant motion. Every compound molecule formed by the combination of the molecules of the simpler substances is continually decomposed, the simpler molecules combining with those which result from the decomposition of other compound molecules, and thus forming new com-

pound molecules, which are in their turn decomposed; this process of decomposition and re-composition continuing indefinitely.

In the state of chemical equilibrium, as we have seen (p. 410), the number of compound molecules of each *kind* remains constantly the same, this constancy resulting according to the statical theory from the absolute *permanence* of each compound molecule, and according to the dynamical theory from the *equivalence* of the decompositions and recompositions of the compound molecules of each kind. So far, either theory is consistent with the phenomenon of chemical equilibrium.

57. I now proceed to consider the *number* of the equations of equilibrium required by each theory respectively.

Let us take for example the case which we have already considered in which the substances present are quinia, brucia, and the hydrochlorates of each of these bases. Then each molecule of hydrochloric acid which is in combination with a molecule of quinia is under the influence of the attractive force of this molecule, tending to retain it in combination, and of the attractive forces of the uncombined molecules of both bases, tending to withdraw it from combination with the molecule to which it is attached. If the molecule of acid be in a state of rest, these forces must equilibrate, and for this equilibrium at least one equation of condition will be necessary. So also for the acid which is in combination with the brucia. The statical theory, therefore, seems to require in this case at least *two* equations of equilibrium. But as we have seen, p. 411, the quantities of the four substances present in the solution, being functions of *three* independent variables, there can be no more than *one* equation between them. The statical theory, therefore, seems to require a greater number of equations than the nature of the case admits.

Let us now consider the number of equations of equilibrium required by the dynamical theory.

Let the number of bases present in the mixture be n, and let a single acid be added in a quantity insufficient to convert the whole into salts. We shall then have in general present in the solution

1. n portions of uncombined bases, which we may denominate $B_1, B_2, \ldots\ldots B_n$

2. n salts, which we may denominate $S_1, S_2, \ldots\ldots S_n$, these latter containing respectively the quantities of acid A_1, $A_2, \ldots\ldots A_n$.

Then the salt S_1 will be subject to decomposing forces emanating from the free bases $B_1, B_2, B_3, \ldots\ldots B_n$. The first of these forces does not disturb equilibrium, as is easily seen ; but for the sake of symmetry, it is desirable to include it in the equations. Let the quantity of acid withdrawn from S_1 by the action of the base B_m during the unit of time be denoted by (A_1B_m). Then it is evident that the total amount of acid withdrawn from S_1 by the action of all the bases may be represented by

$$\Sigma(A_1B_m),$$

m having all values from 1 to n. The quantity of the first base set free by this action will be then,

$$\frac{\beta_1}{\alpha}\,\Sigma_1^n\left(A_1B_m\right).$$

On the other hand, the free base B_1 will exercise a decomposing power on each of the salts $S_1, S_2, \ldots\ldots S_n$, withdrawing from them, and therefore combining with, quantities of acid which are represented by

$$(A_1B_1)\ ,\ (A_2B_1)\ \ldots\ldots (A_nB_1),$$

and, therefore, in the aggregate by

$$\Sigma_1^n(A_mB_1).$$

The quantity of B_1, therefore, which enters into combination, will be

$$\frac{\beta_1}{\alpha}\,\Sigma_1^n\left(A_m\,B\right).$$

Now, in order that equilibrium may be preserved, the quantities of each

base combined and set free, at each instant, must be the same. Hence, for the first base

$$\Sigma_1^n (A_1B_m) = \Sigma_1^n (A_mB_1).$$

Similarly

$$\Sigma_1^n (A_2B_m) = \Sigma_1^n (A_mB_2) \qquad (18)$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

$$\Sigma_1^n (A_nB_m) = \Sigma_1^n (A_mB_n).$$

These equations are evidently n in number. But it is easy to see that only $n-1$ are independent. For if we add together the n equations, we shall have the identity

$$\Sigma (A_{m'}B_m) = \Sigma (A_{m'}B_m),$$

the summation being extended to every value of m' and m from 1 to n. The number of independent equations, therefore, furnished by the dynamical theory agrees with that which may be deduced from the number of the independent variables. For these variables, being—1. The quantities of the several bases ; and 2. The total quantity of acid, are evidently in number $n+1$.

Case of two Bases.

58. In the case of two bases, which we have already examined experimentally, the preceding theory gives a single equation of equilibrium—namely, in the notation which we have adopted,

$$(A_1B_2) = (A_2B_1),$$

where (A_1B_2) denotes the quantity of acid withdrawn in the unit of time from the salt, S_1, by the action of the free base B_2 ; (A_2B_1) having a corresponding signification for S_2 and B_1. Now we have seen experimentally that the equation of equilibrium is

$$A_1B_2 = KA_2B_1.$$

Th ese results are harmonized by assuming

$$(A_1B_2) = CA_1B_2\,, \qquad (A_2B_1) = C'A_2B_1, \tag{19}$$

C, C' being constants. The physical theorem contained in these assumptions is expressed by saying that *the quantity of acid withdrawn in the unit of time from combination with one base by the action of another base, or, as it may be termed, the rate of decomposition of the salt, is proportional to the product formed by multiplying the quantity of the salt by the quantity of the decomposing base.*

It is plain that this assumed law may be equally well expressed as the law of the rate of composition. For if we denote by σ_1 the amount of the salt S_1 which is decomposed in the unit of time by the action of the base B_2, the amount of acid which it contains is

$$\frac{\alpha\sigma_1}{\beta_1 + \alpha},$$

and the amount of salt formed by the union of this acid with B_2 is

$$\frac{\beta_2 + \alpha}{\beta_1 + \alpha}\,\sigma_1\,.$$

Hence it is plain that the quantities (1) of the salt decomposed, (2) of the salt formed, and (3) of the acid which passes from the one base to the other are proportional. We may, therefore, express the assumed law as follows :—

The amount of salt formed in the unit of time by the combination of one base, or, as it may be termed, the velocity of combination of this base, with the acid withdrawn from combination with the other is proportional to the product formed by multiplying the total quantity of the first base with the total quantity of acid combined with the second base.

Now, according to the law of Berthelot,* this same product measures the velocity of combination of a base and an acid when both are free. If, therefore, our assumed law be true, it would follow that the velocity of combination of a base with an acid follows a similar law, with possibly a different constant, whether the acid be free or combined with another base. It remains to inquire whether any other assumption is consistent with the result of the experiment, and whether the constant is really different.

* "Ann. de Chim. et de Phys.," 3e Serie, Tom. lxvi., p. 112.

59. When a salt (AB) is decomposed by the action of a free base B', the acid A leaving the base B to enter into combination with B', the effect may, as the most general supposition which we can make, be conceived to be due to the action of two forces, namely—the decomposing force of B' and the retaining force of B. Now, if A were free, the velocity of combination of this acid with B' would, according to the law of Berthelot, be represented by KAB'. Hence, according to the principles of mechanics, the force by which this combination would be effected is expressed by

$$K \frac{d}{dt} AB'.$$

The law of the retaining force, if such there be, exerted by B upon A is unknown. We may, however, represent it generally by

$$F(A,B)$$

or, since A,B are to each other in a constant ratio, by

$$F(A).$$

The total force, therefore, by which the combination (AB') is effected, will be represented by

$$K \frac{d}{dt} AB' - F(A).$$

Reverting now to our former notation, let us consider the case of two bases B_1, B_2, and two salts S_1, S_2, formed by the combination of the same acid with other portions of these bases. Let it be supposed that these four substances are brought into chemical presence of each other and are in chemical equilibrium. Let a_1, a_2, be the portions of acid which enter into the composition of the salts S_1, S_2 respectively. Then, according to the dynamical theory, there is a constant combination of B_1 with a_2, which, for equilibrium, is balanced by a constant combination of B_2 with a_1. The forces by which these

combinations are effected are, in accordance with the principles which have been laid down, represented by

$$K \frac{d}{dt} a_2 B_1 - F(a_2), \quad K' \frac{d}{dt} a_1 B_2 - F(a_1)$$

respectively. These forces being necessarily equal for equilibrium, we have

$$K \frac{d}{dt} a_2 B_1 - F(a_2) = K' \frac{d}{dt} a_1 B_2 - F(a_1). \tag{20}$$

But we have seen that the form of the equation of equilibrium is

$$a_2 B_1 = \text{const.} \times a_1 B_2 \tag{21}$$

and that there is but one such equation. The two forms (20) and (21) must therefore be identical, and this cannot be unless

$$F(a_1) = 0, \quad F(a_2) = 0.$$

We infer, therefore, that the retaining force does not exist—in other words, that the decomposition of hydrochlorate of brucia by the chemical presence of quinia is not *resisted* by any force residing in the brucia from which the acid is withdrawn. The same is of course true of the decomposition of hydrochlorate of quinia by the chemical presence of brucia. It is also true, as we have seen, where the bases in presence of each other are quinia and codeia, and is probably a general law.

It follows from this principle that (at least in the cases which have been considered) the force which determines the combination of an acid with a base cannot be a force of attraction between the molecules of these substances. For if such a force really existed, it would necessarily act in resisting decomposition no less than in determining combination. If for the purpose of avoiding this difficulty, we suppose the force of attraction to be destroyed, as in the case of oppositely electrified particles, at the moment of combination, it is difficult to understand how the molecule of acid whose attractive force had been thus destroyed, could attract a molecule of another base.

60. Assuming then that the combining power of a given quantity of a base with a given quantity of an acid, whether this latter be combined or free, is represented by

$$K \frac{d}{dt} Ba,$$

where K is a constant depending wholly on the nature of the base and acid, the equations of equilibrium of three bases combined in pairs will be

$$\begin{aligned} K_1 B_1 a_2 &= K_2 B_2 a_1 \\ K_2 B_2 a_3 &= K_3 B_3 a_2 \\ K_3 B_3 a_1 &= K_1 B_1 a_3. \end{aligned} \tag{22}$$

Hence it is easily seen that if we form the values of V, p. 423, for each of these combinations, denoting them by V', V'', V''', we ought to have

$$V' \; V'' \; V''' = 1. \tag{23}$$

We have here then an experimental test to which we can submit the theory. I have given in the table, p. 427, the values of this function for the combinations: 1. Quinia and codeia; 2. Codeia and brucia; and 3. Brucia and quinia, respectively. If these values be multiplied together we shall have

$$V' \; V'' \; V''' = 1{\cdot}026$$

which is as close an approximation as can be expected, remembering the amount of errors to which the calculation of V is necessarily liable.*

The constants K_1, K_2, K_3 may be termed the *intrinsic* combining forces of the several bases with the same acid. We have from equations (22)

$$\frac{K_1}{K_2} = \frac{B_2 a_1}{B_1 a_2} = \frac{\beta_2}{\beta_1} V', \text{\&c., \&c.}$$

These equations give the *ratios* of the intrinsic combining forces of the three

* *Vid.* p. 423.

bases which have been examined. Assuming K_3 (the intrinsic combining force of brucia) $= 1$, we have the following values for these three forces,*

$$\begin{aligned} &\text{Intrinsic combining force of quinia,} = 3{\cdot}84 \\ &\quad\text{,,} \qquad\quad \text{,,} \quad \text{codeia,} = 2{\cdot}21 \\ &\quad\text{,,} \qquad\quad \text{,,} \quad \text{brucia,} = 1 \end{aligned} \tag{24}$$

We have seen p. 435, that the result of the investigations here described, combined with the law of Berthelot, is inconsistent with the supposition of a force which *resists* decomposition. If this be true, the *velocity* of combination of a base with an acid which is already combined with another base is the same as if the acid were free. In either case, therefore, this velocity is represented by the expression

$$KBa,$$

the value of K being the same for both cases.

61. It remains to consider whether the investigation in which we have been engaged gives any information as to the *time* required for the establishment of equilibrium, when the two bases and their corresponding salts are brought together in proportions which do not satisfy the equation (11), p. 421.

Let b_1, b_2, be the total quantities of each base (combined and uncombined) present in the solution. Let also a_1, a_2, be the quantities of acid combined with each base respectively. Then we know that the quantities of free base are

$$b_1 - \frac{\beta_1 a_1}{a}, \quad b_2 - \frac{\beta_2 a_2}{a}$$

respectively.

The diminution which a_1 sustains by the action of the free portion of b_2 is, as we have seen, measured by

$$K_2\left(b_2 - \frac{\beta_2 a_2}{a}\right)a_1.$$

* It remains to determine whether these combining forces vary with the acid used, as also to assign their values for the other organic bases. I hope to return to this subject in a further communication.

Similarly, the augmentation caused by the free portion of b_1 is measured by

$$K_1\left(b_1 - \frac{\beta_1 a_1}{a}\right)a_2 .$$

We have then the equation

$$\frac{da_1}{dt} = K_1\left(b_1 - \frac{\beta_1 a_1}{a}\right)a_2 - K_2\left(b_2 - \frac{\beta_2 a_2}{a}\right)a_2. \tag{25}$$

But since the total quantity of acid remains constant, we have

$$a_1 + a_2 = c.$$

Eliminating a_2 from (25), and arranging,

$$\frac{da_1}{dt} = K_1 b_1 c - \left\{K_1 b_1 + K_2 b_2 + (K_1\beta_1 - K_2\beta_2)\frac{c}{a}\right\}a_1 + \frac{K_1\beta_1 - K_2\beta_2}{a}a_1^2,$$

or, putting for the sake of abbreviation

$$K_1 b_1 + K_2 b_2 = P, \quad K_1\beta_1 - K_2\beta_2 = Qa,$$

$$\frac{da_1}{dt} = K_1 b_1 c - (P + Qc)\,a_1 + Qa_1^2 \tag{26}$$

$$= Q\left(a_1 - \frac{P + Qc}{2Q}\right)^2 - \left\{\frac{(P + Qc)^2}{4Q} - K_1 b_1 c\right\}.$$

If in this equation Q be negative, it is evident that the first term of the right hand member is negative and the second positive. If Q be positive, the first term is positive and the second negative. For

$$(P + Qc)^2 - 4bQK_1 b_1 c = P^2 + Q^2c^2 + 2Qc\,(K_2 b_2 - K_1 b_1)$$
$$= (P - Qc)^2 + 4QK_2 b_2 c > 0.$$

Hence it is evident that in every case these terms have opposite signs. Suppose Q to be positive, and put

$$(P + Qc)^2 - 4QK_1 b_1 c = R^2.$$

We have then

$$\frac{da_1}{dt} = Q\left\{\left(a_1 - \frac{P + Qc}{2Q}\right)^2 - \frac{R^2}{4Q^2}\right\}. \tag{27}$$

62. Before integrating this equation it is necessary to observe that the right hand member will vanish for a value of a_1 intermediate between 0 and c (the extreme values). For if we put $a_1 = 0$, the right hand member becomes K_1b_1c, a positive quantity; and if we put $a_1 = c$, the same member becomes $-K_2b_2c$, a negative quantity. Further, the value of a_1 which causes this member to vanish is the value corresponding to equilibrium. For we have then

$$\frac{da_1}{dt} = 0,$$

whence also

$$\frac{da_2}{dt} = 0,$$

indicating that the transference of acid has ceased.

Let it be supposed that the original value of a_1 is *less* than that which corresponds to equilibrium, and, therefore, that the second member of eq. (26) is positive.* Then we have by integration

$$\frac{R + P + Qc - 2Qa_1}{R - P - Qc + 2Qa_1} = \frac{R + P + Qc - 2Qa'_1}{R - P - Qc + 2Qa'_1}\, e^{Rt} \tag{28}$$

where a'_1 is the original value of a_1.

The value of a_1 corresponding to equilibrium is evidently given by the equation

$$2Qa_1 = P + Qc - R,$$

and it appears from (28) that this value is not absolutely attained for any finite value of t. It may be shown, however, that a close approximation to this value is in general very rapidly attained.

Assume

$$\frac{P + Qc - R}{2Q} = \frac{a_1}{m} = \frac{a'_1}{m'}.$$

* It is evident that this expression only vanishes *once* for values of a_1 between 0 and c.

Eliminating by means of these equations a_1 and a'_1 from (28) we find

$$\frac{2R}{1-m}+P+Qc-R=\left(\frac{2R}{1-m'}+P+Qc-R\right)e^{Rt}. \tag{29}$$

The second member of this equation increases very rapidly on account of the exponential e^{Rt}. The left hand member will, therefore, increase rapidly also, and consequently $1-m$ will rapidly become small.

Hence, as $m=1$ is the condition of equilibrium, we infer that a distribution of acid, not differing sensibly from that which corresponds to equilibrium, is in general very rapidly attained.

63. It is not, of course, possible to determine by this method the actual time necessary for the establishment of equilibrium in a given combination, but we may determine the *ratio* of these periods for two different combinations.

Putting

$$n=\frac{1+m}{1-m},\quad n'=\frac{1+m'}{1-m'}$$

we have from eq. (29)

$$e^{Rt}=\frac{P+Qc+nR}{P+Qc+n'R}. \tag{30}$$

Let t denote the time in which the ratio of the actual value of a_1 to the value required for equilibrium passes from m', its original value, to a certain value, m, this latter being nearly $=1$. Then the coefficients n, n' are both known. Moreover P, Q, R, are homogeneous in K_1, K_2. Hence the foregoing equation may be written

$$e^{Rt}=\phi\left(\frac{K_1}{K_2}\right)$$

all the other quantities entering into ϕ being known. Similarly for another combination, we should have

$$e^{R't'}=\phi'\left(\frac{K_1'}{K'_2}\right)$$

whence

$$\frac{t}{t'}=\frac{R'}{R}\,\frac{\log\phi\left(\frac{K_1}{K_2}\right)}{\log\phi'\left(\frac{K'_1}{K'_2}\right)}$$

the second member of the equation depending upon the *ratios* of the intrinsic combining forces, and therefore being fully determined as in p. 436.

64. As an example of this investigation, let it be supposed that the numbers of atoms:—1. Of the total quantity of each base b_1, b_2; 2. Of the total quantity of acid, c – are the same. We have then

$$\frac{b_1}{\beta_1} = \frac{b_2}{\beta_2} = \frac{c}{a}. \tag{31}$$

Hence

$$P = K_1 b_1 + K_2 b_2,$$

$$Qc = (K_1\beta_1 - K_2\beta_2)\frac{c}{a} = K_1 b_1 - K_2 b_2,$$

$$P + Qc = 2K_1 b_1$$

$$R^2 = (P + Qc)^2 - 4K_1 b_1 Qc = 4K_1 K_2 b_1 b_2.$$

Making these substitutions in (28) we have after some reductions

$$\frac{a_1\sqrt{K_2 b_2} + a_2\sqrt{K_1 b_1}}{a_1\sqrt{K_2 b_2} - a_2\sqrt{K_1 b_1}} = \frac{a_1'\sqrt{K_2 b_2} + a_2'\sqrt{K_1 b_1}}{a'\sqrt{K_2 b_2} - a_2'\sqrt{K b_1}}\, e^{2t}\sqrt{K K_2 b_1 b_2}, \tag{32}$$

or putting

$$a_1 = m a_2,\ a'_1 = m' a'_2,\ \sqrt{K_1 b_1} = n\sqrt{K_2 b_2},\ \sqrt{K_1 K_2 b_1 b_2} = p, \tag{33}$$

$$\frac{m+n}{m-n} = \frac{m'+n}{m'-n}\, e^{2pt},$$

whence putting

$$m = in, \quad m' = i'n,$$

we have

$$i - 1 = \frac{(i+1)(i'-1)}{i'+1}\, e^{-2pt}. \tag{34}$$

The condition of equilibrium, which is

$$i - 1 = 0,$$

is not attained for any finite value of t; but unless p be very small, $i - 1$ will rapidly become small on account of the factor e^{-2pt}. Now from eqs. (31)

$$p = \frac{c}{a}\sqrt{K_1 K_2 \beta_1 \beta_2}$$

This can become small only in consequence of the smallness of c, K_1 or K_2. The first of these suppositions, combined with the proportionality of c_1, b_1, b_2, implies that the original solutions were very weak. The remaining suppositions imply that at least one of the bases possesses a low combining power.

In general, if t denote the time in which i passes from its original value to any given value, it is evident that

$$t \propto \frac{1}{p}.$$

This result may be expressed by saying that—

The time required for the establishment of a distribution of acid, approximating in any given degree to that which is necessary for equilibrium, is inversely proportional to—1. *The strength of the solution.* 2. *The square root of the combining power of each base.* 3. *The square root of the atomic weight of each base; and directly proportional to the atomic weight of the acid.*

If a second combination be made resembling the first in all respects but one, namely that the second of the bases is replaced by a third base, and if we denote by t' the time requisite for the approximate establishment of equilibrium, we have

$$t : t' :: \sqrt{K_3} : \sqrt{K_2}.$$

Hence if one base be retained as a standard, and if we denote by K the combining power of any other base which is united with it, and by t the time requisite for the approximate establishment of equilibrium,

$$Kt^2 = \text{const.}$$

In the cases which we have examined, namely where the bases are—1. Quinia and codeia; 2. Quinia and brucia; 3. Brucia and codeia, the times are nearly as the numbers

$$100 : 151 : 196.$$

65. The results of the general investigation are much simplified if the intrinsic combining forces of the two bases be inversely proportional to their atomic weights. For in that case we shall have

$$Q = K_1\beta_1 - K_2\beta_2 = 0,$$

The general equation (26) becomes, therefore,

$$\frac{da_1}{dt} = K_1 b_1 c - P a_1,$$

giving by integration

$$\frac{K_1 b_1 c - P a_1}{K_1 b_1 c - P a'_1} = e^{-Pt}$$

This equation may be put under the following very symmetrical form.

Let n_1, n_2 denote the numbers of atoms of the two bases respectively. Let also ν_1, ν_2, denote the numbers of atoms of the quantities of acid, a_1, a_2, combined respectively with these bases at the end of the time t, ν'_1, ν'_2 being the values of these numbers at the beginning of the motion.

Then the foregoing equation may be put under the form

$$\frac{\dfrac{\nu_1}{n_1} - \dfrac{\nu_2}{n_2}}{\dfrac{\nu'_1}{n_1} - \dfrac{\nu'_2}{n_2}} = e^{-Pt}.$$

65. If we apply the principles established in the present section to the problem discussed in sections IV. and V., we shall find reason to conclude that some of the results there obtained can only be regarded as approximate.

When a base is capable of forming with an acid more than one combination, it has been assumed that, if the acid be added gradually, the formation of the less acid salt is complete before the formation of the more acid salt begins.

Thus, for example, if sulphuric acid be added in successive small quantities to a solution of quinia, we assume, in conformity with the ordinary notion, that the formation of the common sulphate—whether it is to be termed neutral or basic—must be complete before any part of the bi-sulphate is formed.

But it is evident from the principle of the division of an acid between two bases that this assumption is not mathematically true. For, inasmuch as the ordinary sulphate is itself capable of combining with sulphuric acid, we must look upon it as a base. If, therefore, to a mixture of quinia and sulphate

of quinia we present a small quantity of sulphuric acid, a partition will necessarily take place, part of the acid uniting with the quinia to form the ordinary sulphate, and part uniting with the sulphate of quinia, to form the bi-sulphate.

As this conclusion is true, whatever be the quantity of acid which has been first added, it is plain that, theoretically, we must regard the formation of both salts as commencing and terminating simultaneously.

Experimentally, however, we have no sensible evidence of the truth of this result. In the experiments recorded, p. 402, the values of the quotient tabulated in the third column, which remain sensibly constant until the number of atoms of acid is equal to the number of atoms of base, appear to indicate that the formation of the lower salt is complete before the formation of the higher salt begins. And, as we have seen, p. 401, the occurrence of a *break* in the locus there described, at or near the point where this equality is reached, indicates the completion, at or near this point, of a definite chemical compound.

There is, however, no real opposition between theory and experiment with regard to this question, the explanation of the apparent discrepancy being perfectly analogous to that already given (p. 427), for the mixture, in solution, of an organic and an inorganic base. The lower salt must be regarded as a base whose combining power is very small as compared with that of the alkaloid itself. Hence if acid be added to a solution in which the alkaloid and the lower salt are both present, the effect will be similar to that produced by the addition of acid to a solution containing an alkaloid and one of the inorganic alkalies, such as potassa or ammonia. In both cases the stronger base will seem to take all the acid until enough has been added to convert this base into a salt. We have seen (p. 427) that this is a necessary consequence of the law which governs the partition of an acid between two very unequal bases.

NOTE ON PAGE 391.

It is necessary to observe that, in adopting the coefficient $\frac{22}{19}$, the sugar as determined by the copper solution is reckoned as *crystallizable* grape sugar, containing therefore an atom of water. If this sugar be reckoned as inverted or uncrystallizable sugar the coefficient should be $\frac{20}{19}$.

NOTE ON SECTION V.

The method of determining the rotatory power of a salt, depending upon the addition of a measured quantity, does not in general give results which are theoretically accurate. We have seen (p. 405) that, at least in some cases, the solvent must be considered as a base, determining a *division* of the acid between itself and the alkaloid, if the salt whose rotatory power is sought be the lowest salt, or between itself and a lower salt, if the salt whose rotatory power is sought be not the lowest. Hence, as the whole of the acid does not enter into combination with the alkaline base or lower salt, results obtained upon the assumption that the whole of the acid does so combine cannot be regarded as mathematically accurate. It is therefore important to ascertain in what cases the results so obtained may be regarded as sensibly accurate.

Now it is easy to infer from the law of chemical equilibrium (p. 421), that when there is a partition of acid between two bases, the value of the fraction

$$\frac{r - r'}{a}$$

p. 396, does not remain constant. Conversely, if this fraction remain sensibly constant, we infer that there is no sensible partition of the acid.

It is necessary, therefore, to make three or more experiments according to the method described in p. 398, tabulating the values of this fraction. If these values remain sensibly constant the results obtained are sensibly exact. If on the contrary these values show a sensible variation the method cannot be considered reliable.

Now, in the Table (p. 402), representing the combination of quinia with nitric acid, it is evident that the values of the fraction

$$\frac{r - r'}{a}$$

(Column III.) do remain sensibly constant till the formation of the nitrate is complete. On the contrary, in the Tables (pp. 404, 406) representing the formation of the higher nitrate and hydrochlorate, these values are not sensibly constant, but show a perceptible diminution. We infer, therefore, that the results obtained by the method under consideration may be relied on as close approximations to the truth in the case of the lower (neutral or basic) salts, but that they are sensibly erroneous in the case of the higher salts.

The same considerations show that the results tabulated in Sec. VI. ought to be regarded, not as mathematically accurate, but as close approximations only. Theoretically, the acid is presented not to a mixture of *two* bases, but at first to a mixture of *three* bases, the solvent being one. After the first addition, the bases must be regarded as, theoretically, *five*, namely the two portions of uncombined base, the two portions of salt, and the solvent. But the combining forces of the latter three are so small as compared with the combining forces of the bases themselves, that we are justified in regarding, as we have done in Sec. VI., the partition of the acid as taking place altogether between the bases, this assumption, though not mathematically accurate, being a very close approximation to the truth.

NOTE ON SECTION VI.

In each series of experiments here described, I have endeavoured, as far as possible, to preserve a constant temperature. For the effects of a change of temperature are so complicated that it would be exceedingly difficult to separate them, so as to render the results of different experiments comparable with each other. The rotatory powers of the bases and of their respective salts are in general functions of the temperature, and it is probable that the combining power of each base also contains the same element. Hence it is evident that the quantities A, B, K which we have treated as constants, are in reality variable with the temperature, and this variability would necessarily invalidate the reasoning in the text. Hence the importance of preserving as nearly as possible a constant temperature.

The effect of temperature on the rotatory powers of substances, more especially of the organic alkaloids and their salts, would require a systematic and rather extended investigation. Generally a rise in the temperature is accompanied by a fall in rotatory power. I have given the law of this diminution in the case of inverted sugar where the effect is very marked, and hope to return to the subject in a future communication.

TABLE I.

Rotatory Powers of various Substances.

Name of Substance.	Solvent.	Rotatory Powers.		Temperature, Fahr.
		Right-handed.	Left-handed.	
Vegetable Alkaloids.				
A. Quinia group.				
Quinia.	Proof spirit.		2·97	62
Do.	Rectified spirit.		3·65	62
Cinchonia.	Rectified spirit.	3·98		59
Do.	Absolute alcohol.	3·81		59
Quinidine.	Rectified spirit.	4·69		64
Cinchonidine.	Proof spirit.		2·96	55
B. Morphia group.				
Morphia.	Proof spirit.		2·68	64
Do.	Rectified spirit.		2·59	58
Codeia.	Proof spirit.		2·63	62
Do.	Water.		2·85	62
Narcotine.	Rectified spirit.		2·19	—
Do.	Chloroform.		3·75	58
Thebaia.	Rectified spirit.		2·84	—
Papaverine.	Do.	inert.	inert.	
C. Strychnia group.				
Strychnia.	Proof spirit.		1·84	62
Do.	Rectified spirit.		2·04	57
Brucia.	Proof spirit.		1·66	62
Do.	Rectified spirit.		1·69	61
D. Miscellaneous.				
Aconitina.	Proof spirit.		·32	56
Atropia.	Do.		·17	61
Caffeine.	Rectified spirit.	inert.	inert.	
Digitaline.	Proof spirit.		·52	58
Piperine.	Rectified spirit.	inert.	inert.	—
Solanine.	Chloroform.	inert.	inert.	
Veratria.	Proof spirit.	trace.		
Nicotine.	Rectified spirit.		2·68	54

TABLE I.—*continued.*

Name of Substance.	Solvent.	Rotatory Powers.		Temperature, Fahr.
		Right-handed.	Left-handed.	
MISCELLANEOUS SUBSTANCES.				
Salicine.	Proof spirit.		1	60
Do.	Water.		1·42	60
Arbutine.	Proof spirit.		1·20	64
Helenin.	Do.	3·40		64
Jalapin.	Do.		·49	60
Phloridzine.	Rectified spirit.		1·18	—
Picrotoxine.	Do.		·71	52
Santonine.	Do.		3·42	57
Convolvulin.	Do.		·59	—
Cholesterin.	Do.		·57	—

TABLE II.

CONSTANTS CONNECTED WITH SACCHAROMETRY.

(A).

Value of the coefficient K (eq. (6), p. 389) considered as a function of the temperature.

K.			Temp.
·4051	..	..	43
·3549	..	..	61
·2686	..	..	92

These values may be expressed by the formula

$$K = \cdot 5249 - \cdot 002786 \times t.$$

(B).

Value of the coefficient K' (eq. (6), p. 389).

K'			Temp.
·712	—	—	64

TABLE III.

Table of the Values of

$$N = \frac{\text{Rotatory power of salt}}{\text{Rotatory power of base}}.$$

Base.	Acid.	N.	Temp.	Solvent.
Quinia.	Hydrochloric.	1·344	60°	Proof spirit.
Brucia.	Do.,	·291	60°	Do.
Strychnia.	Do.,	·295	60°	Do.
Codeia.	Do.,	·909	62°	Do.
Morphia.	Do.,	·929	60°	Do.
Atropia.	Do.,	1·149	60°	Do.
Nicotine.	Do.,	– ·138	52°	Rectified spirit.

TABLE IV.

Values of the Constants A, B (p. 416).

Acid—Hydrochloric—Temp. 60°.

A.

	Quinia.	Brucia.	Codeia.	Morphia.	Atropia.
Quinia.	–	·5938	·1810	·1074	– ·0266
Brucia.	·4062	—	– ·1781	– ·0897	– ·0174
Codeia.	·8190	1·178	—	– 1·196	·1050
Morphia.	·8926	1·089	2·193	—	·1773
Atropia.	1·026	1·017	·8950	·8227	—

TABLE IV.—*continued.*

B.

	Quinia.	Brucia.	Codeia.	Morphia.	Atropia.
Quinia.	—	·0476	·0960	·1047	·1143
Brucia.	– ·0476	—	– ·0945	– ·0810	– ·0788
Codeia.	– ·0960	·0945	—	– 1·1659	– ·4750
Morphia.	– ·1047	·0810	·1166	—	– ·8019
Atropia.	– ·1143	·0788	·4750	·8019	—

NOTE.—It is evident from the general values of these constants (p. 416) that the value of A is independent of the particular compensating fluid used in the polariscope, and of the length of the tube, depending solely upon the *ratios* of the rotatory powers of the several substances. On the other hand, B, being a function of the *absolute* rotatory powers, will vary with the nature of the compensating fluid, and with the length of the tube.

Let r be the reading of the scale for any one standard fluid contained in the tube of the polariscope. Then if the compensating fluid and the tube be changed, everything else remaining the same, it is evident that

$$B \propto \frac{1}{r}.$$

In tabulating the values of B, therefore, I have multiplied each of them by the reading of the scale (in inches) for a solution of salicine in proof spirit, containing in each cubic inch ten grains of salicine. Hence, before applying these values to any particular experiment, it is necessary to fill the tube of the instrument used with a similar solution, and, having taken the reading of the scale, to divide the tabulated values by this number.

CORRIGENDA.

Page 375 line 29 *for* $A'B'$ *read* $A'B$.
—— 377 —— 8 *for* θ *read* θ_n.
—— 390 —— 8 *for* $\frac{O_3}{l_1}$ *read* $\frac{l_3}{l_2}$
—— 394 —— 16, 17, *for* $\frac{n}{a}$ *read* $\frac{n'}{a}$.
—— 416 —— 19 *for* $D_2 + D_2$ *read* $D_1 + D_2$.
—— ,, —— 24 *for* substraction *read* subtraction.
—— 438 —— 4 last letter, for a_2 *read* a_1.

XI.—*Report on the Strength of Single-riveted Lap Joints.* *By* Bindon B. Stoney, M. A., M. R. I. A., Memb. Inst. C. E. With Tables and Plate XXV.

Read June 22, 1874.

The experiments described in the following Report were made with a lever testing machine, represented in Plate XXV. The main lever of this machine is of wrought iron, with steel knife edges at the bearings; the longer arm of the lever being 20 feet, and the shorter arm one foot in length. Thus a weight of one cwt., or its fractional parts, in the scale hanging from the end of the longer arm, produces a tension of one ton and its corresponding fractional parts in the specimen under test, which connects the shorter arm of the lever with a powerful adjusting screw which is secured to the base of the machine. The weight of the main lever is balanced by means of a small lever placed above and connected with its centre of gravity, so that the main lever is thus floating as it were in the air—a matter of great convenience, for it obviates the necessity of taking the weight of the lever into consideration when calculating strains. This machine possesses great delicacy, and as there is only one main lever, the friction and complexity due to numerous bearings are avoided.

Table 1 gives the whole series of experiments in the order in which they were made, and Table 2 gives an abstract of some of the more important ones, arranged synoptically for facility of comparison. Several of the earlier experiments were made with the view of feeling the way, and are of comparatively little value. The strains which are in the Tables printed in heavy type indicate how a joint failed, whether by the plates tearing or by the rivets shearing.

The plates experimented on were nominally $\frac{3}{8}$ inch thick boiler plates, and the rivets were nominally $\frac{3}{4}$ inch in diameter, the riveting being carefully done by hand. Plate D is very anomalous; its strength apparently varied widely in different parts of the plate, and experiments on it are worthless as regards

general conclusions, while they show that one plate out of a lot may be of very inferior quality indeed, and yet to outward appearance be as good as the rest. Experiments 2 and 8, 38 and 49, 53 and 57, also indicate a variable quality in other plates, though in a less degree than in D. The "gross area" of a plate is its total transverse area before punching or drilling holes. The "net area" of a plate is the effective area through the line of holes, and is equal to the gross area minus the area of the holes cut out. The useful strength of a joint is measured by the breaking strain of the joint divided by the gross area of the plate.

It will be recollected that the hole for a $\frac{3}{4}$ inch rivet is usually from ·03 to ·06 inch larger than the nominal size of the rivet, in order to permit the latter to enter freely when heated. The bearing area of a rivet is the product of its diameter by the thickness of the plate against which it bears.

It will be convenient to divide the conclusions derivable from the experiments into three classes, according as they affect the strength of the rivets, the strength of the plates, or that of the joints as a whole.

RIVETS.

Shearing strength of Rivets in punched and drilled holes.—In experiments 1, 3, 17, 19, 20, 21, 22, 24, and 25, the shearing strength of rivets in single riveted lap joints varied from 17·78 to 20·14 tons per square inch, and possibly higher, as in several of these experiments the plates failed before the rivets were shorn. If we draw average conclusions from Table 2, where five rivets occurred in a row, we find that the average shearing strain of rivets in punched holes (expts. 19, 20, and 21) = 19·16 tons per square inch, and of those in drilled holes (expts. 22, 24, and 25) = 18·28 tons per square inch; that is, the shearing strength of rivets in drilled holes is about 4 per cent. weaker than in punched holes, which result agrees with Mr. Maynard's experiments, and is attributed by him to the sharp edges of the drilled hole having a greater tendency to shear off the rivet than the slightly rounded edges of the punched hole.

Shearing strength of Rivets in single-riveted and double-riveted lap joints compared.—In experiments 9 and 11 the shearing strength of rivets in double-riveted lap joints were 19·35 and 21·27 tons per square inch.

The apparent conclusion is, that the shearing strength of rivets is greater in double-riveted than in single-riveted lap joints, but further experiments are required to determine their relative proportions.

Pitch of Rivets.—The experiments on Plate C, Table 2, indicate that $\frac{3}{4}$ inch rivets are likely to fail before a $\frac{3}{8}$ inch plate of fair quality when the pitch exceeds three diameters of the rivet hole, and that consequently this pitch should not be exceeded in single-riveted lap joints.

Bearing area of Rivets.—It will be observed that the strain per square inch of bearing area of rivets in the strongest forms of single-riveted lap joint, in Table 2, does not exceed 30 tons, and in general is much less, while it may reach as high as 44·7 tons per square inch, or double the tensile strength of iron, when the rivet is in double shear, as in experiment 14, Table I.

PLATES.

Lap of Plates.—Comparing Plate E with the other plates in Table 2, it appears that the lap of single-riveted lap joints need not exceed $2\frac{1}{2}$ diameters of the hole, and that consequently the boiler-makers' practice of making the lap of single-riveted joints equal to 3 diameters of the rivet is sufficient when the plates are of fair quality. With reference to the lap it may be observed, that the splitting of a plate between the edge and the rivets is not so likely to occur in wide plates where several rivets are in a transverse row, like the experiments in Table 2, as in very narrow plates, like examples 1, 10, 11, and 12 ; for the greater width and stiffness of the wide plates counteracts this tendency to open under severe strain. The lap of narrow single-riveted bars however should not be less than four times the diameter of the rivet.

Pitch of Holes.—The experiments on Plates C and E, in Table 2, indicate that the best pitch in single-riveted lap joints for $\frac{3}{4}$ inch rivets in $\frac{3}{8}$ inch plates is from 2 to $2\frac{1}{2}$ diameters of the rivet hole, and that consequently the shearing area of the rivets will exceed the net area of the plate in the strongest arrangement of single-riveted lap joint. This conclusion is at variance with a common opinion, that a coarser pitch is stronger than the experiments indicate, and it shows that the boiler-makers' practice of making the pitch equal to from $2\frac{1}{2}$ to 3 diameters of the nominal size of the rivet is well founded.

Effect of drilling without riveting on the net strength of Plates.—The following experiments, though not concordant, indicate that drilling iron plates (without riveting) reduces their net strength slightly more than is merely due to the sectional area of the iron drilled out :—

Comparing Experiments 6 and 8, Plate A, the reduction of net strength from drilling was		2·6	per cent.
„ „ 37 and 45, Plate E, „ „ .		3·9	„
„ „ 48 and 53, Plate F, „ „ .		9·2	„
„ „ 48 and 57, Plate F, the increase was .		7·5	„
Average reduction of *net* strength of plates from drilling without riveting		2·05	„

Effect of Punching without riveting on the net strength of Plates.—The following experiments indicate that punching iron plates (without riveting) reduces their net strength considerably more than is merely due to the sectional area of the iron punched out :—

Comparing Experiments 7 and 8, Plate A, the reduction of net strength from punching was		15·6	per cent.
„ „ 37 and 44, Plate E, „ „ .		10·2	„
„ „ 48 and 52, Plate F, „ „ .		7·4	„
„ „ 48 and 56, Plate F, „ „ .		12·6	„
Average reduction of *net* strength of plates from punching without riveting		11·45	„

Effect of riveting on the Plate itself.—Experiments 54 and 55 indicate that the operation of riveting and the subsequent longitudinal contraction of the rivets on cooling do not weaken the plate, but rather the reverse. It should be observed, however, that no benefit is derived from the friction produced between the plates by the contraction of the rivets when a joint is on the point of rupture, for then the rivets have elongated under strain and therefore lost their grip on the plate.

The results of the last three sections are confirmed by Column 14 in

Table 2, where we find that the average tearing unit-strain of the plate between the holes in seven joints with punched holes is 80·1 per cent., and in seven joints with drilled holes 89·8 per cent., of the tearing unit-strain of the original solid plate. From this it follows that the average reduction of net strength of the iron between the holes in joint plates with punched holes as compared with those with drilled holes is 10·8 per cent., which does not differ materially from the proportion previously obtained from perforated plates without riveting.

Putting these results into a tabular form for convenience we have in round numbers,

The tearing unit-strain of the original solid plate, . . .	100
The tearing-unit strain of the iron between the holes in single-riveted lap joints with drilled holes, . . .	90
The tearing unit-strain of the iron between the holes in single-riveted lap joints with punched holes, . . .	80

We thus see that, from the mechanical treatment and from the form of the joint, the iron which remains between the holes in a single-riveted lap joint has lost 10 per cent. of its original strength when the holes are drilled, and 20 per cent. when the holes are punched. To obtain the actual strength of the joint we must make a further reduction for the iron cut out in the rivet holes, which will presently be found to reduce the strength of the joint to nearly one-half the proportions in the foregoing table.

SINGLE-RIVETED LAP JOINTS.

Strength of single-riveted Lap Joints with punched holes.—Column 12, in Table 2, indicates that though the strength of a single-riveted lap joint with punched holes, and of the best proportions, may sometimes approach, or even equal, 50 per cent. of that of the solid plate, yet this cannot be depended on, as it may sometimes descend as low as 37 per cent., while the average strength of the nine joints with punched holes in Table 2 is 43·7 per cent. of that of the original solid plate.

Strength of single-riveted Lap Joints with drilled holes.—Column 12, in Table 2, indicates that though the strength of a single-riveted lap joint

with drilled holes, and of the best proportions, may occasionally reach, or even exceed, 50 per cent. of that of the original solid plate, yet this cannot be depended on, as it may sometimes descend as low as 44 per cent., while the average strength of the nine joints with drilled holes in Table 2 is 47·7 per cent. of the original plate.

Comparing Experiments	19 and 22, Plate C, the reduction of strength of a punched compared a drilled joint is . . .	3·9	per cent.
„ „	38 and 43, Plate E, . . .	4·9	„
„ „	41 and 42, „ . . .	2·36	„
„ „	46 and 47, „ . . .	9·5	„
„ „	49 and 50, Plate F., . . .	16·3	„
Average reduction of strength of single-riveted lap joints with punched holes compared with those with drilled holes .		7·56	„

GENERAL CONCLUSIONS.

Putting the foregoing results into a concise form, we have the following rules for practical guidance in single-riveted lap joints :—

1. The strongest pitch for single-riveted lap joints is about two and a-half, and does not exceed three, diameters of the nominal size of the rivet.

2. The lap of single-riveted joints need not exceed three diameters of the nominal size of the rivet for wide plates of fair quality and four diameters for bars.

3. If the breaking weight of the original solid plate is 100
The breaking weight of a single-riveted lap joint of good proportions with drilled holes will be 48
Ditto, with punched holes will be 44

4. In single-riveted lap joints the difference of strength in favour of drilled work over punched work is about 9 per cent. of the latter.

It may perhaps be thought that in framing practical rules we should take

the lowest in place of average percentages from Column 12, Table 2, inasmuch as a joint might in practice be as weak as, or even weaker than, the weakest of those in the Table; but it will be recollected that the engineer always adopts a co-efficient, or factor of safety, for the actual working strain in any structure, and this co-efficient for riveted work generally varies from one-fourth to one-seventh of the breaking strain, depending upon the nature of the work. Thus a sufficient margin is left to ensure safety in working practice.

Hitherto it has been customary to assume that the strength of single-riveted lap joints is equal to 56 per cent. of that of the solid plate, and this practice has been chiefly based on the great authority of the late Sir Wm. Fairbairn, who many years since made a few experiments on $\frac{1}{4}$ inch plates connected by two or three rivets of $\frac{3}{8}$ inch and $\frac{1}{2}$ inch diameter.* In these experiments it was found that the strength of a single-riveted lap joint was about 46 per cent. of that of the original solid plate; a result which agrees very closely with those obtained by the author. Sir Wm. Fairbairn, however,† "looking at the increase of strength which might be gained by having a greater number of rivets in combination, and the adhesion of the two surfaces of the plate in contact," assumed that if the strength of the original solid plate was 100, that of the single-riveted lap joint would be 56; that is, the percentage of strength derived from the experiments was raised from 46 to 56—an assumption which is not supported by anything in the author's experiments or by Sir Wm. Fairbairn's previous reasoning, for a few pages before‡ he writes as follows:—"It may be said that the pressure or adhesion of the two surfaces of the plates would add to the strength; but this is not found to be the case, to any great extent, as in almost every instance the experiments indicate the resistance to be in the ratio of their sectional area, or nearly so."

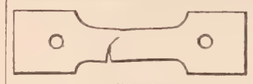
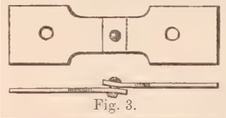
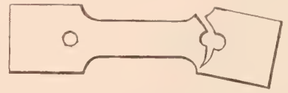
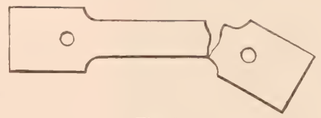
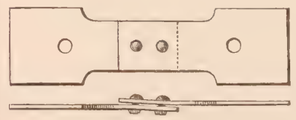
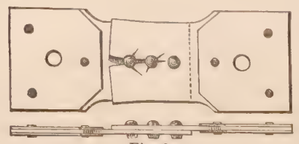
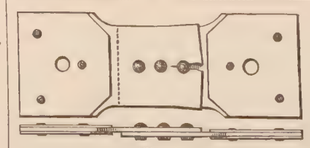
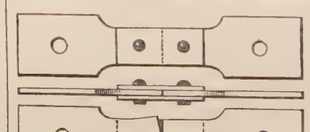
It is indeed possible that the longitudinal seams of a cylindrical boiler in which the plates break joint are somewhat strengthened by this arrangement as well as by the extra iron in the cross seams, though I am not aware of any experiments to test this, but at all events such reasoning does not apply to the cross seams of the boiler, or to the fore-and-aft seams of a ship, or to nu-

* Appendix to Fairbairn's Useful Information for Engineers. First Series.

† *Idem*, p. 284. ‡ *Idem*, p. 271.

merous other structures common in engineering; and it seems therefore hazardous practice to assume that all single-riveted lap joints have a strength of 56 per cent. of the solid plate, when the best forms of experimental joints have on an average only from 44 to 48 per cent., according as the holes are punched or drilled, and in many cases in actual practice the rivets are not pitched so close as to give the strongest form of joint, in which case a most dangerous error might be incurred by reckoning on 56 per cent. of the strength of the solid plate.

TABLE I.

No.	Date.	With or cross grain.	Nominal thickness of Plate.	Actual dimensions of Plate.	Gross Area of Plate.	Net Area of Plate.	Lap.	Rivets, nominal size.	Rivet Head.	Hole Punched or Drilled.	Mean Diameter of Rivet Holes.	Shearing Area of Rivets	No. of Rivets.	Single or Double Shear.	Bearing Area of Rivets.	Pitch of Rivets.	Breaking Weight.	Strain per s. inch of rivet area.	Strain per s. inch of **Net** plate area.	Strain per s. inch of **Gross** plate area.	Ultimate elongation, or set after fracture.	Strain per s. inch of Bearing area.	Letter of Plate.	Plate or Rivets gave way.	OBSERVATIONS.
			inch.	inches.	s. inch.	s. inch.	inch.	nch.			inch.	s. inch.			s. inch.	inch.	tons.	tons.	tons.	tons.	per cent.	tons.			
1	1870. Nov. 23	with	$\frac{3}{8}$	2·96 × ·39	1·154	0·8502	$2\frac{1}{2}$	$\frac{3}{4}$	snap.	punched.	·78	·478	1	single.	·304	. .	9·625	20·14	**11·32**	8·34	. .	31·6	A.	Plate split.	Fig. 1. Rivet hole $\frac{3}{4}$ inch diameter at small end, $\frac{1}{16}$ more at larger end. End of one plate opened, and cracks appeared at the sides of the hole.
2	Dec. 3	with	$\frac{3}{8}$	3 × ·40	1·2	1·2	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	17·0	. .	**14·17**	**14·17**	1·5	. .	A.	Plate torn.	 Fig. 2. Tore partially across and along a laminated and flawed surface.
3	,,	with	$\frac{3}{8}$	3 × ·39	1·17	0·87	$2\frac{3}{4}$	$\frac{3}{4}$	snap.	punched.	·78	·478	1	single.	·304	. .	9·06	**18·95**	10·41	7·74	. .	29·8	A.	Rivet shorn.	 Fig. 3. Lap seems sufficient.
4	,,	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	Abortive experiment, plate tore through one of the connecting holes.
5	1871. Jan. 31	with	$\frac{3}{8}$	2·98 × ·4	1·19	1·19	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	19·5	. .	16·39	16·39	2·5	. .	A.	. .	 Fig. 4. Plate tore through one side of the connecting hole where the width was $2\frac{1}{8}$ inches. This gives 16·36 tons per square inch for strain at narrow part of plate at time of rupture.
6	,,	with	$\frac{3}{8}$	3 × ·4	1·2	0·9	. .	$\frac{3}{4}$	. .	drilled.	·75	. .	. .	. .	. .	. .	16·0	. .	**17·78**	13·3	. .	. .	A.	Plate torn thro' drilled hole.	Fig. 5. To test effect of drilling. Compared with 8, drilling reduced net strength 2·6 per cent.
7	Feb. 13	with	$\frac{3}{8}$	3 × ·39	1·17	0·87	. .	$\frac{3}{4}$	. .	punched.	·78	. .	. .	. .	. .	. .	13.4	. .	**15·4**	11.5	. .	. .	A.	Plate torn thro' punched hole.	See Fig. 5. To test effect of punching. Compared with 8, punching reduced net strength 15·6 per cent.
8	Apr. 12	with	$\frac{3}{8}$	3 × ·4	1·2	1·2	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	21·9	. .	**18·25**	**18·25**	6	. .	A.	Plate torn.	 Fig. 6. To test strength of iron in plate A.
9	May 2	with	$\frac{3}{8}$	4 × ·4	1·6	1·3	$4\frac{3}{4}$	$\frac{3}{4}$	snap.	punched.	·78	·956	2	single.	·624	1·95	18·5	**19·35**	14·23	11·56	. .	29·9	A.	Rivets shorn.	 Fig. 7. Drew joints into a straight line, and sheared both rivets. Compared with experiment 3, this indicates a little additional shearing strength in rivets with double riveting. Lap was sufficient. One of the plates showed indications of being on the limit of rupture, cracks commencing at each side of one of the rivet holes.
10	May 23	with	$\frac{3}{8}$	6·05 × ·4	2·42	2·1	$6\frac{3}{8}$	$\frac{3}{4}$	snap.	punched.	·78	1·434	3	single.	·936	$1\frac{3}{4}$	23·5	16·39	**11·19**	9·71	. .	25·1	A.	Plate split.	 Fig. 8. Two auxiliary plates riveted with 3 half-inch rivets on each end of sample to ensure its not tearing through connecting holes. One plate split out in front of rivets, the split extending from rivet to rivet and the plate spreading.
11	June 21	with	$\frac{3}{8}$	6 × ·4	2·4	2·09	$7\frac{1}{4}$	$\frac{3}{4}$	snap.	punched.	·78	1·434	3	single.	·936	2	30·5	**21·27**	**14·6**	12·71	. .	32·5	B.	Plate split and one rivet shorn.	 Fig. 9. One rivet shorn and one plate split; well-proportioned joint, as both rivets and plate seemed to go simultaneously.
12	July 18	with	$\frac{3}{8}$	3 × ·4	1·2	0·888	$3\frac{1}{2}$	$\frac{3}{4}$	snap.	punched.	·78	·956	1 on each side of joint.	double.	·312	. .	13·25	13·86	**14·92**	11·04	. .	42·4	B.	Plate split.	 One plate split out in front of rivet; length of part split = $1\frac{1}{2}$. Covers same thickness as plate and not injured.

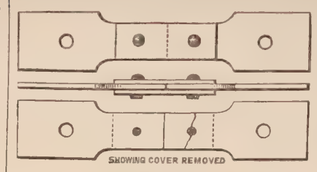
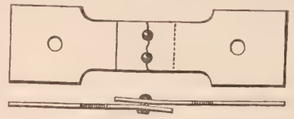
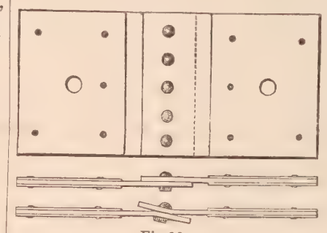
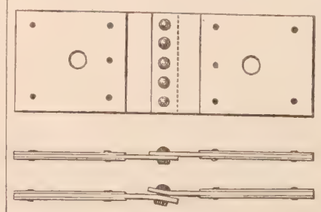
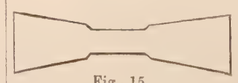
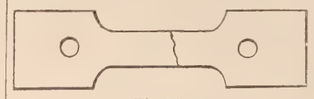
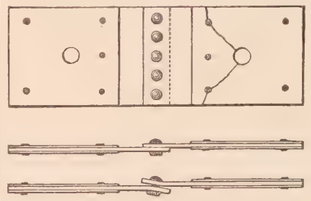
13	"	with	$\frac{3}{8}$	3 × ·4	1·2	1.2	..	..	..	..	..	..	..	..	..	..	20·5	..	17·1	17·1	..	..	B.	..	See Fig. 4. Tore through connecting hole, where net width = 1·70 + 1·55 = 3·25 inches. Set = 2 per cent.
14	July 26	with	$\frac{3}{8}$	3 × ·38	1·14	0·843	4	$\frac{3}{4}$	snap.	punched.	·78	·956	1 on each side of joint.	double	·2964	..	13·25	13·86	**15·71**	11·62	..	44·7	B.	Plate torn thro' the rivet hole.	 Fig. 11. At $3\frac{1}{2}$ tons one plate began to slip between covers, showing that friction was not of much value; the other plate began to slip at $12\frac{1}{2}$ tons: friction probably not to be depended on unless there are several rivets, as some may fail to produce sufficient grip, or fail to fill holes perfectly. Lap sufficient, no splitting of plate as in Fig. 10.
15	"	with	$\frac{3}{8}$	3 × ·4	1·2	1·2	..	..	..	.	..	..	..	..	..	..	22·25	..	**18·54**	**18·54**	6	..	B.	Plate torn right across.	See Fig. 6. To test iron; width reduced 4 per cent. at point of fracture. A flash of light appeared at moment of fracture, possibly from paint of testing machine, which was violently struck by the sample at moment of fracture; see experiments 19 and 24. See also experiment 8, where plate A appeared to be of the same strength and quality of iron.
16	Aug. 9	with	$\frac{3}{8}$	3 × ·4	1·2	0·888	4·5	$\frac{3}{4}$	snap.	punched.	·78	·956	2	single.	·624	$1\frac{3}{4}$	15·75	16·48	**17·74**	13·12	..	25·24	B.	Plate tore thro' one of the rivet holes.	See Fig. 7. After rupture the sliding of plates past each other was only ·05 inch.
17	"	with	$\frac{3}{8}$	4·5 × ·39	1·755	1·147	4·5	$\frac{3}{4}$	snap.	punched.	·78	·956	2	single.	·608	2	18·5	19·35	**16·13**	10·54	..	30·43	B.	Plate tore thro' the rivet holes.	 Fig. 12. Two rivets in one transverse row.
18	Aug. 23	with	$\frac{3}{8}$	10·5 × ·4	4·2	2·64	4·5	$\frac{3}{4}$	snap.	punched.	·78	2·39	5	single.	1·56	2	38·75	16·21	**14·67**	9·23	..	24·84	B.	Plate tore thro' the rivet holes.	 Fig 13. Strengthening covers at ends. Size and pitch of rivets very nearly Boilermakers' practice.
19	Sept. 6	with	$\frac{3}{8}$	12 × ·41	4·92	3·32	4·5	$\frac{3}{4}$	snap.	punched.	·78	2·39	5	single.	1·599	$2\frac{3}{8}$	47·3	19·79	**14·24**	9·61	..	29 56	C.	Plate tore thro' the rivet holes.	See Fig. 13. A flash of light appeared at moment of fracture. See experiments 15 and 24.
20	Oct. 18	with	$\frac{3}{8}$	13 × ·39	5·07	3·55	4·5	$\frac{3}{4}$	snap.	punched.	·78	2·39	5	single.	1·521	$2\frac{1}{2}$	42·5	**17·78**	11·79	8·38	..	27·94	C.	Rivets sheared, beginning at one edge.	See Fig. 13. Rivets did not shear simultaneously, but that near one edge first gave way, and then all the others, except the last, tore in succession. Rivets were apparently good rivet iron.
21	Oct. 23	with	$\frac{3}{8}$	13 × ·39	5·07	3·49	4·5	$\frac{3}{4}$	snap.	punched.	·81	2·576	5	single.	1·58	$2\frac{1}{2}$	51·5	**19·90**	14 75	10·15	..	32·6	C.	Rivets sheared.	See Fig. 13. Same plates as last; plates slightly warmed and straightened and riveted again. Holes in plates having been a little enlarged in last experiment, the diameter of rivet hole is taken to be 0·81 inches. Plates on the point of giving way, several cracks commencing at rivet holes.
22	Oct. 31	with	$\frac{3}{8}$	12 × ·4	4·8	3·18	4·5	$\frac{3}{4}$	snap.	drilled.	·81	2·5765	5	single.	1·62	$2\frac{3}{8}$	48·0	**18·63**	15·10	10·0	..	29·63	C.	Rivets sheared.	See Fig. 13. Compared with experiment 19, punching reduced strength of joint 3·9 per cent. compared with drilling.
23	Nov. 18	with	$\frac{3}{8}$	10·9 × ·41	4·47	2·81	4·5	$\frac{3}{4}$	snap.	drilled.	·81	2·5765	5	single.	1·66	$2\frac{1}{4}$	43·75	16·97	**15·57**	9·79	..	26·35	C.	Both plates tore.	See Fig. 13.
24	Dec. 6	with	$\frac{3}{8}$	10·5 × ·39	4·095	2·535	4·5	$\frac{3}{4}$	snap.	drilled.	·80	2·513	5	single.	1·56	2	45	**17·9**	17·75	10·99	..	28·84	C.	Rivets sheared.	See Fig. 13. A flash of light appeared at moment of fracture. Strain seemed to pass very fairly through axis of plates, so that it was uniformly distributed. Compared with experiment 23, 2-inch pitch seems stronger than $2\frac{1}{4}$-inch pitch.
25	Dec. 18	with	$\frac{3}{8}$	9·5 × ·4	3·8	2·2	4·5	$\frac{3}{4}$	snap.	drilled.	·80	2·513	5	single.	1·6	$1\frac{3}{4}$	46	**18·30**	**20·90**	12·10	..	28·75	C.	Rivets sheared, and one plate showed incipient cracks at rivet holes.	 Fig. 14 Plate area well proportioned to rivet area. Compared with experiments 23 and 24, $1\frac{3}{4}$-inch pitch seems stronger than $2\frac{1}{4}$ or 2-inch pitch.
26	1872. Jan. 27	with	$\frac{3}{8}$	2 × ·4	0·8	0·8	..	..	..	..	..	..	..	..	..	..	..	..	..	..	4·5.	..	C.	..	 Fig.. 15 At $15\frac{1}{2}$ tons (= 19·4 tons per square inch), the tapered ends narrowed and upset so as to slip through the clamps. The sample, however, seemed on the point of rupture as an incipient crack showed on one edge. Set = 4·5 per cent.

TABLE I.—CONTINUED.

No.	Date.	With or cross grain.	Nominal thickness of Plate.	Actual dimensions of Plate.	Gross Area of Plate.	Net Area of Plate.	Lap.	Rivets, nominal size.	Rivet Heads.	Hole Punched or Drilled.	Mean Diameter of Rivet Holes.	Shearing Area of Rivets.	No. of Rivets.	Single or Double Shear.	Bearing Area of Rivets.	Pitch of Rivets.	Breaking Weight.	Strain per s. inch of rivet area.	Strain per s. inch of **Net** plate area.	Strain per s. inch of **Gross** plate area.	Ultimate elongation, or set after fracture.	Strain per s. inch of Bearing area.	Letter of Plate.	Plate or Rivets gave way.	OBSERVATIONS.
			inch.	inches.	s. inch.	s. inch.	inch.	inch.			inch.	s. inch.			s. inch.	inch.	tons.	tons.	tons.	tons.	per cent.	tons.			
27	1872. Feb. 19	with	$\frac{5}{8}$	1·5 × ·4	0·6	0·6	..	..	..	..	..	..	..	..	..	..	13·2	..	**22**	**22**	11	..	C.	Plate torn	 Fig. 16. Experiment to test iron in plate C. Good tough iron.
28	,,	with	$\frac{5}{8}$	8 × ·34	2.72	1·36	4·5	$\frac{3}{4}$	snap.	drilled.	·8	2·513	5	single.	1·36	$1\frac{1}{2}$	31·2	12·41	**22·94**	11·47	..	22·93	D.	Plate tore thro' rivet holes.	See Fig. 14.
29	July 29	with	$\frac{5}{8}$	9·5 × ·34	3·23	1·87	3·0	$\frac{3}{4}$	snap.	drilled.	·8	2·513	5	single.	1·36	$1\frac{3}{4}$	34·0	13·53	**18·18**	10·53	..	25·0	D.	Plate tore thro' rivet holes.	See Fig. 14.
30	1873. Jan. 22	with	$\frac{5}{8}$	9·5 × ·34	3.23	1·87	2·0	$\frac{3}{4}$	snap.	drilled.	·8	2·513	5	single.	1·36	$1\frac{3}{4}$	31·8	12·65	**17·01**	9·85	..	23·4	D.	Plate tore thro' rivet holes.	See Fig. 14. Differs from last merely in lap being 2′ in place of 3″. Reduced lap seems to have reduced strength of joint. See remark on experiment 32.
31	Jan. 27	with	$\frac{5}{8}$	8 × ·34	2·72	1·36	2·0	$\frac{3}{4}$	snap.	drilled.	·8	2·513	5	single.	1·36	$1\frac{1}{2}$	25	9·95	18·4	9·2	..	18·4	D.	Plate tore thro' pin hole at end.	 Fig. 17. Length of crooked line of fracture measured through solid plate = 5·4 inches.
32	Feb. 17	with	$\frac{5}{8}$	8 × ·35	2·80	1·4	2·0	$\frac{3}{4}$	snap.	drilled.	·8	2.513	5	single.	1·4	$1\frac{1}{2}$	27	10·74	**19·3**	9·64	..	19·3	D.	Plate tore thro' rivet holes.	See Fig. 14. Compared with experiment 28 the greater lap seems to have produced greater strength. Plate D, judging from experiments 33, 34, and 36, must have been very variable in quality.
33	,,	with	$\frac{5}{8}$	3 × ·35	1·05	1·05	...	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	D.	Plate torn.	See Fig. 16. Broke as 13 tons (=12·4 tons per square inch) was lowered on it; must be very bad iron. Weight not laid on by degrees, but all at once.
34	Feb. 24	with	$\frac{5}{8}$	3 × ·35	1·05	1·05	..	..	..	..	..	..	..	..	..	..	12·5	..	**11·81**	**11·81**	0	..	D.	Plate tore fairly across.	See Fig. 16. To test Plate D. Very careful experiment; $12\frac{1}{2}$ tons very slowly and slightly stretched the iron till it broke with a dull sound. Fracture hackly and free from any crystal of a bright appearance. No ultimate set after fracture. Bending test over $\frac{1}{2}$ inch radius = $14\frac{1}{2}°$.
35	Mar. 11	with	$\frac{5}{8}$	8 × ·34	2·72	1·33	2·0	$\frac{3}{4}$	snap.	punched.	·82	2·6405	5	single.	1·394	$1\frac{1}{2}$	21·75	8·24	**16·36**	8·0	..	15·6	D.	Plate tore thro' rivet holes.	See Fig. 14. These experiments with plate D seem to show that a single-riveted joint may appear to be much stronger than 50 per cent. of the unpunched plate when the latter is bad iron of variable quality.
36	Mar. 28	with	$\frac{5}{8}$	1·5 × ·35	0·525	0·525	..	..	..	..	..	..	..	..	..	..	7·75	..	**14·76**	**14·76**	..	..	D.	Plate tore fairly across.	See Fig. 16. To test plate D. Broke as before with a dull sound; fracture was hackly and had specks of darker cindery-looking matter here and there. Compared with experiments 28, 33, and 34, this plate must have been of very unequal texture and strength in different parts.
37	,,	with	$\frac{5}{8}$	1·5 × ·35	0·525	0·525	..	..	..	..	..	..	..	..	..	..	11·25	..	**21·43**	**21·43**	2	..	E.	Plate tore fairly across.	See Fig. 16. Sold as "Best Boiler" plate. Hard iron, judging from small percentage of set after fracture.
38	,,	with	$\frac{5}{8}$	8 × ·36	2·88	1·404	2·0	$\frac{3}{4}$	snap.	punched.	·82	2·6405	5	single.	1·476	$1\frac{1}{2}$	28·75	10·89	**20·43**	9·99	..	19·5	E.	Plate tore thro' rivet holes.	See Fig. 14.
39	,,	with	$\frac{5}{8}$	8 × ·36	2·88	1·404	1·5	$\frac{3}{4}$	snap.	punched.	·82	2·6405	5	single.	1·476	$1\frac{1}{2}$	27·75	10·51	**19·76**	9·64	..	18·8	E	Plate tore thro' rivet holes.	See Fig. 14. One plate split out to edge at the end rivet. Compared with experiment 38, the smaller lap seems to have been very nearly as strong as the other.
40	April 9	with	$\frac{5}{8}$	8 × ·38	3·04	1·482	1·5	$\frac{3}{4}$	snap.	drilled.	·82	2·6405	5	single.	1·558	$1\frac{1}{2}$	32·0	12·12	**21·592**	10·526	..	20·6	E.	Plate tore thro' rivet holes.	See Fig. 14. Fine grained iron with fine crystal well developed. Compared with experiment 37, the net strength of the iron was not diminished by drilling. Compared with experiment 39, punching reduced strength of joint 8·4 per cent. compared with drilling.
41	April 21	with	$\frac{5}{8}$	9·5 × ·37	3·515	2.0	1·5	$\frac{3}{4}$	snap.	punched.	·82	2·6405	5	single.	1·517	$1\frac{3}{4}$	32	12·12	**16·0**	9·10	..	21·1	E.	Plate tore thro' the rivet holes.	See Fig. 14. Compared with experiment 39, $1\frac{1}{2}$ inch pitch seems stronger than $1\frac{3}{4}$ inch.
42	April 29	with	$\frac{5}{8}$	9·5 × ·37	3·515	2·0	1·5	$\frac{3}{4}$	snap.	drilled.	·82	2·6405	5	single.	1·517	$1\frac{3}{4}$	32·75	12·41	**16·37**	9·32	..	21·59	E.	Plate tore thro' the rivet holes.	See Fig. 14. Compared with experiment 40, $1\frac{1}{2}''$ pitch seems stronger than $1\frac{3}{4}$ inch. Compared with experiment 41, punching reduced strength of joint 2·36 per cent. compared with drilling.
43	May 7	with	$\frac{5}{8}$	8 × ·36	2·88	1·404	2·0	$\frac{3}{4}$	snap.	drilled.	·82	2·6405	5	single.	1·476	$1\frac{1}{2}$	30·25	11·46	**21·54**	10·503	..	20·5	E.	Plate tore thro' the rivet holes.	See Fig. 14. Compared with experiment 37, the net strength of the iron was not reduced by drilling. Compared with experiment 40, $1\frac{1}{2}''$ lap seems to be quite as strong as $2''$ lap. Compared with experiment 38, punching reduced strength of joint 4·9 per cent. compared with drilling.
44	May 27	with	$\frac{5}{8}$	4 × ·38	1·52	0·8968	..	..	..	punched.	·82	..	2 holes.	..	..	$1\frac{1}{2}$	17·25	..	**19·24**	11·35	..	..	E.	Plate tore thro'	

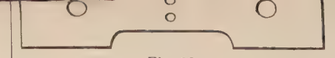
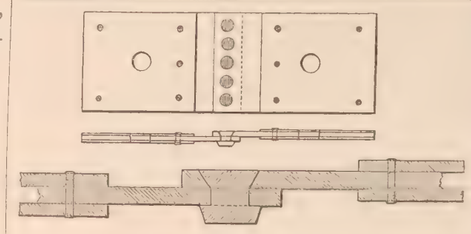
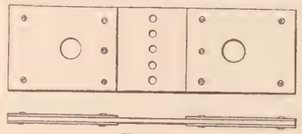
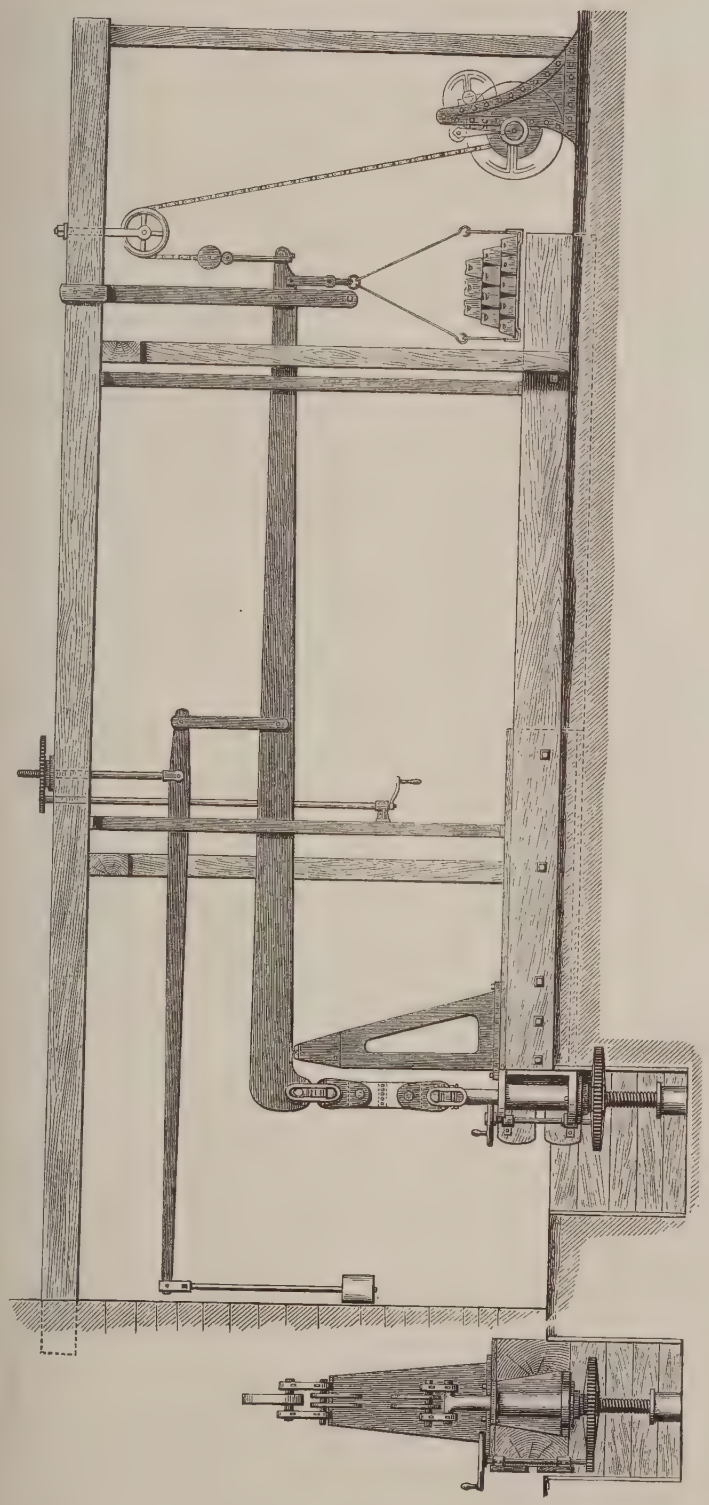
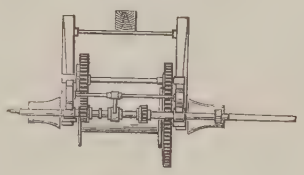
																								periment 37, punching reduced net strength 10·2 per cent.  Fig. 18.
45	,,	with	$\frac{5}{8}$	4 × ·37	1·48	0·8732	..	..	..	drilled.	·82	..	2 holes.	..	..	$1\frac{1}{2}$	18·0	..	**20·60**	12·16	..	..	E.	Plate tore thro' holes as before.	See Fig. 18. To try effect of *drilling* holes without rivets. Compared with experiment 44, punched holes are weaker than drilled holes. Compared with 37, drilling reduced net strength nearly 4 per cent.
46	June 9	with	$\frac{5}{8}$	9·5 × ·37	3·515	1·998	2·0	$\frac{3}{4}$	snap.	drilled.	·82	2·6405	5	single.	1·517	$1\frac{3}{4}$	37·0	14·01	**18·52**	10·53	..	24·39	E.	Plate tore thro' rivet holes.	See Fig. 14. Compared with experiment 42, 2-inch lap appears stronger than $1\frac{1}{2}$ inch. Compared with experiment 43, $1\frac{1}{2}$ inch pitch appears as strong as $1\frac{3}{4}$ inch pitch.
47	,,	with	$\frac{5}{8}$	9·5 × ·37	3·515	1·998	2·0	$\frac{3}{4}$	snap.	punched.	·82	2·6405	5	single.	1·517	$1\frac{3}{4}$	33·5	12·69	**16·77**	9·53	..	22·08	E.	Plate tore thro' rivet holes.	See Fig. 14. Compared with experiment 41, 2-inch lap appears a little stronger than $1\frac{1}{2}$ inch. Compared with experiment 46, punching reduced strength of joint 9·5 per cent. compared with drilling. Compared with 38, $1\frac{1}{2}$ inch pitch appears stronger than $1\frac{3}{4}$ inch pitch.
48	July 2	with	$\frac{5}{8}$	1·5 × ·38	0·57	..	..	..	..	..	..	..	..	..	..	..	13·7	..	**24·0**	**24·0**	6	..	F.	..	See Fig. 16. Strong iron, but laminated. Fracture silvery-looking for iron.
49	,,	with	$\frac{5}{8}$	8 × ·39	3·12	1·52	2·0	$\frac{3}{4}$	snap.	punched.	·82	2·6405	5	single.	1·6	$1\frac{1}{2}$	27·7	10·49	**18·22**	8·88	..	17·31	F.	Plate tore thro' rivet holes.	See Fig. 14. Laminated plate, but good iron. See experiment 38, where a similar joint in a plate of apparently inferior quality bore a higher strain.
50	,,	with	$\frac{5}{8}$	8 × ·38	3·04	1·52	2·0	$\frac{3}{4}$	snap.	drilled.	·80	2·513	5	single.	1·52	$1\frac{1}{2}$	32·25	12·83	**21·22**	10·61	..	21·22	F.	Plate tore thro' rivet holes.	See Fig. 14. Laminated plate, but good iron. Compared with experiment 49, punching reduced strength of joint 16·3 per cent. compared with drilling.
51	Aug. 12	with	$\frac{5}{8}$	8 × ·38	3·04	..	2·0	$\frac{3}{4}$	flush with plate on countersunk side.	punched & countersunk in one plate.	·82	..	5	single.	..	$1\frac{1}{2}$	24·5	..	..	8·06	..	..	F.	Plate tore thro' the countersunk holes.	 Fig. 19. Angle of the countersunk drill = 60°; hole in countersunk plate countersunk to within $\frac{1}{16}''$ of surface, not quite thro'. Compared with 49, countersinking reduced strength of punched joint 9.2 per cent. Compared with 48, the strength of this countersunk joint is only 33·6 per cent. of that of the original plate.
52	,,	with	$\frac{5}{8}$	8 × ·37	2·96	1·44	..	..	..	punched.	·82	..	5 holes.	..	..	$1\frac{1}{2}$	32		**22·22**	10·81	..	.	F.	Tore thro' holes.	 Fig. 20. Perforated plate without rivets, *punched* to see the effect of punching holes out of a plate. Compared with experiment 48, punching reduced net strength 7·4 per cent.
53	,,	with	$\frac{5}{8}$	8 × ·38	3·04	1·48	..	..	..	drilled.	·82	..	5 holes.	..	..	$1\frac{1}{2}$	32·25	..	**21·8**	10·61	..	..	F.	Tore thro' holes.	See Fig. 20. Perforated plate without rivets, *drilled* to see the effect of drilling holes out of a plate. Compared with experiment 48, net strength was reduced by drilling 9·2 per cent. Compare 57, where drilling seems to increase net strength.
54	Aug. 26	with	$\frac{5}{8}$	8 × ·39	3·12	1·52	..	$\frac{3}{4}$	snap.	punched.	·82	..	5	..	..	$1\frac{1}{2}$	38·25	..	**25·16**	12·26	..	..	F.	Plate tore thro' holes.	Fig. 21. Perforated plate, *punched* holes filled with rivets to see if the latter had any effect on the plate. Fracture rather steely-looking in parts. See experiment 48.
55	,,	with	$\frac{5}{8}$	8 × ·40	3·2	1·56	..	$\frac{3}{4}$	snap.	drilled.	·82	..	5	..	..	$1\frac{1}{2}$	45·0	..	**28·34**	14·06	..	..	F.	Tore thro' holes.	See Fig. 21. Perforated plate, *drilled* holes filled with rivets. Compared with 54 the drilled plate is, as usual, stronger than the punched plate. Comparing 54 and 55 with 48, it seems probable that the contraction of the rivets materially strengthened the iron, by their heads holding together the parts on either side of the row of holes.
56	Oct. 22	with	$\frac{5}{8}$	8 × ·36	2·88	1·404	..	..	..	punched.	·82	..	5 holes.	..	..	$1\frac{1}{2}$	30·75	..	**21·97**	10·68	..	..	F.	Tore thro' holes.	See Fig. 20. Perforated plate with *punched* holes, similar to 52, and without rivets. Compared with experiment 48, punching reduced net strength 12·6 per cent. Compared with 57, punching reduces strength very considerably as compared with drilling.
57	,,	with	$\frac{5}{8}$	8 × ·36	2·88	1·404	..	..	..	drilled.	·82	..	5 holes.	..	..	$1\frac{1}{2}$	36·25	..	**25·8**	12·58	..	..	F.	Tore thro' holes.	See Fig. 20. Perforated plate with drilled holes similar to 53, and without rivets. Compared with experiment 48, drilling appeared to *increase* net strength 7·5 per cent. Compared with 53, this experiment indicates a variable quality in the plate.

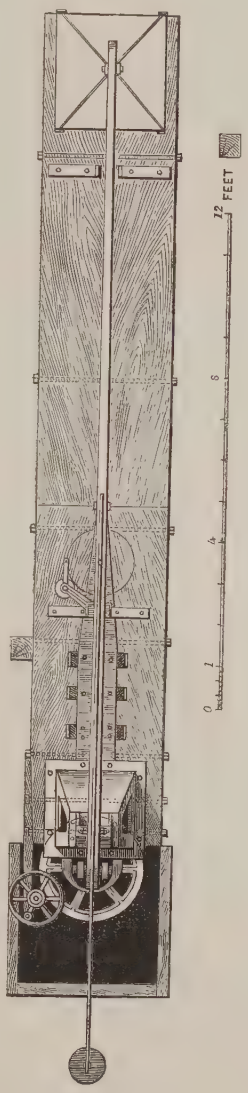
TABLE II.

EXPERIMENTS ON SINGLE-RIVETED LAP-JOINTS WITH $\frac{3}{8}$-INCH PLATES, AND FIVE $\frac{3}{4}$-INCH RIVETS IN EACH JOINT.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
No. of Experiment.	Letter of Plate.	Diameter of Rivet Holes.	Gross area of Plate.	Net area of Plate.	Shearing area of Rivets.	Bearing area of Rivets.	Lap of Plates.	Pitch of Rivets.	Breaking weight.	Breaking strain of joint per square inch of gross plate area.	Ditto as a percentage of the tearing strain per square inch of the original solid plate in column 17.	Breaking strain of joint per square inch of net plate area.	Ditto as a percentage of the tearing strain per square inch of the original solid plate in column 17.	Shearing strain per square inch of rivet area.	Strain per square inch of bearing area of rivets.	Tearing strain of the original solid plate per square inch.	Ultimate elongation, or tensile set after fracture of the original plate.	OBSERVATIONS.
		Inch.	Sq. inch.	Sq. inch.	Sq. inch.	Sq. inch.	Inch.	Inch.	Tons.	Tons.	Per cent.	Tons.	Per cent.	Tons.	Tons.	Tons.	Per cent.	
Original Plates.																		
15	B.	..	1·2	..	..	..	..	..	22·25	..	..	..	..	..	..	**18·54**	6	Pretty tough iron, but of inferior tensile strength.
27	C.	..	·6	..	..	..	..	..	13·2	..	..	..	..	..	..	**22·0**	11	Strong and tough iron.
37	E.	..	·525	..	..	..	..	..	11·25	..	..	..	..	..	..	**21·43**	2	Hard iron, judging from the small set after fracture.
48	F.	..	·57	..	..	..	..	..	13·7	..	..	..	..	..	..	**24·0**	6	Strong and pretty tough iron.
Joints with Punched Holes.																		
18	B.	·78	4·2	2·64	2·39	1·56	4·5	2	38·75	9·23	50	**14·67**	79	16·21	24·84	..	..	One plate tore through line of rivet holes.
19	C.	·78	4·92	3·32	2·39	1·6	4·5	2·375	47·3	9.61	44	**14·24**	65	19·79	29·56	..	..	Ditto.
20	,,	·78	5·07	3·55	2·39	1·521	4·5	2·5	42·5	8·38	38	11·97	..	**17·78**	27·94	..	..	Rivets sheared in succession.
21	,,	·81	5·07	3·49	2·576	1·58	4·5	2·5	51·5	10·15	46	14·75	..	**19·90**	32·60	..	..	Rivets sheared simultaneously; both plates on point of rupture.
38	E.	·82	2·88	1·4	2·64	1·476	2·0	1·5	28·75	9·99	47	**20·48**	96	10·89	19·5	..	..	One plate tore through line of rivet holes.
39	,,	·82	2·88	1·4	2 64	1·476	1·5	1·5	27·75	9·64	45	**19·76**	92	10·51	18·8	..	..	Ditto.
41	,,	·82	3·315	2·0	2·64	1·517	1·5	1·75	32	9·10	42	**16·00**	75	12·12	21·1	..	..	Ditto.
47	,,	·82	3·315	2·0	2·64	1·517	2·0	1·75	33·5	9·53	44	**16·77**	78	12·69	22·08	..	..	Ditto.
49	F.	·82	3·12	1·52	2·64	1·6	2·0	1·5	27·7	8·88	37	**18·22**	76	10·49	17·31	..	..	Ditto.
											Average = 43·7.		Average = 80·1.					
Joints with Drilled Holes.																		
22	C.	·81	4·8	3·18	2·576	1·62	4·5	2·375	48·0	10·0	45	15·10	..	**18·63**	29·63	..	..	Rivets sheared.
23	,,	·81	4·47	2·81	2·576	1·66	4·5	2·25	43·75	9·79	44	**15·57**	71	16·97	26·35	..	..	Both plates tore through line of rivet holes.
24	,,	·80	4·095	2·535	2·513	1·56	4·5	2·00	45	10·99	50	17·75	..	**17·90**	28·84	..	..	Rivets sheared.
25	,,	·80	3·8	2·2	2·513	1·6	4·5	1·75	46	12·10	55	**20·90**	95	**18·30**	28·75	..	..	Rivets sheared, and one plate on the point of tearing.
40	E.	·82	3·04	1·48	2·64	1·558	1·5	1·5	32	10·526	49	**21·592**	101	12·12	20·60	..	..	One plate tore through line of rivet holes.
42	,,	·82	3·515	2·0	2·64	1·517	1·5	1·75	32·75	9·32	44	**16·37**	76	12·41	21·59	..	..	Ditto.
43	,,	·82	2·88	1·4	2·64	1·476	2·0	1·5	30·25	10·503	49	**21·54**	100·5	11·46	20·50	..	..	Ditto.
46	,,	·82	3·515	2·0	2·64	1·517	2·0	1·75	37·0	10·53	49	**18·52**	86	14·01	24·39	..	..	Ditto.
50	F.	·80	3·04	1·52	2·513	1·52	2·0	1·5	32·25	10·61	44	**21·22**	99	12·83	21·22	..	..	Ditto.
											Average = 47·7.		Average = 89·8.					






0
2
4
6
12 FEET

XII.—*On the first Comet of* 1845. *By* Dr. W. Doberck, Astronomer at Colonel Cooper's Observatory, Markree, Co. Sligo.

Read, January 11th, 1875.

The first Comet, which appeared in the year 1845, was discovered in Berlin by d'Arrest as a rather bright object. In Mr. Bishop's observations the Comet is described as a faint nebulous object, the light, however, being much brighter towards the centre. On January 18 the diameter of the whole was 185″, and of the more condensed part 77″. On January 27, a star-like nucleus appeared, the Comet being circular without trace of tail. On January 31 the Comet was noticed as a large and bright nebulous object, perhaps 5′–6′ in diameter, nucleus very undefined and some indication of tail. The diameter was 133″ on February 3, and that of the nucleus, which was then nearly if not quite circular and of 10–11 magnitude 26″. The Comet was a "broad parabola" on February 5, and on February 6 a kind of tail appeared opposite to the Sun 6′–8′ long, nucleus being excentric. In Greenwich the Comet was seen as a faint nebulosity on January 27, but brighter in the centre, and the diameter was about 60″. In the early part of February the Comet gradually increased in size, but was very diffuse. On February 6 and 7 a bright point was noticed. Afterwards it decreased, and the last observations, especially in Kremsmünster, are considerably less accurate owing to the faintness of the Comet.

Nicolai remarked, that the brightness of the Comet did not increase to the same extent, as we should have expected from calculations made in conformity with the ordinary theory, in which the intrinsic brightness of the Comet is presumed to be constant. This circumstance seems to prove that the apparent brightness of a Comet is more influenced by the distance of the Comet from the Sun than by its distance from the Earth.

I have undertaken the determination of the definitive orbit of this Comet

at the request of Professor Bruhns of Leipzig, who is an eminent authority in this department of astronomy.

Though the systems of elements which have been hitherto published are based upon a smaller number of observations than are now available, yet as these different systems agree very well together, I have not thought it necessary to determine a new set of provisional elements. The parabola which shall serve as the basis of my calculations has been derived from the comparison of the best systems, and is as follows:—

$$T = 1845 \text{ Jan. } 8{\cdot}196817 \text{ Mean Time Berlin.}$$

$$\left.\begin{array}{ll} \Omega & 336^\circ\ 44'\ 28''{\cdot}0 \\ \iota & 46\ \ 50\ \ 38\ \ {\cdot}0 \\ \pi & 91\ \ 19\ \ 41\ \ {\cdot}0 \end{array}\right\} \text{ M. Eq. } 1845{\cdot}0.$$

$$\log q \quad 9{\cdot}9567490$$

Motion direct.

T is the time of the perihelion passage.

Ω, the longitude of the ascending node on the ecliptic.

ι, the inclination of the plane of the orbit to the ecliptic.

π, the longitude of the perihelion, is the angular distance of the perihelion from a point in the orbit whose angular distance back from the ascending node is equal to the longitude of the node.

q the distance of the perihelion from the Sun expressed with the Earth's mean distance from the Sun as unity.

I calculated by the aid of these provisional elements the perturbations arising from the influence of Jupiter and the Earth, the action of the other planets being insensible.

The perturbations are computed in the form of corrections, which are to be applied to the co-ordinates of the Comet calculated on the hypothesis of undisturbed motion. They are obtained by the mechanical quadrature, according to Encke's method, of the differential equations of the second order, which are obtained by deducting the three equations for undisturbed motion round the Sun from the analogous expressions for the disturbed motion. When, as in this instance, the squares of the disturbing forces are to be neglected, the resulting expressions are:

$$\frac{d^2\xi}{dt^2} = mk^2\left(\frac{x'-x_0}{\rho_0^3} - \frac{x'}{r'^3}\right) + \frac{k^2}{r_0^3}\left(3\,\frac{x_0}{r_0}\,\delta r_0 - \xi\right),$$

$$\frac{d^2\eta}{dt^2} = mk^2\left(\frac{y' - y_0}{\rho_0^{\,3}} - \frac{y'}{r'^3}\right) + \frac{k^2}{r_0^{\,3}}\left(3\,\frac{y_0}{r_0}\,\delta r_0 - \eta\right),$$

$$\frac{d^2\zeta}{dt^2} = mk^2\left(\frac{z' - z_0}{\rho_0^{\,3}} - \frac{z'}{r'^3}\right) + \frac{k^2}{r_0^{\,3}}\left(3\,\frac{z_0}{r_0}\,\delta r_0 - \zeta\right),$$

where ξ, η, ζ, are the perturbations to be applied to the rectangular co-ordinates of the Comet, of which x_0 is directed towards the mean vernal equinox of 1845·0, and the axis of y_0 lies in the ecliptic; r_0 is the distance of the Comet from the Sun; x', y', z', r' are the planet's co-ordinates, and distance from the Sun; ρ_0 is the planet's distance from the Comet; all quantities being expressed with the Earth's mean distance from the Sun taken as unity; and m the mass of the planet is expressed in parts of the Sun's mass. In the following Table are the perturbations expressed as units of the seventh place of decimals:—

TABLE I.

PERTURBATIONS OF THE ECLIPTIC COORDINATES.

Date.	ξ	η	ζ
1844.			
Dec. 28	+ 0	− 0	− 0
1845.			
Jan. 2	0	0	1
,, 7	0	0	2
,, 12	− 0	1	3
,, 17	− 1	2	5
,, 22	2	4	8
,, 27	3	6	12
Feb. 1	5	8	17
,, 6	7	10	24
,, 11	8	13	33
,, 16	9	16	46
,, 21	9	22	63
,, 26	6	28	79
March 3	− 3	38	98
,, 8	+ 2	49	116
,, 13	+ 8	− 62	− 134

For computation of the ephemeris, the position is referred to three rectangular axes x, y, z, passing through the Sun, where x is directed towards the mean vernal equinox for 1845·0, and y lies in the plane of the equator. For the undisturbed orbit we have the following formulæ :—

$$x = [9{\cdot}9811889]\ r \sin (188^\circ\ 12'\ 14''{\cdot}82 + v).$$
$$y = [9{\cdot}6642388]\ r \sin (\ 62^\circ\ 53'\ \ 1''{\cdot}18 + v).$$
$$z = [9{\cdot}9697433]\ r \sin (104^\circ\ 53'\ \ 1''{\cdot}30 + v).$$

in which r is the distance of the Comet from the Sun, v the true anomaly of the Comet, while the numbers in brackets denote logarithms.

By the aid of Hansen-Olufsen's solar tables the geocentric co-ordinates were determined, and thence after applying precession and nutation but not aberration, and after allowing for the perturbations, the following ephemeris (Table II.) was computed.

Owing to the very irregular motion of this Comet it was found necessary to compute the ephemeris for intervals of 12 hours. α is the apparent right ascension, δ the apparent declination, and Δ the distance of the Comet from the Earth.

TABLE II.

Apparent right Ascensions and Declinations of Comet I 1845 for Noon and Midnight, Berlin mean time, from 1st Jan., 1845, to 15th March, 1845, derived from the provisional elements.

Date.	α		δ		Log Δ.
1845.	0h	12h	0h	12h	0h
Jan. 1	293° 14′ 16″·2	293° 6′ 3″·3	+ 37° 40′ 54″·7	+ 37° 52′ 24″·7	9·8833
,, 2	292 57 39 ·4	292 49 4 ·1	38 3 59 ·7	38 15 40 ·0	9·8772
,, 3	292 40 17 ·8	292 31 20 ·2	38 27 25 ·1	38 39 15 ·3	9·8709
,, 4	292 22 10 ·5	292 12 48 ·3	38 51 10 ·6	39 3 10 ·8	9·8644
,, 5	292 3 13 ·3	291 53 25 ·1	39 15 16 ·2	39 27 27 ·1	9·8576
,, 6	291 43 23 ·5	291 33 7 ·8	39 39 43 ·7	39 52 6 ·1	9·8506
,, 7	291 22 37 ·6	291 11 52 ·4	40 4 34 ·6	40 17 9 ·8	9·8434
,, 8	291 0 51 ·2	290 49 34 ·1	40 29 51 ·7	40 42 38 ·9	9·8359
,, 9	290 38 0 ·3	290 26 9 ·2	40 55 33 ·2	41 8 35 ·1	9·8282
,, 10	290 14 0 ·3	290 1 32 ·8	41 21 44 ·1	41 35 1 ·0	9·8203
,, 11	289 48 46 ·1	289 35 39 ·3	41 48 25 ·7	42 1 58 ·8	9·8120
,, 12	289 22 11 ·8	289 8 22 ·7	42 15 40 ·4	42 29 29 ·9	9·8036
,, 13	288 54 10 ·9	288 39 35 ·1	42 43 28 ·4	42 57 36 ·7	9·7948
,, 14	288 24 35 ·2	288 9 9 ·9	43 11 54 ·9	43 26 24 ·4	9·7857
,, 15	287 53 17 ·9	287 36 57 ·9	43 41 4 ·4	43 55 55 ·0	9·7764
,, 16	287 20 8 ·7	287 2 48 ·7	44 10 57 ·4	44 26 11 ·9	9·7667
,, 17	286 44 57 ·1	286 26 32 ·0	44 41 39 ·3	44 57 19 ·7	9·7568
,, 18	286 7 32 ·2	285 47 55 ·5	45 13 13 ·8	45 29 21 ·6	9·7465
,, 19	285 27 39 ·7	285 6 42 ·6	45 45 44 ·7	46 2 23 ·9	9·7359
,, 20	284 45 2 ·7	284 22 37 ·3	46 19 19 ·5	46 36 32 ·6	9·7248
,, 21	283 59 24 ·4	283 35 20 ·6	46 54 3 ·1	47 11 52 ·0	9·7137
,, 22	283 10 23 ·7	282 44 29 ·8	47 29 59 ·8	47 48 27 ·1	9·7020
,, 23	282 17 35 ·4	281 49 35 ·7	48 7 15 ·2	48 26 25 ·5	9·6900
,, 24	281 20 29 ·6	280 50 11 ·0	48 45 58 ·3	49 5 54 ·9	9·6776
,, 25	280 18 35 ·5	279 45 35 ·3	49 26 15 ·2	49 46 59 ·9	9·6648
,, 26	279 11 8 ·0	278 35 5 ·9	50 8 10 ·2	50 29 47 ·3	6·6516
,, 27	277 57 23 ·6	277 17 51 ·9	50 51 51 ·6	51 14 24 ·4	9·6379
,, 28	276 36 23 ·8	275 52 48 ·7	51 37 25 ·7	52 0 56 ·7	9·6239
,, 29	275 6 58 ·4	274 18 40 ·5	52 24 57 ·8	52 49 27 ·9	9·6095
,, 30	273 27 47 ·4	272 34 3 ·4	53 14 29 ·0	53 39 59 ·9	9·5946
,, 31	271 37 14 ·6	270 37 5 ·2	54 5 59 ·7	54 32 29 ·1	9·5793
Feb. 1	269 33 18 ·7	268 25 38 ·1	54 59 24 ·9	55 26 46 ·4	9·5636
,, 2	267 13 41 ·3	265 57 6 ·2	55 54 32 ·4	56 22 38 ·4	9·5475
,, 3	264 35 29 ·4	263 8 25 ·7	56 51 0 ·8	57 19 35 ·1	9·5310
,, 4	261 35 25 ·0	259 55 58 ·5	57 48 14 ·4	58 16 51 ·3	9·5142
,, 5	258 9 34 ·6	256 15 39 ·1	58 45 15 ·9	59 13 17 ·4	9·4971

TABLE II.—(*continued*).

Date.	α		δ		Log. Δ.
1845.	0h	12h	0h	12h	0h
Feb. 6	254° 13′ 39″·3	252° 2′ 59″·5	+ 59° 40′ 42″·3	+ 60° 7′ 14″·7	9·4799
,, 7	249 43 5 ·8	247 13 26 ·9	60 32 35 ·0	60 56 21 ·2	9·4625
,, 8	244 33 36 ·6	241 43 15 ·5	61 18 7 ·8	61 37 27 ·0	9·4452
,, 9	238 42 8 ·9	235 30 25 ·0	61 53 44 ·6	62 6 28 ·4	9·4280
,, 10	232 8 21 ·7	228 36 36 ·4	62 14 58 ·3	62 18 34 ·8	9·4113
,, 11	224 56 9 ·1	221 8 21 ·1	62 16 37 ·4	62 8 26 ·0	9·3953
,, 12	217 14 57 ·7	213 17 57 ·9	61 53 25 ·0	61 31 3 ·4	9·3802
,, 13	209 19 33 ·2	205 22 1 ·6	61 0 53 ·0	60 22 33 ·6	9·3665
,, 14	201 27 40 ·6	197 38 35 ·3	59 35 59 ·3	58 41 13 ·2	9·3546
,, 15	193 56 31 ·0	190 22 54 ·6	57 38 22 ·2	56 27 43 ·1	9·3447
,, 16	186 58 54 ·6	183 45 17 ·7	55 9 46 ·7	53 45 5 ·6	9·3374
,, 17	180 42 26 ·6	177 50 26 ·5	52 14 20 ·2	50 38 19 ·7	9·3336
,, 18	175 9 12 ·6	172 38 26 ·7	48 57 45 ·8	47 13 27 ·9	9·3313
,, 19	170 17 41 ·6	168 6 26 ·1	45 26 15 ·6	43 36 53 ·2	9·3330
,, 20	166 4 9 ·7	164 10 18 ·2	41 46 10 ·0	39 54 48 ·4	9·3378
,, 21	162 24 16 ·4	160 45 29 ·9	38 3 30 ·4	36 12 50 ·1	9·3456
,, 22	159 13 27 ·3	157 47 36 ·7	34 23 21 ·4	32 35 31 ·7	9·3561
,, 23	156 27 31 ·2	155 12 43 ·4	30 49 46 ·7	29 6 25 ·0	9·3690
,, 24	154 2 47 ·7	152 57 22 ·3	27 25 46 ·7	25 47 59 ·4	9·3838
,, 25	151 56 5 ·9	150 58 40 ·8	24 13 16 ·5	22 41 39 ·2	9·4002
,, 26	150 4 47 ·6	149 14 11 ·2	21 13 17 ·3	19 48 9 ·0	9·4178
,, 27	148 26 37 ·6	147 41 54 ·6	18 26 16 ·1	17 7 34 ·5	9·4363
,, 28	146 59 47 ·1	146 20 5 ·4	15 52 4 ·2	14 39 39 ·5	9.4554
Mar. 1	145 42 39 ·3	145 7 20 ·6	13 30 15 ·6	12 23 46 ·4	9·4748
,, 2	144 33 59 ·7	144 2 29 ·4	11 20 7 ·2	10 19 11 ·4	9·4944
,, 3	143 32 42 ·2	143 4 31 ·4	9 20 53 ·1	8 25 6 ·0	9·5140
,, 4	142 37 52 ·6	142 12 40 ·0	7 31 42 ·4	6 40 36 ·3	9·5334
,, 5	141 48 46 ·9	141 26 8 ·4	5 51 42 ·2	5 4 52 ·4	9·5527
,, 6	141 4 42 ·0	140 44 23 ·3	4 20 5 ·9	3 37 11 ·4	9·5716
,, 7	140 25 8 ·1	140 6 52 ·9	2 56 5 ·3	2 16 42 ·9	9·5902
,, 8	139 49 34 ·5	139 33 10 ·0	1 38 58 ·3	+ 1 2 46 ·9	9·6085
,, 9	139 17 37 ·0	139 2 53 ·4	+ 0 28 4 ·5	− 0 5 12 ·0	9·6264
,, 10	138 48 55 ·3	138 35 40 ·9	− 0 37 10 ·4	1 7 51 ·3	9·6438
,, 11	138 23 8 ·8	138 11 17 ·1	1 37 19 ·2	2 5 37 ·2	9·6609
,, 12	138 0 3 ·5	137 49 26 ·2	2 32 49 ·8	2 59 0 ·0	9·6776
,, 13	137 39 23 ·7	137 29 54 ·4	3 24 40 ·6	3 48 23 ·7	9·6938
,, 14	137 20 57 ·3	137 12 31 ·0	4 11 43 ·3	4 34 11 ·4	9·7097
,, 15	137 4 33 ·9	136 57 4 ·9	− 4 55 50 ·2	− 5 16 41 ·3	9·7252

I compared the ephemeris with all published observations using W. Struve's constant of aberration $497^{s}\cdot78$. To the computed right ascensions and declinations the parallax has been applied assuming the solar parallax equal to $8''\cdot9$*, and I introduced new comparison-stars, whenever it appeared necessary. The stars, which were used for comparison in Koenigsberg, were afterwards observed by Wichmann himself. In the Greenwich observations the comparison-stars were taken out of the catalogues of that observatory, or from the British Association Catalogue. Hind principally used Argelander's Zone-observations. Valz made use of Lalande's stars, which certainly are sufficiently accurate for his observations. Rümker, Petersen and Bruderer determined their own comparison-stars.

In making the interpolations for the purpose of comparing the ephemeris with the observations, the differential coefficients derived by the aid of the numerical differences up to and including the fourth order were taken into account. During a part of February when the motion was extremely rapid differential coefficients of the fifth order were also included. The interpolated co-ordinates are then determined by the formula:

$$\alpha = \alpha_0 + \Delta\alpha = \alpha_0 + t\left(\frac{d\alpha}{dt}\right)_0 + \frac{t^2}{1.2}\left(\frac{d\,\alpha}{dt^2}\right)_0 + \frac{t^3}{1.2.3}\left(\frac{d^3\alpha}{dt^3}\right)_0 + \frac{t^4}{1.2.3.4}\left(\frac{d^4\alpha}{dt^4}\right)_0$$

where t is the time between the epoch of the ephemeris and the epoch of the observation expressed in parts of 12^{h}.

* Except for the Greenwich and Genf observations, which were already reduced for parallax.

TABLE III.

COMPARISON OF THE OBSERVATIONS OF COMET I 1845 WITH THE EPHEMERIS DERIVED FROM THE PROVISIONAL ELEMENTS.

Mean Time. Berlin.	Observed Values.		Calculated Values.		Observed Value minus Calculated Value.	
	α	δ	α	δ	$\Delta\alpha$	$\Delta\delta$
1845.						
KOENIGSBERG:						
Jan.						
13·36695	288°43′ 3″·4	+42°53′32″·9	288°43′24″·0	+42°53″37′·0	−20″·6	−4″·1
13·73159	288 32 17 ·2	43 4 16 ·5	288 32 52 ·7	43 4 5 ·0	−35 ·5	+11 ·5
14·30200	288 14 35 ·5	43 20 25 ·4	288 15 9 ·4	43 20 27 ·1	−33 ·9	−1 ·7
14·73770	288 1 20 ·6	43 33 8 ·5	288 1 52 ·2	43 33 13 ·9	−31 ·6	−5 ·4
15·28972	287 44 11 ·5	43 53 26 ·5	287 43 43 ·0	43 49 27 ·6	[+28 ·5]	[+238 ·9]
16·34555	287 7 11 ·2	44 21 5 ·4	287 8 5 ·4	44 21 14 ·5	−54 ·2	−9 ·1
Feb.						
8·33008	242 41 30 ·1	61 30 3 ·0	242 42 35 ·9	61 30 42 ·6	−65 ·8	−39 ·6
8·37410	242 26 25 ·6	61 31 38 ·6	242 27 33 ·8	61 32 25 ·0	−68 ·2	−46 ·4
11·30610	222 36 58 ·2	62 10 58 ·2	222 37 53 ·6	62 11 54 ·8	−55 ·4	−56 ·6
12·32051	214 43 12 ·2	61 38 22 ·2	214 43 54 ·5	61 39 31 ·2	−42 ·3	−69 ·0
14·37331	198 36 40 ·4	58 54 17 ·0	198 36 45 ·7	58 55 35 ·0	−5 ·3	−78 ·0
25·31646	151 20 16 ·5	23 13 19 ·1	151 19 38 ·0	23 14 33 ·9	+38 ·5	−74 ·8
Mar.						
5·36069	141 33 4 ·1	5 16 31 ·5	141 32 25 ·0	5 17 24 ·5	+39 ·1	−53 ·0
7·53871	140 5 46 ·7	2 12 44 ·8	140 5 21 ·1	2 13 26 ·4	+25 ·6	−41 ·6
9·37091	139 7 4 ·3	0 2 29 ·5	139 6 38 ·7	0 2 57 ·7	+25 ·6	−28 ·2
GENF:						
Feb.						
7·49837	247 12 33 ·9	60 55 26 ·6	247 13 52 ·5	60 56 16 ·3	−78 ·6	−49 ·7
27·47508	147 44 36 ·8	17 10 22 ·6	147 44 4 ·4	17 11 26 ·5	+32 ·4	−63 ·9
Mar.						
9·39554	139 6 5 ·1	+0 0 52 ·8	139 5 53 ·6	+0 1 37 ·5	+11 ·5	−44 ·7
12·45546	137 50 34 ·2	−2 57 18 ·7	137 50 21 ·5	−2 56 42 ·6	+12 ·7	−36 ·1

TABLE III.—(*continued*).

Mean Time Berlin.	Observed Values.		Calculated Values.		Observed Value minus Calculated Value.	
	α	δ	α	δ	$\Delta\alpha$	$\Delta\delta$
1845.						
BERLIN:						
Jan.						
10·26932	290° 6′ 55″·3	+41°28′ 47″·1	290° 7′ 9″·0	+41°28′ 43″·7	− 13″·7	+ 3″·4
11·26804	289 41 25 ·3	41 55 24 ·0	289 41 35 ·7	41 55 31 ·6	− 10 ·4	− 7 ·6
11·29296	289 40 29 ·5	41 56 7 ·5	289 40 56 ·8	41 56 11 ·3	− 27 ·3	− 3 ·8
13·26410	288 45 47 ·7	42 50 38 ·2	288 46 19 ·5	42 50 46 ·0	− 31 ·8	− 7 ·8
14·32373	288 13 49 ·5	43 20 56 ·4	288 14 28 ·7	43 21 4 ·8	− 39 ·2	− 8 ·4
27·35619	277 28 24 ·5	51 7 15 ·6	277 29 18 ·0	51 7 33 ·0	− 53 ·5	− 17 ·4
28·44052	275 57 15 ·9	51 57 33 ·4	275 57 59 ·0	51 57 46 ·9	− 43 ·1	− 13 ·5
30·34342	272 50 18 ·2	53 31 9 ·3	272 51 4 ·9	53 31 35 ·8	− 46 ·7	− 26 ·5
Feb.						
4·36133	260 23 14 ·6	58 7 53 ·2	260 24 16 ·0	58 8 29 ·5	− 61 ·4	− 36 ·3
25·51487	150 57 42 ·6	22 37 28 ·4	150 56 57 ·5	22 38 41 ·4	+ 45 ·1	− 73 ·0
Mar.						
3·46038	143 7 12 ·6	8 28 17 ·6	143 6 40 ·5	8 29 7 ·2	+ 32 ·1	− 49 ·6
6·48243	140 45 29 ·4	3 37 27 ·3	140 45 0 ·6	3 38 22 ·5	+ 28 ·8	− 55 ·2
8·38236	139 37 21 ·9	+ 1 10 10 ·1	139 37 0 ·2	+ 1 10 53 ·2	+ 21 ·7	− 43 ·1
12·45039	137 50 46 ·0	− 2 57 27 ·3	137 50 24 ·9	− 2 56 42 ·0	+ 21 ·1	− 45 ·3
GOETTINGEN:						
Jan.						
11·23873	..	+41 54 36 ·7	..	+41 54 45 ·4	..	− 8 ·7
11·24163	289 41 52 ·7	..	289 42 16 ·8	..	− 24 ·1	..
12·24982	289 14 45 ·5	..	289 15 8 ·4	..	− 22 ·9	...
12·25246	..	42 22 15 ·4	..	42 22 30 ·2	..	− 14 ·8
KREMSMÜNSTER:						
Jan.						
28·32823	276 6 27 ·0	+51 52 6 ·6	276 7 50 ·7	+51 52 24 ·1	− 1′ 23″·7	− 17 ·5
30·33736	272 50 3 ·0	53 31 0 ·9	272 51 43 ·0	53 31 17 ·0	− 1 40 ·0	− 16 ·1
Feb.						
2·32776	266 22 9 ·0	56 11 44 ·1	266 23 54 ·6	56 12 31 ·1	− 1 45 ·6	− 47 ·0
2·61064	265 37 43 ·4	56 27 47 ·0	265 40 0 ·6	56 28 42 ·8	− 2 17 ·2	− 55 ·8
7·43062	247 30 58 ·5	60 51 57 ·9	247 35 16 ·3	60 52 45 ·8	− 4 17 ·8	− 47 ·9
8·31610	242 44 31 ·5	61 29 39 ·8	242 47 17 ·8	61 30 9 ·3	− 2 46 ·3	− 29 ·5
8·38987	242 17 45 ·0	61 32 26 ·4	242 22 9 ·1	61 32 55 ·6	− 4 24 ·1	− 29 ·2
9·39905	236 8 22 ·5	62 3 2 ·9	236 10 34 ·2	62 3 48 ·1	− 2 11 ·7	− 45 ·2
10·52448	228 25 6 ·0	62 17 20 ·2	228 26 49 ·6	62 18 29 ·3	− 1 43 ·6	− 1 9 ·1
11·73916	219 13 16 ·5	61 58 53 ·7	219 17 9 ·2	62 2 16 ·5	− 3 52 ·7	− 3 22 ·8
12·72928	211 27 46 ·5	61 16 32 ·3	211 28 27 ·6	61 18 20 ·9	− 0′ 41″·1	− 1′ 48″·6

TABLE III.—(*continued*).

Mean Time Berlin.	Observed Values.		Calculated Values.		Observed Value minus Calculated Value.	
	α	δ	α	δ	$\Delta\alpha$	$\Delta\delta$
Feb.						
13·71797	203°37′ 36″·0	+60° 0′ 29″·4	203°39′ 3″·6	+60° 3′ 23″·2	− 1′ 27″·6	− 2′ 53″·8
28·35161	146 32 16 ·5	14 59 29 ·1	146 31 51 ·1	15 0 32 ·2	+ 0 25 ·4	− 1 3 ·1
Mar.						
1·29683	145 21 28 ·5	12 48 29 ·8	145 21 42 ·9	12 50 7 ·8	− 0 14 ·4	− 1 38 ·0
6·38990	140 47 30 ·0	3 45 4 ·8	140 48 49 ·7	3 46 12 ·5	− 1 19 ·7	− 1 7 ·7
7·36862	140 12 25 ·5	2 25 52 ·2	140 11 40 ·2	2 26 37 ·3	+ 0 45 ·3	− 0 45 ·1
8·35791	139 39 6 ·0	1 12 37 ·9	139 37 50 ·1	1 12 39 ·3	+ 1 15 ·9	− 0 1 ·4
9·35843	139 7 34 ·5	0 1 2 ·5	139 7 3 ·5	0 3 48 ·9	+ 0 31 ·0	− 2 46 ·4
PADUA:						
Jan.						
29·33597	274 34 33 ·5	52 40 10	274 40 17 ·5	52 41 2 ·6	..	− 52 ·6
30·29133	272 40 36 ·0	53 28 50	272 56 33 ·9	53 28 57 ·7	− 0 57 ·9*	− 7 ·7
Feb.						
2·47924	265 59 25 ·5	56 20 7	266 0 42 ·9	56 21 6 ·3	− 1 17 ·4	− 59 ·3
3·31039	263 39 42 ·5	57 7 57	263 41 55 ·6	57 8 18 ·4	− 2 13 ·1	− 21 ·4
4·55108	259 44 42 ·0	58 14 36	259 46 1 ·2	58 19 30 ·5	− 1 19 ·2	..
5·29936	257 0 12 ·1	59 1 28	257 2 8 ·9	59 1 37 ·9	− 1 56 ·8	− 9 ·9
7·34484	247 58 48 ·1	60 48 42	248 1 5 ·1	60 48 40 ·6	− 2 17 ·0	+ 1 ·4
11·48226	221 15 52 ·0	62 6 55	221 17 25 ·6	62 8 39 ·5	− 1 33 ·6	− 1 44 ·5
12·51220	213 28 11 ·7	61 29 15	213 13 0 ·3	61 30 20 ·9	..	− 1 5 ·9
13·48101	205 31 24 ·5	60 22 49	205 31 51 ·7	60 24 4 ·4	− 27 ·2	− 1 15 ·4
25·33538	151 17 54 ·7	23 9 45	151 17 32 ·6	23 11 11 ·5	+ 22 ·1	− 1 26 ·5
27·33601	147 57 8 ·9	17 31 19	147 56 34 ·9	17 32 45 ·1	+ 34 ·0	− 1 26 ·1
BONN:						
Jan.						
10·31921	290 5 21 ·2	41 30 8 ·4	290 5 54 ·5	41 30 2 ·1	− 33 ·3	+ 6 ·3
15·30274	287 42 24 ·4	43 49 40 ·7	287 43 15 ·7	43 49 51 ·5	− 51 ·3	− 10 ·8
15·32068	287 42 7 ·8	43 50 15 ·3	287 42 40 ·5	43 50 23 ·5	− 32 ·7	− 8 ·2
18·29376	285 55 13 ·0	45 22 17 ·8	285 55 52 ·2	45 22 29 ·0	− 39 ·2	− 11 ·2
18·31489	285 54 23 ·5	45 22 48 ·2	285 55 2 ·3	45 23 10 ·0	− 38 ·8	− 21 ·8
Feb.						
8·38794	242 20 47 ·2	61 31 45 ·9	242 22 53 ·3	61 32 57 ·0	− 2′ 6″·1	− 1′ 11″·1
8·47596	241 50 52 ·8	61 35 17 ·8	241 52 12 ·9	61 36 10 ·3	− 1 20 ·1	− 52 ·5
9·39909	236 9 7 ·2	62 2 57 ·3	236 10 27 ·3	62 3 46 ·6	− 1 20 ·1	− 49 ·3

* The minute of time corrected.

Table III.—(*continued*).

Mean Time Berlin.	Observed Values.		Calculated Values.		Observed Value minus Calculated Value.	
	α	δ	α	δ	$\Delta\alpha$	$\Delta\delta$
Altona :						
Jan.						
3·37865	292°33′ 6″·2	..	292°33′24″·1	..	− 17″·9	..
10·32461	..	+41°30′14″·6	..	+41°30′10″·3	..	+ 4″·3
10·32528	..	41 30 10 ·4	..	41 30 11 ·3	..	− 0 ·9
10·32724	290 5 19 ·0	..	290 5 43 ·4	..	− 24 ·4	..
10·33143	290 5 14 ·1	..	290 5 37 ·1	..	− 23 ·0	..
11·39207	289 37 40 ·6	41 58 38 ·2	289 38 23 ·7	41 58 50 ·2	− 43 ·1	− 12 ·0
Hamburg :						
Jan.						
3·31787	292·34 5 ·8	..	292 34 28 ·5	..	− 22 ·7	..
3·32310	..	38 35 4 ·7	..	38 34 54 ·8	..	+ 9 ·9
10·34087	290 4 58 ·4	..	290 5 27 ·5	..	− 29 ·1	..
10·34429	..	41 30 38 ·4	..	41 30 41 ·1	..	− 2 ·7
11·35029	289 38 38 ·3	41 57 51 ·1	289 39 27 ·8	41 57 43 ·2	− 49 ·5	+ 7 ·9
11·68038	289 30 35 ·3	42 6 47 ·5	289 31 0 ·4	42 6 43 ·6	− 25 ·1	+ 3 ·9
12·31989	289 12 55 ·8	42 24 17 ·2	289 13 13 ·5	42 24 19 ·9	− 17 ·7	− 2 ·7
13·28682	288 45 24 ·8	42 51 19 ·3	288 45 40 ·3	42 51 23 ·8	− 15 ·5	− 4 ·5
30·50762	272 32 52 ·5	53 39 43 ·8	272 33 26 ·9	53 40 3 ·9	− 34 ·4	− 20 ·1
Feb.						
3·36627	263 31 5 ·8	57 11 5· 7	263 32 16 ·0	57 11 30 ·3	− 1 10 ·2	− 24 ·6
3·67124	262 36 38 ·1	57 28 39 ·3	262 37 43 ·7	57 29 16 ·4	− 1 5 ·6	− 37 ·1
5·45027	256 25 51 ·0	59 9 31 ·7	256 27 40 ·9	59 10 6 ·4	− 1 49 ·9	− 34 ·7
6·41466	252 24 11 ·8	60 1 44 ·2	252 26 14 ·4	60 2 22 ·1	− 2 2 ·6	− 37 ·9
7·37644	247 50 19 ·6	60 49 42 ·1	247 51 33 ·6	60 50 11 ·6	− 1 14 ·0	− 29 ·5
8·38441	242 22 25 ·6	61 31 44 ·4	242 23 56 ·5	61 32 46 ·7	− 1 30 ·9	− 62 ·3
9·58748	234 54 57 ·6	62 7 37 ·6	234 56 26 ·9	62 8 11 ·4	− 1 29 ·3	− 33 ·8
10·44195	230 43 51 ·0	62 18 44 ·6	229 2 20 ·7	62 18 5 ·1	..	..
18·73655	171 30 21 ·9	46 21 8 ·8	171 30 7 ·7	46 22 49 ·8	+ 14 ·2	− 1 41 ·0
19·73188	167 8 56 ·6	42 43 59 ·7	167 8 9 ·3	42 45 23 ·1	+ 47 ·3	− 1 23 ·4
24·34103	153 18 52 ·7	26 17 12 ·9	153 18 2 ·9	26 18 25 ·1	+ 49 ·8	− 1 12 ·2
26·48064	149 16 53 ·6	19 49 53 ·6	149 16 5 ·6	19 51 5 ·6	+ 48 ·0	− 1 12 ·0
27·42185	147 49 30 ·4	17 18 16 ·4	147 48 50 ·2	17 19 21 ·6	+ 40 ·2	− 1 5 ·2
Mar.						
1·42977	145 13 0 ·3	12 31 37 ·2	145 12 15 ·1	12 32 37 ·2	+ 45 ·2	− 1 0 ·0
5·48264	141 27 27 ·9	5 5 33 ·2	141 26 51 ·4	5 6 9 ·8	+ 36 ·5	− 36 ·6
5·47570	141 27 44 ·2	5 6 18 ·8	141 27 10 ·8	5 6 48 ·3	+ 33 ·4	− 29 ·5
8·38336	139 37 3 ·0	1 9 13 ·2	139 36 58 ·9	1 10 48 ·1	+ 4 ·1	− 1 34 ·9

TABLE III.—(*continued*).

Mean Time Berlin.	Observed Values.		Calculated Values.		Observed Value minus Calculated Value.	
	α	δ	α	δ	$\Delta\alpha$	$\Delta\delta$
MANNHEIM:						
Jan.						
9·27674	290°31′ 3″	+41° 2′ 53″	+290°31′17″·5	+41° 2′37″ ·0	− 14″·5	+ 16″·0
Feb.						
4·33356	260 28 44 ·2	58 6 11 ·6	260 29 43 ·5	58 6 48 ·9	− 59 ·3	− 37 ·3
8·33157	242 40 36 ·5	61 29 37 ·6	242 41 58 ·6	61 30 44 ·8	− 1 22 ·1	− 1 7 ·2
10·32399	229 50 37 ·0	62 16 22 ·3	229 52 30 ·9	62 17 23 ·7	− 1 53 ·9	− 1 1 ·4
25·35642	151 15 17 ·9	23 5 37 ·3	151 15 6 ·9	23 7 20 ·6	+ 11 ·0	− 1 43 ·3
28·34758	146 32 40 ·8	15 0 1 ·0	146 32 11 ·6	15 1 6 ·0	+ 29 ·2	− 1 5 ·0
Mar.						
6·33575	140 51 6 ·0	3 50 15 ·5	140 51 6 ·0	3 50 48 ·1	+ 0 ·0	− 32 ·6
7·33665	140 13 11 ·8	2 28 29 ·1	140 12 53 ·2	2 29 6 ·8	+ 18 ·6	− 37 ·7
PARIS:						
Jan.						
10·30905	290 5 31	41 29 37	290 6 8 ·7	41 29 47 ·1	− 37 ·7	− 10 ·1
11·33616	289 39 20	41 57 9	289 39 48 ·0	41 57 21 ·4	− 28 ·0	− 12 ·4
MARSEILLE:						
Jan.						
17·76559	286 16 44 286 16 41	45 5 10 45 5 18	286 16 46 ·9	45 5 38 ·4	− 4 ·4	− 24 ·4
18·75871	285 37 39 285 37 50	45 37 43 45 37 49	285 37 48 ·0	45 37 42 ·0	− 3 ·5	+ 4 ·0
20·75193	284 11 8 284 11 9	46 45 3 46 44 51	284 11 19 ·0	46 45 13 ·3	− 10 ·5	− 16 ·3
25·30160	279 57 57 279 57 58	49 37 52 49 37 35	279 58 34 ·1	49 38 28 ·6	− 36 ·6	− 45 ·1
Feb.						
3·31203	263 40 27 263 40 51	57 8 35 57 8 7	263 41 34 ·9	57 8 24 ·9	− 55 ·9	− 3 ·9
4·34543	260 26 7 260 26 7	58 7 11 58 7 19	260 27 20 ·1	58 7 33 ·6	− 1 13 ·1	− 18 ·6
5·32121	256 55 42	59 1 52	256 57 8 ·0	59 2 50 ·8	− 1 26 ·0	− 58 ·8
6·69630	251 8 31	60 16 59	251 9 46 ·3	60 17 23 ·7	− 1 15 ·3	− 24 ·7
7·34356	247 59 40 248 0 37	60 48 21 60 49 8	248 1 23 ·2	60 48 36 ·7	− 1 14 ·5	+ 7 ·8
10·44514	229 4 41 229 0 56 228 56 41 228 51 56 229 4 41	62 15 47 .. .. .. 62 15 49	229 1 6 ·2	62 18 5 ·5	− 1 19 ·2	− 2 17 ·5

Table III.—(*continued*).

Mean Time Berlin.	Observed Values.		Calculated Values.		Observed Value minus Calculated Value.	
	α	δ	α	δ	Δα	Δδ
Feb.						
11·42784	221°47′42″	+62° 8″36	221°42′26″·0	+62° 9′39″·2	− 53″·0	− 38″·2
	221 41 12					
	221 34 27	..				
	221 48 9	62 9 26				
	221 41 54	..				
	221 34 54	..				
13·44343	205 45 57	60 26 2	205 49 43 ·4	60 27 5 ·3	− 41 ·8	− 53 ·3
	205 38 27	60 24 34				
	206 2 44	60 28 0				
14·52371	197 30 23	58 37 38	197 28 36 ·7	58 38 26 ·4	− 2 51 ·2	− 1 50 ·7
	197 22 53	58 36 4				
	197 19 23	58 35 35				
	197 30 23	58 37 6				
15·42590	190 56 13	56 38 23	190 54 52 ·7	56 38 29 ·1	− 45 ·2	− 55 ·4
	190 53 58	56 37 45				
	190 51 58	56 36 47				
	190 56 22	56 38 14				
	190 54 7	56 37 32				
	190 52 7	56 36 35				
17·75438	176 27 0	49 46 10	176 26 26 ·5	49 47 35 ·9	+ 45 ·5	− 1 13 ·9
	176 27 24	49 46 34				
18·75126	171 29 0	46 20 43	171 25 52 ·2	46 18 55 ·7	− 25 ·2	− 47 ·7
	171 22 46	46 15 21				
	171 28 9	46 20 55				
	171 21 53	46 15 33				
19·74905	167 7 8	42 43 51	167 3 50 ·0	42 41 39 ·6	+ 38 ·0	− 36 ·3
	167 4 23	42 40 56				
	167 1 53	42 38 23				
24·36933	153 15 28	26 14 4	153 14 27 ·0	26 13 0 ·9	− 16 ·6	− 1 27 ·7
	153 14 43	26 12 43				
	153 13 58	26 11 22				
	153 13 13	26 9 48				
	153 13 30	26 9 49				
25·36041	151 15 9	23 6 41	151 14 42 ·9	23 6 39 ·5	+ 3 ·6	− 40 ·0
	151 14 24	23 5 18				
27·35496	147 56 12	17 30 29	147 54 55 ·1	17 29 48 ·8	+ 11 ·9	− 1 15 ·1
	147 55 27	17 29 17				
	147 54 12	17 26 25				
	147 55 52	17 30 3				
	147 55 7	17 28 53				
	147 53 52	17 26 15				
28·36092	146 31 5	14 58 42	146 31 10 ·6	14 59 13 ·6	− 17 ·1	− 32 ·1
	146 30 42	14 58 41				

TABLE III.—(*continued*).

Mean Time Berlin.	Observed Values.		Calculated Values.		Observed Value minus Calculated Value.	
	α	δ	α	δ	Δα	Δδ
Mar.						
1·36778	145° 16′ 40″	+ 12° 40′ 20″	145° 16′ 42″·9	+ 12° 40′ 48″·5	− 9″·9	− 29″·5
	145 16 26	12 40 18				
3·34190	143 13 24	8 40 58	143 13 30 ·6	8 42 11 ·3	− 6 ·1	− 57 ·3
	143 13 25	8 41 30				
4·36752	142 18 47	6 38 49	142 19 23 ·3	6 53 39 ·5	− 22 ·8	..
	142 19 14	6 38 18				
5·36882	141 31 48	5 18 45	141 32 7 ·7	5 16 42 ·6	− 13 ·7	+ 56 ·9
	141 32 0	5 16 34				
7·41932	140 9 45	2 21 39	140 9 48 ·9	+ 2 22 42 ·5	− 0 ·9	− 56 ·5
	140 9 51	+ 2 21 53				
11·43627	138 13 7	− 2 2 14	138 12 45 ·4	− 2 2 17 ·0	+ 25 ·1	− 4 ·0
	138 13 14	− 2 2 28				
12·37091	137 51 38	− 3 7 58	137 52 12 ·2	− 2 52 32 ·3	− 26 ·2	..
	137 51 54	− 3 7 14				
MR. BISHOP'S OBSERVATORY:						
Jan.						
13·31877	288 43 57 ·8	42 52 16 ·3	288 44 43 ·6	42 52 18 ·2	− 45 ·8	− 1 ·9
18·66656	285 40 39 ·4	45 34 28 ·1	285 41 27 ·2	45 34 35 ·1	− 47 ·8	− 7 ·0
18·69983	285 39 49 ·5	45 35 26 ·6	285 40 7 ·9	45 35 41 ·7	− 18 ·4	− 15 ·1
20·28538	284 31 38 ·7	46 28 48 ·9	284 32 5 ·6	46 28 55 ·9	− 26 ·9	− 7 ·0
27·30573	277 32 14 ·9	51 4 59 ·7	277 33 10 ·1	51 5 19 ·4	− 55 ·2	− 19 ·7
31·56549	270 28 46 ·7	54 35 10 ·2	270 29 16 ·7	54 35 40 ·8	− 30 ·0	− 30 ·6
31·60920	270 23 6 ·5	54 37 48 ·2	270 23 52 ·3	54 38 4 ·1	− 45 ·8	− 15 ·9
Feb.						
2·29762	266 27 12 ·6	56 10 23 ·0	266 28 23 ·8	56 10 52 ·0	− 1′ 11″·2	− 29 ·0
3·57881	262 53 35 ·0	57 23 10 ·6	262 54 39 ·4	57 23 49 ·6	− 1 4 ·4	− 39 ·0
3·59371	262 50 52 ·9	57 24 4 ·7	262 51 56 ·7	57 24 40 ·8	− 1 3 ·8	− 36 ·1
5·62063	255 57 10 ·8	59 16 54 ·4	255 47 33 ·9	59 19 46 ·2	..	..
6·60924	251 32 26 ·0	60 11 41 ·2	251 33 51 ·7	60 12 42 ·7	− 1 25 ·7	− 1′ 1″·5
7·60297	246 40 30 ·0	60 59 58 ·0	246 42 2 ·7	61 0 50 ·6	− 1 32 ·7	− 52 ·6
11·41230	221 47 43 ·7	62 8 45 ·7	221 49 24 ·8	62 9 56 ·4	− 1 41 ·1	− 1 10 ·7
11·59955	220 21 44 ·9	62 4 58 ·0	220 22 56 ·2	62 5 58 ·3	− 1 11 ·3	− 1 0 ·3
GREENWICH:						
Jan.						
27·32465	277 30 44 ·0	51 6 5 ·5	277 31 56 ·1	..	− 1 37 ·2	..
27·32412	277 29 53 ·8	51 6 30 ·9	..	51 6 25 ·4	..	− 7 ·1

Table III.—(*continued*).

Mean Time Berlin.	Observed Values.		Calculated Values.		Observed Value minus Calculated Value.	
	α	δ	α	δ	Δα	Δδ
Feb.						
2·30944	266°15′ 47″·0	+ 56°10′ 57″·0	266°26′ 32″·8	+ 56°11′ 57″·2	[− 6′ 50″·3]	− 29″·5
2·31149	266 23 38 ·0	56 11 58 ·5				
4·35521	260 23 49 ·0	58 8 12 ·5	260 25 27 ·8	58 8 33 ·1	− 1 38 ·8	− 20 ·6
	252 50 44 ·0	59 58 25 .8				
	252 48 27 ·3	59 57 5 ·3				
6·32567	252 46 56 ·2	59 57 36 ·7	252 49 34 ·2	59 57 55 ·4	− 1 41 ·1	− 15 ·8
	252 45 25 ·2	59 57 30 ·6				
	247 50 7 ·0	60 50 27 ·5				
	247 49 5 ·7	..				
	247 46 51 ·2	..	247 46 17 ·6		− 2 1 ·3	
7·39291	247 45 33 ·3	..				+ 1 ·2
7·39712	247 44 39 ·7	60 50 57 ·7		60 51 6 ·5		
	247 37 42 ·3	..				
	247 35 56 ·0	60 51 57 ·8				
	221 39 25 ·7	..				
	221 34 9 ·5	62 8 18 ·4				
11·45162	221 26 22 ·7	..	221 30 39 ·1	62 6 58 ·6	− 1 9 ·8	+ 1 9 ·3
	221 23 22 ·2	62 7 57 ·4				
	221 24 6 ·3	..				
Mar.						
	..	8 34 1 ·8				
	143 9 41 ·2	8 33 7 ·8				
3·42427	..	8 32 37 ·7		8 33 23 ·2		
3·42547	143 9 5 ·4	8 32 11 ·7	143 8 37 ·4		+ 46 ·1	+ 39 ·4
	..	8 31 39 ·8				
5·38308	141 31 35 ·8	5 15 51 ·5	141 31 19 ·8	+ 5 15 38 ·9	+ 16 ·0	− 2 ·3
	..	+ 5 15 30 ·8				
	..	− 2 53 37 ·4				
12·39707	137 51 41 ·8	− 2 54 32 ·4	137 51 34 ·4		+ 7 ·4	
12·40007	..	− 2 54 52 ·7		− 2 53 51 ·1		+ 42 ·4
	..	− 2 55 11 ·3				
Mr. Lassell's	Observatory,	Starfield:				
Jan.						
19·82880	284 51 28 ·5	46 13 31 ·0	284 52 45 ·0	46 13 24 ·2	− 1 16 ·5	+ 6 ·8
20·36614	284 27 46 ·4	..	284 28 22 ·8	..	− 36 ·4	..
20·36920	..	46 32 1 ·0	..	46 31 39 ·9	..	+ 21 ·1
24·32828	..	48 58 44 ·2	..	48 58 46 ·3	..	− 2 ·1
24·33065	280 59 45 ·0	..	281 0 21 ·5	..	− 36 ·5	..
Feb.						
8·50548	..	61 36 23 ·0	..	61 37 19 ·0	..	− 56 ·0

TABLE III.—(*continued*).

Mean Time Berlin.	Observed Values.		Calculated Values.		Observed Value minus Calculated Value.	
	α	δ	α	δ	$\Delta\alpha$	$\Delta\delta$
Feb.						
8·50899	241°39′23″ ·7	..	241°40′42″ ·5	..	− 1′18″ ·8	..
14·51927	197 30 33 ·3	..	197 30 33 ·3	..	0 ·0	..
14·51971	..	58 37 31 ·5	..	58 38 46 ·7	..	− 1 15 ·2
Mar.						
11·39866	..	− 2 16 13 ·5	..	− 2 0 14 ·7	..	..
11·40089	138 14 3 ·4	..	138 13 38 ·1	..	+ 25 ·3	..

I have excluded the Berlin observations of 28th December, 1844, and of 30th March, 1845, as they have been noted by the observers as unsatisfactory, besides being completely isolated. The first observation immediately after the discovery had to be made in a hurry as the sky was becoming overcast, and at the last the comet was so faint, that it was scarcely visible. Several very imperfect Greenwich observations were also excluded. Wichmann states, that the circumstance of this being the first comet he ever observed, as well as its faintness and indistinctness, has detracted from the value of his observations.

I endeavoured to determine the probable errors committed by the different observers, before deducing the normal places from the deviations, and by a graphical method I found the following result (Table IV.) I did not however always assume the weight to be in accordance with the value of the probable error, but altered it occasionally according to my own judgment.

Some of the observations seem to indicate slight constant differences between the results of the different observers. These differences chiefly arise from the circumstance that different observers choose different points of such a nebulous object as a comet from which to measure to the comparison stars. The orbit finally sought is that described by the mean point of the comet, and this is best obtained by allowing the point which each observer has adopted to contribute its influence to the final result.

In the case of the minor planets, in which there can be no uncertainty about the point from which to measure, constant differences between the results of different observers have occasionally been noticed. From whatever cause this difference has arisen, it would be impossible in the case of a comet to disentangle it from the differences arising from the uncertainty of the point chosen, and hence the former is assumed to be evanescent.

TABLE IV.

Probable Errors r_α and r_δ and Weights P_α and P_δ of a single Observation in Right Ascension and Declination, made by the separate Observers.

	r_α.	P_α.	r_δ.	P_δ.
Goettingen,	3″·0	$2\frac{1}{2}$	6″·5	$2\frac{1}{2}$
Paris,	5″·8	$2\frac{1}{2}$	7″·0	$2\frac{1}{2}$
Altona,	5″·6	$2\frac{1}{2}$	4″·4	$2\frac{1}{2}$
Bonn,	12″·0	$\frac{2}{3}$	8″·2	$1\frac{1}{2}$
Bishop's,	13″·3	$\frac{2}{3}$	7″·7	$1\frac{1}{2}$
Berlin,	11″·7	$\frac{2}{3}$	12″·2	$\frac{2}{3}$
Koenigsberg,	15″·0	$\frac{1}{2}$	14″·7	$\frac{1}{2}$
Mannheim,	13″·4	$\frac{1}{2}$	18″·8	$\frac{1}{3}$
Hamburg,	20″·3	$\frac{1}{3}$	12″·6	$\frac{1}{2}$
Lassell's,	20″·2	$\frac{1}{3}$	16″·0	$\frac{1}{3}$
Genf,	18″·0	$\frac{1}{5}$	21″·5	$\frac{1}{5}$
Padua,	24″·2	$\frac{1}{5}$	19″·7	$\frac{1}{5}$
Marseilles,	25″·1	$\frac{1}{10}$	29″·0	$\frac{1}{10}$
Greenwich,	21″·3	$\frac{1}{10}$	48″·3	$\frac{1}{20}$
Kremsmünster,	55″·3	$\frac{1}{25}$	39″·2	$\frac{1}{25}$

The next step is the determination of the normal deviations. The observations were first divided into five groups, each extending over about a fortnight. The deviations of all the observations in each group were multiplied by the weights, and the sum of the products divided by the sum of the weights is adopted as the mean deviation corresponding to the epoch, which is computed in the same manner from all the times of the observations.

TABLE V.

NORMAL DEVIATIONS WITH THEIR PROBABLE ERRORS.

Mean Time Berlin.	Observed Right Ascension minus Calculated.	Observed Declination minus Calculated.
1845.		
Jan. 11½	− 28″·81 ± 1″·80	− 5″·10 ± 1″·75
,, 24½	− 43 ·32 ± 3 ·07	− 15 ·44 ± 2 ·46
Feb. 7½	− 73 ·78 ± 2 ·22	− 50 ·46 ± 1 ·94
,, 24½	+ 29 ·37 ± 4 ·30	− 75 ·03 ± 4 ·12
Mar. 7	+ 20 ·16 ± 3 ·13	− 44 ·55 ± 3 ·47

The linear equations by which the variations of the geocentric right ascensions and declinations of the Comet are obtained as functions of the variations of the elements of the orbit, have been given by Gauss, Bessel, and by others. They are:

$$0 = \cos\delta \,.\, d\alpha + \frac{\sin\alpha}{\Delta}\, dx - \frac{\cos\alpha}{\Delta}\, dy$$

$$0 = d\delta + \frac{\cos\alpha \sin\delta}{\Delta}\, dx + \frac{\sin\alpha \sin\delta}{\Delta}\, dy - \frac{\cos\delta}{\Delta}\, dz$$

where x, y, z are, as before, the heliocentric co-ordinates of the Comet referred to the equator, and belonging to the same time t, for which the equations are to be computed.

In these equations are to be substituted:

$$dx = \left[\frac{dx}{dv}\frac{dv}{dT} + \frac{dx}{dr}\frac{dr}{dT}\right] dT + \left[\frac{dx}{dv}\frac{dv}{d\log q} + \frac{dx}{dr}\frac{dr}{d\log q}\right] d\log q$$
$$+ \left[\frac{dx}{dv}\frac{dv}{de} + \frac{dx}{dr}\frac{dr}{de}\right] de + \frac{dx}{d\pi}\, d\pi + \frac{dx}{d\Omega}\, d\Omega + \frac{dx}{di}\, d\iota,$$

and analogous expressions for dy and dz.

The partial differential coefficients in these expressions have been computed by the following formulæ, where $\log k = 8{\cdot}23558$, $\log m = 0{\cdot}36222$ and ϵ is the obliquity of the ecliptic.

$$\frac{dx}{dv} = x \cot(188^\circ 12' + v) = \frac{dx}{d\pi},\ \frac{dx}{dr} = \frac{x}{r},\ \frac{dx}{d\Omega} = -\frac{dx}{dv} - y \cos\epsilon - z \sin\epsilon$$

$$\frac{dx}{d\iota} = r \sin u \cos\ (\log \sin = 9{\cdot}9812)$$

$$\frac{dy}{dv} = y \cot(62^\circ 53' + v) = \frac{dy}{d\pi},\ \frac{dy}{dr} = \frac{y}{r},\ \frac{dy}{d\Omega} = -\frac{dy}{dv} + x \cos\epsilon$$

$$\frac{dy}{d\iota} = r \sin u \cos\ (\log \sin = 9{\cdot}6642)$$

$$\frac{dz}{dv} = z \cot(104^\circ 53' + v) = \frac{dz}{d\pi},\ \frac{dz}{dr} = \frac{z}{r},\ \frac{dz}{d\Omega} = -\frac{dz}{dv} + x \sin\epsilon$$

$$\frac{dz}{d\iota} = r \sin u \cos\ (\log \sin = 9{\cdot}9697)$$

$$\frac{dv}{dT} = -\frac{k\sqrt{2q}}{r^2}\,\frac{dv}{d\log q} = \frac{3}{2}\,m(t-T)\,\frac{dv}{dT} \qquad \frac{dv}{de} = -\frac{9}{20}(t-T)\,\frac{dv}{dT} - \frac{2}{5}\,tg\,\frac{1}{2}\,v$$

$$\frac{dr}{dT} = -\frac{k \sin v}{\sqrt{2q}}\,\frac{dr}{d\log q} = mq \cos v \qquad \frac{dr}{de} = -\frac{9}{20}(t-T)\,\frac{dr}{dT} + \frac{r}{10}\,tg^2\,\frac{1}{2}\,v.$$

Substituting in the equations for $d\alpha$ and $d\delta$ the normal deviations given in Table V., and taking as values of t the corresponding times in the same table, the equations connecting the variations of the elements with the normal deviations have been deduced. Finally, multiplying each equation by the square root of the weight of the corresponding normal deviation, and expressing all the numbers by their logarithms we have the following equations of condition.

α	$0 = -1{\cdot}7195$	$-0{\cdot}9556\Delta\frac{T}{100}$	$-9{\cdot}5646\Delta lq$	$+0{\cdot}4521\Delta\pi$	$-9{\cdot}2470\Delta\Omega$	$+0{\cdot}2320\Delta\iota$	$+9{\cdot}8110\Delta\frac{e}{10}$
	$0 = -1{\cdot}6913$	$-0{\cdot}8970$,,	$-9{\cdot}9632$,,	$+0{\cdot}4381$,,	$+8{\cdot}2534$,,	$+0{\cdot}0061$,,	$+0{\cdot}5294$,,
	$0 = -1{\cdot}9040$	$-1{\cdot}1615$,,	$+0{\cdot}1600$,,	$+0{\cdot}6058$,,	$-0{\cdot}2361$,,	$-9{\cdot}3773$,,	$+1{\cdot}1588$,,
	$0 = +1{\cdot}4223$	$+0{\cdot}5118$,,	$+0{\cdot}6533$,,	$-0{\cdot}2802$,,	$-0{\cdot}4372$,,	$-9{\cdot}4752$,,	$-0{\cdot}3953$,,
	$0 = +1{\cdot}4544$	$+0{\cdot}5744$,,	$+0{\cdot}5663$,,	$-0{\cdot}2546$,,	$-0{\cdot}4023$,,	$+9{\cdot}5394$,,	$-0{\cdot}7431$,,
δ	$0 = -1{\cdot}0967$	$+0{\cdot}1584\Delta\frac{T}{100}$	$-0{\cdot}8807\Delta lq$	$-9{\cdot}4281\Delta\pi$	$+9{\cdot}5665\Delta\Omega$	$+9{\cdot}8329\Delta\iota$	$-9{\cdot}1228\Delta\frac{e}{10}$
	$0 = -1{\cdot}4271$	$-9{\cdot}8154$,,	$-0{\cdot}8760$,,	$+9{\cdot}9814$,,	$+9{\cdot}6511$,,	$+9{\cdot}9347$,,	$-9{\cdot}7368$,,
	$0 = -2{\cdot}0524$	$-0{\cdot}9872$,,	$-1{\cdot}2169$,,	$+0{\cdot}7615$,,	$+0{\cdot}2475$,,	$+0{\cdot}2823$,,	$+0{\cdot}5335$,,
	$0 = -1{\cdot}8752$	$-0{\cdot}8955$,,	$-1{\cdot}0096$,,	$+0{\cdot}6476$,,	$-0{\cdot}4489$,,	$-9{\cdot}1004$,,	$+0{\cdot}8151$,,
	$0 = -1{\cdot}7993$	$-0{\cdot}7487$,,	$-1{\cdot}0635$,,	$+0{\cdot}6564$,,	$-0{\cdot}4445$,,	$+9{\cdot}1783$,,	$+0{\cdot}2528$,,

To verify the equations I computed the places of the Comet with slightly modified elements, and the differences between these places and those derived from the original elements, agreed perfectly with the differences as computed from the equations of condition.

From the ten equations I computed, by the method of least squares, the following normal equations, the correctness of which was checked in the usual manner.

$0 = +3{\cdot}6288$	$+2{\cdot}7549\Delta\frac{T}{100}$	$+2{\cdot}5015\Delta lq$	$-2{\cdot}3730\Delta\pi$	$+1{\cdot}4600\Delta\Omega$	$-1{\cdot}5757\Delta\iota$	$-2{\cdot}5621\Delta\frac{e}{10}$
$0 = +3{\cdot}5825$	$+2{\cdot}5015$,,	$+2{\cdot}8203$,,	$-2{\cdot}3245$,,	$+0{\cdot}2017$,,	$-1{\cdot}6593$,,	$-2{\cdot}1840$,,
$0 = -3{\cdot}3012$	$-2{\cdot}3730$,,	$-2{\cdot}3245$,,	$+2{\cdot}0542$,,	$-1{\cdot}0866$,,	$+1{\cdot}2654$,,	$+2{\cdot}1471$,,
$0 = +2{\cdot}2365$	$+1{\cdot}4600$,,	$+0{\cdot}2017$,,	$-1{\cdot}0866$,,	$+1{\cdot}5560$,,	$+0{\cdot}6051$,,	$-1{\cdot}3366$,,
$0 = -2{\cdot}5634$	$-1{\cdot}5757$,,	$-1{\cdot}6593$,,	$+1{\cdot}2654$,,	$+0{\cdot}6051$,,	$+0{\cdot}9598$,,	$+0{\cdot}7289$,,
$0 = -3{\cdot}4066$	$-2{\cdot}5621$,,	$-2{\cdot}1840$,,	$+2{\cdot}1471$,,	$-1{\cdot}3366$,,	$+0{\cdot}7289$,,	$+2{\cdot}4974$,,

From the six normal equations, the values of the unknown quantities are determined in terms of Δe, which is of course equal to $e - 1$.

$$\begin{aligned}
\Delta T &= -\ 0{\cdot}002834 + 18{\cdot}5400\Delta e \\
\Delta lq &= -\ 0{\cdot}0000144 + 0{\cdot}0587\Delta e \\
\Delta\Omega &= -\ 0''{\cdot}3766 - 0{\cdot}0334\Delta e \\
\Delta\iota &= +\ 2''{\cdot}2225 + 0{\cdot}4037\Delta e \\
\Delta\pi &= -\ 0''{\cdot}4525 + 0{\cdot}3027\Delta e.
\end{aligned}$$

Assuming the eccentricity to be 1, the parabolic elements referred to the mean equinox of 1845·0 are:

T 1845, Jan. 8·193983 Mean Time Berlin.
Ω 366° 44′ 27″·62
ι 46° 50′ 40″·22
π 91° 19′ 40″·55
log q 9·9567346.
Motion direct.

The residual errors on this hypothesis corresponding to the five normal places of Table V., are:

	$\Delta\alpha \cos\delta$	$\Delta\delta$
I.	+ 1″·64	+ 1″·28
II.	+ 0 ·42	− 0 ·45
III.	− 0 ·63	− 2 ·75
IV.	− 4 ·69	+ 0 ·06
V.	− 1 ·32	+ 2 ·47

The most probable corrections to be applied to the assumed elements are:

$\Delta T = +\ 0{\cdot}001\ 741$
$\Delta lq = +\ 0{\cdot}000\ 000\ 08$
$\Delta e = +\ 0{\cdot}000\ 246\ 70$
$\Delta\Omega = -\ 2''{\cdot}08$
$\Delta\iota = +\ 22''{\cdot}76$
$\Delta\pi = +\ 14''{\cdot}95$

By the residuals left when these values are substituted in the differential equations, the probable error of a normal place, whose weight is 1, was determined to be = 2″·73. For the parabola it is 3″·09.

The elements with their probable errors are:

T = 1845, Jan. 8·198558 ± 0·001693 Mean Time Berlin.

$$\left.\begin{aligned} \Omega &= 336°44'25''{\cdot}92 \pm 0''{\cdot}88 \\ \iota &= 46\ 51\ \ 0\ {\cdot}76 \pm 8\ {\cdot}81 \\ \pi &= 91\ 19\ 55\ {\cdot}95 \pm 5\ {\cdot}88 \end{aligned}\right\}\text{M. Eq. } 1845{\cdot}0$$

$$\begin{aligned} \log q &= 9{\cdot}956\ 7491 \pm 0{\cdot}000\ 0055 \\ e &= 1{\cdot}000\ 2467 \pm 0{\cdot}000\ 1047 \end{aligned}$$

And they represent the normal places as follows:

	$\Delta\alpha \cos\delta$	$\Delta\delta$	P
I.	+ 0″·00	+ 1″·05	6
II.	+ 2 ·20	+ 0 ·57	3
III.	− 0 ·18	− 1 ·94	5
IV.	− 4 ·06	− 0 ·79	1
V.	+ 0 ·08	+ 1 ·96	2

These residuals are much smaller than the probable errors given in Table V. resulting from the discussion of the observations. This affords a check of the exactness of the entire calculation.

Among the numerous systems of provisional elements given of this Comet, there is only one in which the orbit has not been assumed to be a parabola. Sievers, employing but three observations, of which the last is on February 10, obtained for the orbit a hyperbola of eccentricity 1·0003323, which is remarkably close to the value I have obtained from the complete discussion of all the available observations.

XIII.—*On μ^2 Bootis (considered as a Revolving Double Star).* *By* Dr. W. Doberck, Astronomer at Colonel Cooper's Observatory, Markree, Co. Sligo.

Read February 8, 1875.

μ^2 Bootis is a very close and highly interesting binary star, which has been diligently observed by those astronomers who possess means for dealing with so difficult an object. It was discovered in 1781 by Sir William Herschel, when its components (of 8 and $8\frac{1}{2}$ magnitude, both greenish white) were nearly 2″ apart, but the measures became soon very difficult, from the rapid diminution in its distance. Its proper motion agreeing with that of μ Bootis, suggested the existence of a physical connexion between these two stars. Actual measures have, however, rendered such a supposition more than doubtful.

TABLE I.

Measures of Distances and Angles of Position between μ and μ^2 Bootis.

Observer.	Epoch.	Position.	Distance.
W. Herschel, . . .	1781·81	170°25′	—
W. Struve, ...	1819·85	172 54	—
,,	1821·78	172 36	1′48″·73
J. Herschel and South,	1821·35	171 51	1 48 ·54
O. Struve, ...	1840·95	171 5	1 48 ·77
,,	1847·69	171 37	1 48 ·67
,,	1850·45	171 42	1 48 ·80
,,	1851·47	171 30	1 48 ·68
,,	1852·59	171 47	1 48 ·71
,,	1854·65	171 43	1 48 ·60
,,	1856·58	171 38	1 48 ·59
,,	1859·09	171 42	1 48 ·59
,,	1861·60	171 35	1 48 ·40
,,	1866·22	171 36	—

I have determined the orbit of the revolving double star by Sir J. Herschel's* first method. This method is described in a paper on ξ Ursæ Majoris recently laid before the Royal Irish Academy by Professor R. S. Ball, F. R. S.†

It is exclusively founded on the angles of position, with exclusion of the distances, which could not be so accurately measured with the means which the Herschels had at their disposal, and even in our days the errors in the distances expressed in parts of the mean distance are, in such extremely near objects as μ^2 Bootis, very large quantities, which are best preserved for affording a final check of the accuracy of the elements arrived at by the aid of the angles. The major semi-axis cannot, of course, be determined without the distances. The graphic method, which is considered to be not sufficiently accurate for the determination of many double stars, is quite accurate enough in those cases, where, as here, the distance is always below 2″, and commonly only part of a second.

Reading off the interpolating curve, laid very nearly through all the observations, I obtained the angles of position and distances, as exhibited in Table II. By the aid of these I derived the first set of elements, as exhibited in Table III.

* Memoirs Royal Astronomical Society, Vol. v., p. 1.

† Proceedings Royal Irish Academy, Vol. i., Series 2, p. 316.

TABLE II.

THE ANGLES OF POSITION AND THE DISTANCES IN THE PROJECTED ELLIPSE, DEDUCED FOR THE CORRESPONDING EPOCHS BY THE INTERPOLATING CURVE.

No.	Epoch.	Interpolated Angle of Position.	Distance.
1	1780	357°·9	1·93
2	1785	356 ·1	1·75
3	1790	354 ·2	1·61
4	1795	352 ·2	1·50
5	1800	349 ·9	1·39
6	1805	347 ·0	1·29
7	1810	343 ·8	1·20
8	1815	340 ·0	1·13
9	1820	335 ·7	1·05
10	1825	330 ·4	0·97
11	1830	324 ·4	0·89
12	1835	317 ·3	0·79
13	1840	308 ·0	0·67
14	1845	294 ·0	0·56
15	1850	273 ·0	0·47
16	1855	246 ·0	0·42
17	1860	217 ·0	0·40
18	1865	189 ·5	0·43
19	1870	167 ·3	0·48
20	1875	147 ·5	0·51

TABLE III.

FIRST APPROXIMATIVE ELEMENTS OF μ^2 BOOTIS.

Position of Node,	183°38′.
Inclination,	ι 47° 7′.
Position of Perihel.,	π 196° 3′.
Angle between Axis Major and line of Nodes,	λ 17°55′.
Eccentricity,	e 0·6097.
Period in Years,	P 254·41.
Mean annual Motion,	n – 1°·415.
Perihelion Passage,	T 1865·66.

In Table IV. will be found the comparison of the observed angles of position with the positions as computed after the approximative elements.

TABLE IV.

COMPARISON OF APPROXIMATIVE ELEMENTS WITH OBSERVATION.

No.	Epoch.	Observ. minus Calc. Angle of Position.	No.	Epoch.	Observ. minus Calc. Angle of Position.
	SIR W. HERSCHEL.			O. STRUVE—*cont.*	
1	1782·68	+ 5° 9′	28	1858·56	− 2° 4′
2	1802·66	+ 2 19	29	1859·63	− 2 40
			30	1861·61	− 4 9
	SIR J. HERSCHEL.		31	1866·74	− 5 20
3	1830·24	− 0 19	32	1868·56	− 5 52
4	1832·56	+ 0 19	33	1870·52	+ 2 19
			34	1872·60	+ 3 54
	SIR J. SOUTH.				
5	1825·46	+ 3 52		MAEDLER.	
			35	1835·65	− 9 25
	SIR J. H. AND SIR J. S.		36	1836·45	− 7 8
6	1823·41	+ 2 24	37	1841·47	+ 0 43
			38	1842·40	− 0 41
	W. STRUVE.		39	1842·66	− 0 18
7	1821·78	− 0 35	40	1843·54	− 1 36
8	1822·21	− 1 33	41	1847·22	− 3 21
9	1826·77	− 1 27	42	1847·80	− 7 14
10	1829·73	− 1 32	43	1848·37	− 5 49
11	1833·34	− 0 28	44	1850·61	− 1 59
12	1833·39	− 0 49	45	1851·27	− 10 29
13	1834·82	− 2 8	46	1851·79	− 9 25
14	1835·55	+ 0 12	47	1852·21	− 9 56
15	1836·65	− 1 52	48	1852·70	− 7 31
16	1837·70	− 0 9	49	1853·33	− 5 20
17	1839·83	− 0 13	50	1854·70	− 8 47
			51	1855·53	+ 6 15
	O. STRUVE.		52	1857·38	+ 1 2
18	1840·46	+ 3 39	53	1858·57	+ 4 1
19	1842·23	− 2 48	54	1859·38	+ 2 30
20	1846·68	− 6 59			
21	1850·46	− 6 36		SMYTH.	
22	1851·48	− 11 43	55	1832·31	− 1 18
23	1852·65	− 0 1	56	1834·56	− 0 3
24	1854·65	− 8 45	57	1837·29	− 1 4
25	1855·57	− 3 36	58	1839·32	− 1 43
26	1856·57	− 1 42	59	1842·52	+ 0 31
27	1857·65	+ 1 16	60	1853·60	− 7 41

Table IV.—*continued.*

No.	Epoch.	Observ. minus Calc. Angle of Position.	No.	Epoch.	Observ. minus Calc. Angle of Position.
	Dawes.			Talmage.	
61	1833·39	− 1° 37′	80	1866·25	+ 13° 24′
62	1837·39	− 0 51	81	1871·74	+ 11 23
63	1840·39	− 4 15			
64	1841·66	− 4 20			
65	1842·40	− 4 54		Kaiser.	
66	1847·30	− 5 14	82	1841·67	− 4 16
67	1848·52	− 7 34			
68	1849·44	− 7 44			
69	1851·42	− 8 6		Secchi.	
70	1852·52	− 6 44	83	1857·45	− 6 9
71	1853·71	− 7 25	84	1866·58	− 1 21
72	1854·41	− 8 32			
				Wilson.	
			85	1873·5	+ 3 18
	Dembowski.				
73	1863·22	− 2 20		Engelmann.	
74	1865·13	− 2 58	86	1865·78	+ 1 47
75	1866·94	− 1 7	87	1867·57	+ 2 36
76	1868·38	+ 1 54			
77	1870·91	+ 3 13		Hind.	
78	1872·87	+ 1 35	88	1847·08	− 11 34
				Jacob.	
	Romberg.		89	1853·23	+ 0 19
79	1863·63	− 1 49	90	1854·05	− 4 18

With the aid of these deviations, I drew the interpolating curve anew, and derived the more correct elements.

TABLE V.

THE ANGLES OF POSITION AND THE DISTANCES DEDUCED FOR THE CORRESPONDING EPOCHS BY THE CORRECTED INTERPOLATING CURVE.

No.	Epoch.	Interpolated Angle of Position.	Distance.
1	1780	358°·3	1·679
2	1790	353 ·9	1·406
3	1800	348 ·2	1·274
4	1810	341 ·7	1·178
5	1820	333 ·8	1·076
6	1830	324 ·5	0·947
7	1840	309 ·9	0·706
8	1850	277 ·0	0·463
9	1855	246 ·0	0·421
10	1860	217 ·0	0·404
11	1865	189 ·5	0·426
12	1870	167 ·3	0·494
13	1875	147 ·5	0·515

TABLE VI.

THE SECOND ELEMENTS OF μ^2 BOOTIS.

Node,	182° 59′
ι	44° 26′
π	195° 48′
λ	17° 41′
e	0·6174
P	290·07 years.
n	− 1°·241
Per. Pass.	1863·51
a	1″·500

In Table VII. will be found the comparison of the angles of position and distances with those given by the observations.

TABLE VII.

COMPARISON OF THE ANGLES OF POSITION AND DISTANCES CALCULATED FROM THE SECOND ELEMENTS OF μ^2 BOOTIS, WITH OBSERVATION.

No.	Epoch.	Observ. Position.	Calc. Position.	Difference O – C.	Observ. Distance.	Calc. Distance.	Difference O – C.
	SIR W. HERSCHEL.						
1	1782·68	357°14′	356°44′	+ 0°30′	—	—	—
2	1802·66	346 14	347 56	– 1 42	—	—	—
	SIR J. HERSCHEL.						
3	1830·24	324 46	325 56	– 1 10	0 ·89	1 ·09	– 0 ·20
4	1832·56	322 43	322 38	+ 0 5	1 ·04	1 .03	– 0 ·01
	SIR J. SOUTH.						
5	1825·46	333 32	331 33	+ 1 59	1 ·42	1 ·22	+ 0 ·20
	SIR J. H. AND SIR J. S.						
6	1823·41	333 42	333 33	+ 0 9	1 ·65	1 ·26	+ 0 ·39
	W. STRUVE.						
7	1821·78	332 3	335 5	– 3 2	—	—	
8	22·21	330 42	334 41	– 3 59	—	—	—
9	26·77	327 0	330 5	– 3 5	1 ·39	1 ·19	+ 0 ·20
10	29·73	324 3	326 34	– 2 31	1 ·25	1 ·10	+ 0 ·15
11	33·34	321 0	321 27	– 0 27	1 ·18	1 ·00	+ 0 ·18
12	33·39	320 36	321 22	– 0 46	1 ·08	0 ·99	+ 0 ·09
13	34·82	317 30	319 9	– 1 39	1 ·31	0 ·96	+ 0 ·35
14	35·55	318 38	317 53	+ 0 45	1 ·10	0 ·94	+ 0 ·16
15	36·65	315 4	315 49	– 0 45	1 ·06	0 ·90	+ 0 ·16
16	37·70	315 0	313 42	+ 1 18	0 ·90	0 ·87	+ 0 ·03
17	39·83	311 9	309 4	+ 2 5	—	—	—
	O. STRUVE.						
18	1840·46	313 46	307 32	+ 6 14	1 ·03	0 ·80	+ 0 ·23
19	42·23	303 28	302 50	+ 0 38	0 ·88	0 ·74	+ 0 ·14
20	46·68	287 7	287 47	– 0 40	0 ·56	0 ·64	– 0 ·08
21	50·46	272 49	270 24	+ 2 25	0 ·48	0 ·58	– 0 ·10
22	51·48	262 40	264 56	– 2 16	0 ·43	0 ·55	– 0 ·12
23	52·65	268 12	258 19	+ 9 53	0 ·44	0 ·52	– 0 ·08
24	54·65	247 39	246 29	+ 1 10	0 ·43	0 ·51	– 0 ·08
25	55·57	246 51	240 54	+ 5 57	0 ·47	0 ·50	– 0 ·03
26	56·57	241 57	234 47	+ 7 10	0 ·50	0 ·51	– 0 ·01

TABLE VII.—*continued.*

No.	Epoch.	Observ. Position.	Calc. Position.	Difference O – C.	Observ. Distance.	Calc. Distance.	Difference O – C.
	O. STRUVE—*continued.*						
27	1857·65	237°32′	228°13′	+ 9°19′	0″·48	0″·51	– 0″·03
28	58·56	227 38	222 45	+ 4 53	0 ·48	0 ·51	– 0 ·03
29	59·63	219 30	216 34	+ 2 56	0 ·49	0 ·52	– 0 ·03
30	61·61	205 27	205 42	– 0 15	0 ·52	0 ·54	– 0 ·02
31	66·74	175 33	180 37	– 5 4	0 ·63	0 ·58	+ 0 ·05
32	68·56	165 51	172 32	– 6 41	0 ·56	0 ·59	– 0 ·03
33	70·52	164 35	164 4	+ 0 31	0 ·60	0 ·61	– 0 ·01
34	72·60	156 37	155 26	+ 1 11	0 ·70	0 ·62	+ 0 ·08
	MAEDLER.						
35	1835·65	309 4	317 4	– 8 40	—	—	—
36	36·45	310 7	316 13	– 6 6	—	—	—
37	41·47	308 43	304 56	+ 3 47	0 ·83	0 ·77	+ 0 ·06
38	42·40	305 11	302 21	+ 2 50	0 ·72	0 ·74	– 0 ·02
39	42·66	304 56	301 35	+ 3 21	0 ·78	0 ·73	+ 0 ·05
40	43·54	301 33	298 58	+ 2 35	0 ·76	0 ·71	+ 0 ·05
41	47·22	288 41	285 36	+ 3 5	0 ·56	0 ·63	– 0 ·07
42	47·80	283 4	283 7	– 0 3	0 ·52	0 ·62	– 0 ·10
43	48·37	282 26	280 38	+ 1 48	0 ·42	0 ·61	– 0 ·19
44	50·61	276 44	269 36	+ 7 8	0 ·40	0 ·57	– 0 ·17
45	51·27	264 58	266 5	– 1 7	0 ·32	0 ·56	– 0 ·24
46	51·79	263 23	263 13	+ 0 10	0 ·34	0 ·54	– 0 ·20
47	52·21	260 42	260 52	– 0 10	0 ·37	0 ·53	– 0 ·16
48	52·70	260 25	258 2	+ 2 23	0 ·42	0 ·52	– 0 ·10
49	53·78	256 14	251 43	+ 4 31	0 ·32	0 ·52	– 0 ·20
50	54·70	247 12	246 10	+ 1 2	0 ·44	0 ·51	– 0 ·07
51	55·53	256 57	241 7	+ 15 50	0 ·42	0 ·50	– 0 ·08
52	57·38	239 14	229 50	+ 9 24	0 ·35	0 ·51	– 0 ·16
53	58·57	236 19	225 22	+ 10 57	0 ·32	0 ·52	– 0 ·20
54	59·38	226 25	217 58	+ 8 27	0 ·43	0 ·52	– 0 ·09
	SMYTH.						
55	1832·31	321 24	323 0	– 1 36	1 ·3	1 ·03	+ 0 ·27
56	34·56	319 54	319 33	+ 0 21	1 ·2	0 ·97	+ 0 ·23
57	37·29	314 48	314 32	+ 0 16	1 ·0	0 ·89	+ 0 ·11
58	39·32	310 36	310 13	+ 0 23	0 ·9	0 ·83	+ 0 ·07
59	42·52	306 6	302 0	+ 4 6	0 ·8	0 ·74	+ 0 ·06
60	53·60	255 0	252 46	+ 2 14	0 ·5	0 ·53	– 0 ·03
	DAWES.						
61	1833·39	319 47	321 22	– 1 35	1 ·15	1 ·00	+ 0 ·15
62	37·37	314 52	314 22	+ 0 30	<1 ·0	0 ·88	—
63	40·39	306 0	307 42	– 1 42	0 ·83	0 ·80	+ 0 ·03
64	41·66	303 14	304 24	– 1 10	0 ·86	0 ·76	+ 0 ·10

TABLE VII.—*continued.*

No.	Epoch.	Observ. Position.	Calc. Position.	Difference. O – C.	Observ. Distance.	Calc. Distance.	Difference. O – C.
	DAWES—*continued.*						
65	1842·40	300°58′	302°21′	– 1°23′	0″·85	0″·74	+ 0″·11
66	47·30	286 31	215 16	+ 1 15	0 ·65	0 ·61	+ 0 ·04
67	48·52	280 2	279 53	+ 0 9	0 ·65	0 ·61	+ 0 ·04
68	49·44	276 12	275 31	+ 0 41	0 ·68	0 ·60	+ 0 ·08
69	51·42	266 34	265 15	+ 1 19	0 ·52	0 ·55	– 0 ·03
70	52·52	262 13	259 4	+ 3 9	0 ·55	0 ·52	+ 0 ·03
71	53·71	254 37	252 8	+ 2 29	0 ·50	0 ·52	– 0 ·02
72	54·41	249 18	247 55	+ 1 23	0 ·47	0 ·51	– 0 ·04
	DEMBOWSKI.						
73	1863·22	197 40	197 19	+ 0 21	0 ·5	0 ·50	0 ·00
74	65·13	186 17	187 57	– 1 40	0 ·5	0 .57	– 0 ·07
75	66·94	178 44	179 43	– 0 59	0 ·5	0 ·58	– 0 ·08
76	68·38	174 32	173 20	+ 1 12	0 ·5	0 ·60	– 0 ·10
77	70·91	163 38	162 24	+ 1 14	0 ·66	0 ·62	+ 0 ·04
78	72·89	153 7	154 14	– 1 7	0 ·75	0 ·62	+ 0 ·13
	ROMBERG.						
79	1863·63	195 48	195 14	+ 0 34	0 ·75	0 ·55	+ 0 ·20
	TALMAGE.						
80	1866·25	196 49	182 50	+ 13 59	0 ·85	0 ·58	+ 0 ·27
81	71·74	168 0	158 58	+ 9 2	—	—	—
	KAISER.						
82	1841·67	303 18	340 24	– 1 6	0 ·82	0 ·75	+ 0 ·07
	SECCHI.						
83	1857·45	231 33	229 26	+ 2 7	0 ·48	0 ·51	– 0 ·03
84	66·58	180 18	181 20	– 1 2	0 ·30	0 ·58	– 0 ·28
	WILSON.						
85	1873 ·5	152	151 43	+ 0 17	0 ·60	0 ·62	– 0 ·02
	ENGELMANN.						
86	1865·78	187 28	184 58	+ 2 30	0 ·57	0 ·57	0 ·00
87	67·57	179 18	176 55	+ 2 23	0 ·70	0 ·58	+ 0 ·12
	HIND.						
88	1847·08	281 17	286 13	– 4 56	—	—	—
	JACOB.						
89	1853·23	265 8	254 56	+ 10 12	0 ·45	0 ·52	– 0 ·07
90	54·05	255 44	250 7	+ 5 37	0 ·50	0 ·51	– 0 ·01

The larger discrepancies about the time of nearest approach cannot surprise us, as the components were then only half a second apart, besides O. Struve's observations are known to suffer from constant errors. Maedler's observations will sometimes be found rather uncertain, but there is no reason to believe that they are affected by personal differences. On the whole, the elements represent fairly the observations taken by experienced astronomers. Later measures taken by commencers must of course be of less accuracy.

Finally, I subjoin elements of the same binary star obtained lately by several astronomers. I have been able to employ a larger number of observations especially after the perihel. passage.

T	Node.	λ	γ	*P*	*a*	*e*	Authority.
1865·5	169° 0′	23°39′	46°22′	198$^{yrs.}$·93	—	0·4957	KLINKERFUES.
1860·88	163 11	54 27	41 52	314 ·761	1″·761	0·6832	HIND.
1865 ·2	172 0	20 5	45 0	200 ·4	—	0·51	WILSON.
1866·00	166 8	23 1	47 31	182 ·6	1 ·165	0·491	WINAGRADSKIJ.
1863·51	182 59	17 41	44 26	290 ·07	1 ·500	0·6174	DOBERCK.

XIV.—*Report on the Anatomy of Insectivorous Edentates.* By ALEXANDER MACALISTER, M. B., Professor of Comparative Anatomy in the University of Dublin. [With Plates XXVI. & XXVII.]

Read January 25, 1875.

PART FIRST.

INTRODUCTION.

HAVING expended in the purchase of specimens of the above-named family,* the grant voted to me by the Royal Irish Academy last year, I beg hereby to present a report on the anatomical structure of the specimens of this series.

The Anteaters† are a well circumscribed group of entomophagous Edentates characterized by having no teeth, small oral rictus, a cylindrical tongue capable of enormous protrusion, long powerful tails, and (except Cyclothurus) no clavicles.

They are divided into two well marked genera:—Manis, with a clothing of scales which are imbricated, with small valve-like external ears and pentadactylous feet; Myrmecophaga with a clothing of coarse (rarely soft) hair, and rounded prominent ears. They both progress on the outside of the forefeet, and hence the claws are incurved.

Manis includes four subgenera: (α) Manis, with the tail longer than the body, forefeet bristly, the inner claw being smaller and bent behind the others; (β) Pholidotus has the forefeet scaly, and the tail is as long as the body and is as broad as the body at its base; the inner claw

* In addition to the Anteaters I purchased a two-toed sloth which was dissected by Mr. Mackintosh and myself, and the record of whose dissection has been prepared and published by Mr. Mackintosh, in Proceedings Royal Irish Academy, 2nd Series, Science, Vol. ii., p. 66.

† I except Orycteropidæ.

is equal to the outer; the inferior lateral scales of the 15--19 rows are keeled; (γ) Phatages, like the last, has unkeeled scales in eleven rows; (δ) Smutsia has a tail with a feebler base, a shorter head, and is distinguished also by the middle row of scales stopping short of the end of the tail.

Myrmecophaga includes three subgenera: (α) Myrmecophaga with pentadactylous manus and pes, without expanded ribs, and having the choanæ bounded below by the confluence of the pterygoids. The back and tail have a median bristly mane. (β) Tamandua is maneless, and has a permanently prehensile tail (this character is transitory in very young Myrmecophagæ) covered with circles of scales at its extremity. (γ) Cyclothurus is claviculate with only two claws on its forefoot, a prehensile tail, expanded ribs, and with the long palatine fissure separating the pterygoids and indenting the palatines.

With the aid of Mr. Mackintosh I have made complete dissections of *Manis multiscutatus* (Gray) from West Africa; also of *Myrmecophaga jubata* (a magnificent specimen, quite fresh), and of *Cyclothurus didactylus* (Gray). I have also had some parts of *Tamandua bivittata*, *Pholidotus Indicus*, and *Cyclothurus fulvus*, and the following monograph is compiled from this material, together with the records of the dissection of *Myrmecophaga jubata* by Owen* and Pouchet†, of *Tamandua tetradactyla* by Rapp‡ and Cuvier, § of *Cyclothurus didactylus* by Meckel,‖ Humphry,¶ Galton,** Cuvier,†† and of *Pholidotus Dalmanni* by Humphry.‡‡ Thus my material includes five of the seven subgenera, and seven species.

* Transactions Zoological Society, Vol. iv., 1854, p. 117.

† Mémoire sur le grand Fourmilier, Paris, 1867. Journal de l'Anatomie, Vol. iii., 1866.

‡ Anatomische Untersuchungen über die Edentaten; Tübingen, 1852.

§ Cuvier et Laurillard, Myologie Comparée, 1855.

‖ Archiv., Bd. v., 1819, Anatomie Comparee, Riester and Sauson's Tr., Vol. vi.

¶ Journal of Anatomy and Physiology, Vol. iv., p. 17.

** Annals and Magazine of Natural History, Oct., 1869.

†† Op. cit., Pl. 257–258.

‡‡ Loc. cit.

§ I.—*Muscles of the Upper Extremity.*

Trapezius in Myrmecophaga arises from the four lower cervical spines and from the seven upper dorsals; in Manis from a small portion (inner third) of the middle curved line of the occiput, from all the cervical spines, and from the ten upper dorsals. In Cyclothurus it arises from the inner half of the occipital line, from the median line of the neck, and from the seven or eight upper dorsal spines. In Tamandua it has no occipital origin. The muscle is a continuous sheet in Myrmecophaga and in Tamandua, partly divided opposite the hinder edge of the spine of the scapula in Cyclothurus, completely divided in Manis. It is inserted in all into the spine of the scapula, its inferior portion being tendinous at its bony attachment. In Cyclothurus it is inserted also into the outer fourth of the clavicle, and into the margin of the mesoscapular segment. It only occupies three-fourths of the scapular spine in this genus. The middle portion opposite the anterior edge of the prescapula is thick in all, and in Manis, muscular fibres pass over the median line from one side to the other, similar to those of the "common pectoral" in Ungulates. In Manis the anterior fibres are inserted into the tendinous inscription which represents the clavicle. The spinal accessory nerve sinks into the anterior cervical border near this line.

Rhomboideus arises in Myrmecophaga from the six lower cervical and four upper dorsal spines, but not at all from the atlas or occiput. It is inserted into the scapular border as usual. In Tamandua it does not rise so high in the neck, and in Cyclothurus it only occupies about half of the cervical region arising from the three lower vertebræ,* as well as from the four upper dorsals. In my *Cyclothurus didactylus* it received a slip from the levator scapulæ, but as this is unnoticed by the four other authors, it is probably only an individual anomaly. The lowest fibres cross those of serratus magnus at the posterior angle of the scapula. In none of these is there any division into major and minor. In Manis there is an occipital

* Mr. Galton does not notice the cervical origin.

rhomboid arising from the occiput and from the upper two-thirds of the middle line of the neck. This portion is inserted into nearly the whole length of the anterior edge of the spine of the scapula, under cover of the superior trapezius. The dorsal segment (rhomboideus major) arises from the four upper dorsal spines, and has a short insertion into the posterior half of the superscapula.

Pectoralis major in Myrmecophaga consists of three lamella: 1st, an anterior superficial part from the presternum and one or two sternebers of the mesosternum (in Cyclothurus from presternum alone, and by a few fibres from the omosternum, but not from the clavicle); 2nd, a posterior superficial portion on the same plane as the last, and extending as far back as the xiphoid cartilage. This portion unites with the last, and they are inserted together into the pectoral crest on the humerus, the first being a little superficial and inserted rather lower down. In Myrmecophaga the tendon is below connected with the fascia of the arm. The third portion arises in Myrmecophaga from the 2–7 sternebræ, and is inserted into the upper part of the lesser humeral tuberosity. This portion has the anterior thoracic nerve distributed to its lower surface. In Cyclothurus and Tamandua this portion is similar; in the former its origin extends from the broad wing of the presternum to the cartilage of the first rib as well as over the four upper mesosternebers. In Manis the separate presternal slip extends to directly above the inner condyle of the humerus, and the two other segments correspond to those of the Anteaters. The deep stratum in Myrmecophaga shows a tendency to divide, but there is no true pectoralis minor. This deep part as it is thick at its upper margin bulges under the clavicle in Cyclothurus, thus functionally replacing the subclavius which is absent in all.*

Latissimus dorsi arises in Myrmecophaga from the fascia over the 7th–10th dorsal spines, and from the 6th–10th ribs; in Cyclothurus from the same extent of spines, and from the 4th–11th ribs; in Manis from the seven lowest ribs. The insertion is fused with that of the teres major in Myrmecophaga, Tamandua, and Manis, separate in Cyclothurus. In the last-

* Subclavius is present in all other Edentates.

named the tendon gives off, as in the others, an "achselbogen," and then passing behind the axillary artery and median nerve, runs over the insertion of the teres major to a point further out than the insertion of that muscle. The upper edge of the "achselbogen" is continued as a fascia over the parts in the axilla to the strong bilaminar costocoracoid membrane with which it is continuous. This membrane is continuous outwards into the semivagina of the shoulder joint, which thus the latissimus dorsi can make tense.

In Manis an accessory slip of the latissimus arises from the eighth rib and remains distinct up to its insertion, which is directly below the main tendon.

Serratus magnus is a remarkable muscle in all. In Myrmecophaga its origin is very extensive, occupying the transverse processes of the six lowest cervical vertebræ, and nearly the whole extent of the outer surface of the infra-angular segments of the first three ribs; also from three inches of the outer surface of the fourth and fifth ribs, and by strong slips from the seventh and eighth ribs. The fibres converge and are inserted into a wide triangular area at the superior angle of the scapula, and into a smaller similar space at the inferior angle, as well as into the margin of the bone between these. In no other Mammal has this muscle so extensive surfaces of origin. In consequence of this wide surface of origin and of the shortness of its fibres, we found that when all the muscles of the shoulder were divided except the serratus, the scapula could not be abducted from the side, though it could be rotated through an arc of about 50°. In Cyclothurus this muscle is much smaller, being completely sundered into three component parts—1st, a cervical part or levator scapulæ, arising from the fifth, sixth, and seventh cervical transverse processes, and inserted into the upper angle and anterior edge of the scapula; 2nd, a thin sheet from the first rib and outer margin of the presternum to the anterior half of the subscapular aspect of the vertebral border of the bone; 3rd, the inferior strongest part, arising widely from the 2nd and 3rd ribs, and by slips from the 4th–7th; this portion is inserted into the inferior angle of the scapula and into a triangular area on the surface of the subscapular fascia, covering the muscle of that name which separates its fibres here from the bone. In Manis the serratus arises from the six lower

cervical vertebræ, from the six upper ribs, and is inserted into the inferior part of the superscapula. The cervical part was not separate in this animal.

The acromio-trachelian is absent in all the Myrmecophagidæ. The deltoid in Myrmecophaga consists of a single band extending from below the metacromion to the humerus below the great tuberosity. This crosses over the infraspinatus and teres minor. Possibly the clavicular deltoid is fused with that part of the great pectoral which is inserted into the fascia of the forearm. In Cyclothurus the clavicular deltoid is separate, arising from the outer two-thirds of the clavicle, and is inserted into the anterior part of the pectoral ridge; it crosses over the supraspinatus, and the insertion of the pectoralis major.

The acromial part is joined to the clavicular and arises from the acromion process. Its fibres are the lowest at their insertion, being behind the origin of the supinator longus. The scapular portion is quite separate, and its fibres run more transversely to be inserted under cover of the acromial portion. Its origin only covers the middle one-third of the spine of the scapula.

In Manis there is a clavicular deltoid from the clavicular tendinous inscription at the insertion of the clavicular trapezius and cleidomastoid, which is inserted into the deltoidal crest on the humerus below the attachment of the acromial part and into the fascia over the humerus. The second part or acromial deltoid is very short and passes from the acromion process to the upper part of the deltoidal crest. Behind this is a largely developed muscle which looks like a scapular deltoid which arises from the whole surface of the infraspinous fascia, and from the outer border of the scapular spine. This portion, instead of taking its usual course, passes down the back of the humerus and is inserted into the back of the supracondyloid process, as well as into the long head of the triceps, and with it into the olecranon process. That this portion is a scapular deltoid is very problematical, for, although there is under it a separate teres major, yet its relation to the teres major about to be described in Cyclothurus would seem to point towards a terete, rather than a deltoidean, affinity in the aberrant muscle. In my Manis I failed to detect the nerve supplying this muscle which would have satisfactorily settled the question.

Teres major in Myrmecophaga arises from all that portion of the scapula which lies below the second spine, except so much as is occupied by the separate slip of the subscapularis. In Manis its origin is from the lower half of the axillary border of the scapula, and if the last-described portion also belong to this series, then it should be described as double and of great strength. In Cyclothurus this muscle is enormous and occupies the ground covered by the two portions of muscle in Manis. It arises from the upper three-fourths of the scapular spine, overlapping the scapular deltoid, as well as from the axillary edge of the scapula for nearly its whole extent, thus making a perfect pocket for the infraspinatus. The fibres run into the humerus for the whole extent of the inner edge from the inner tuberosity to the supra-condyloid process. The lowest fibres are inserted opposite the upper edge of the anconeus internus. None of its fibres were continued into the triceps in Mr. Galton's specimens and in mine, although this was the case in Meckel's. The insertion of this muscle is fused with that of the latissimus in Myrmecophaga and Manis, not in Cyclothurus, although in Cuvier's specimen it seemed so united. Its nerve is the second subscapular.

In Tamandua it is as in Myrmecophaga.

Teres minor in Myrmecophaga arises from a small space above the second scapular spine, and is inserted into the lowest part of the outer humeral tuberosity. In Cyclothurus it is more closely united to the infraspinatus, but can be separated from it with ease on careful dissection. It springs from the second fourth of the axillary edge of the scapula. In Manis it is small and marginal with a fleshy insertion.

Coraco-brachialis is variable. In Myrmecophaga it is the long variety arising from the coracoid head of the biceps close to its origin, and is inserted into the upper edge of the inner condyle of the humerus. In Tamandua the corresponding part arises from the upper edge of the scapula in common with the coracoid head of the biceps; it is inserted by a wide flat band above the inner condyle. This portion is covered by the superior segment of the subscapularis at its origin in both these species. In Tamandua there is besides a very slender coraco-brachialis brevis inserted above the latissimus dorsi tendon, and also covered by the subscapularis, but no such slip existed in Myrmecophaga. In Cyclothurus there is no vestige of coraco-brachialis

at all. Manis has a long coraco-brachialis arising by a tendon separate from the biceps, from the coracoid region, as well as by a fascial expansion from the surface of the subscapularis. It has no short slip, and is not covered by any part of the subscapularis.

Supraspinatus is large and normal in all. Infraspinatus is large in all, and somewhat bilaminar in Cyclothurus, the spinal (mesoscapular) part lying rather superficial to the postscapular. The capsule is very thin under the latter, but there is no representative of the bursa of Hagen such as exists in Orycteropus.

Subscapularis is a complex muscle in both Myrmecophaga and Tamandua. In the former it is intersected by ten tendinous planes, and fills up the whole of the subscapular fossa, excepting the area reserved for the serratus magnus. The body of the muscle is as usual, but there are two accessory slips superadded, one of which springs from the dorsum of the scapula below the postscapular ridge and external to the teres major. This part represents the subscapulo-humeral sometimes present as the result of a dismemberment of the subscapularis. The second portion is triangular and arises from the portion of the fossa above the subscapular nerve. This is the part which overlies the coraco-brachialis as above described, and it is inserted separately below and is part of the rest of the muscle. The subscapulo-humeral slip joins the rest of the muscle at its insertion. In Tamandua these portions are not so distinct as in the larger Anteater, and in Cyclothurus it is thick and wide with three sets of fibres, but forming one indivisible muscular sheet. In Manis it is normal also, had no bursa under its tendon, and seemed heavier than the combined supra- and infra-spinati.

On raising this muscle in Myrmecophaga, the capsule of the shoulder is seen to be deficient beneath it; elsewhere this ligament is weak, the glenoid cavity and ligament very shallow. There is a weak spino-glenoid ligament, and a flat band-like acromio-glenoid ligament. In Cyclothurus the spino-glenoid ligament is very strong and its outer edge is continued as a spino-humeral accessory ligament over the capsule of the shoulder.

In Cyclothurus likewise the tendon of the long head of the biceps is inseparably united to the capsular ligament, especially to the gleno-humeral band below it. There is in this animal a slender flat coraco-clavicular ligament arising from the inferior angle of the superior scapular costa.

Dorsi-epitrochlearis is in Cyclothurus very thick at its origin, which is from a tendinous line on the latissimus dorsi opposite the angle of the scapula. Its fibres expand and pass over the inner condyle, to be inserted into the fascia of the forearm, giving origin to the palmaris longus. This muscle is not so large proportionally in Myrmecophaga, and has no relation to any of the forearm muscles: it is similar in Manis. Its origin is close to the scapula in Myrmecophaga, and in Tamandua it receives some fibres from that bone. Pouchet describes this as being the case in Myrmecophaga also, but it certainly was not the case in our specimen, although, from the close proximity of its origin to that bone, a hasty observation might have led to that opinion.

Triceps longus arises in Manis from the middle two-thirds of the axillary costa of the scapula, and is inserted, as usual, into the olecranon by a tendon easily separable from the rest of the muscle. In Cyclothurus its origin is from the upper fourth of the same edge, and its tendon of insertion is either separate or joined to (Galton) the rest of the muscle.

Triceps externus and internus are fused in Manis, and in Cyclothurus the external head is large, it is normal in Myrmecophaga, but the internal triceps in that species is very singularly disposed, arising, as usual, from the inner side of the back of the humerus, below the musculo-spinal groove, it then becomes tendinous, and passes through a groove on the inner side of the humerus behind the inner condyle, round which it passes as a pulley. The tendon, having arrived in the forearm, is continued into the origin of the flexor digitorum—in fact forms a digastric muscle like the rectus (ambiens of Garrod) in the leg of a bird. This very remarkable arrangement, which will be referred to again, must give an enormous additional power to the flexor of the claws. The only approach to this arrangement known to me is in the Orycterope, where there are fibres of the inner head of the triceps directly continuous into this muscle, but in that animal there is not at all the definite expansion and elongation of the triceps tendon possessed by the Anteater.

Anconeus externus is large and closely tied to the outer head of the triceps in Myrmecophaga; it is quite inseparable from the triceps in Cyclothurus.

Anconeus internus is distinct in all, and in all, passes as usual from the inner condyle to the olecranon; in none is it fused with the triceps, contrasting thus strongly with the outer anconeus. The curious slip described by Pouchet in Myrmecophaga, which ascended from the olecranal insertion of this muscle, and crossed the ulnar nerve to end in a tendon over the biceps, was not represented by any similar arrangement in either arm of our Anteater, nor is it present in Tamandua, nor Cyclothurus.

Biceps in Myrmecophaga is two-headed. The long head arises, as usual, forms a fleshy belly, which bifurcates below, sending one slip to the ulna along with the brachialis anticus, and a second, which joins the insertion of the biceps coracoidalis. In Manis the long head is similar in origin, but single, and its entire insertion is ulnar. No coracoid head is present in this animal. In Cyclothurus the long head of the biceps bifurcates, as in the larger Anteater, and is inserted partly into the radius, partly into the ulna, but quite separate from, and anterior to, the brachialis anticus. In Mr. Galton's specimen the brachialis and the ulnar slip of the biceps were fused. The insertion of the radial part is into the tubercle of that bone, by a strong tendon. There is no coracoid head in this species. The coracoid head is distinct and strong in Myrmecophaga and in Tamandua, and is radial in insertion, fused here with the radial element of the glenoidal biceps. Rapp found a humeral head in Tamandua, but I found no trace of this in any species.

Brachialis anticus is absent in Myrmecophaga, and Tamandua; present, but small, in Cyclothurus, arising only from the lower edge of the deltoidean tuberosity, and not extending to its outer or hinder side: its ulnar insertion is behind that of the biceps (which see). In Manis it arises from the outer side of the humerus, rising higher than in the other, and is inserted partly into the radius, partly into the ulna, thus apparently compensating for the absence of a radial insertion of the biceps.

The forearms of these animals are all exceptionally powerful and muscular. The motions of supination and pronation are free; hence the muscles producing these actions are strong. The carpal flexors and extensors are not so well developed, but the digital flexor muscles are enormous.

Pronator radii teres, arising, as usual, in all, is inserted in Myrmeco-

phaga into the marginal ridge of the lower half of the radius: in Manis it extends for the lower two-thirds of that bone; in Cyclothurus for the lower third. In all its origin is a little above the inner condyle.

Flexor carpi radialis, as definite as usual, but small, arising below the last, and inserted into the base of the second metacarpal. In Cyclothurus it went to the base of the rudiment of the second digit, and is very feeble.

Palmaris longus arises in Myrmecophaga by two heads—one from the inner condyle, the other from the olecranon process. In Manis the origin is similar, and it is inserted by a single tendon into the palmar fascia. In Myrmecophaga, the tendon of insertion, passing under the strong superficial annular ligament, divided into two parts, inserted one into each side of the first phalanx of the powerful middle digit. The muscle is thus specialized as a flexor of the functional class. In Cyclothurus the two heads of this muscle were distinct for their whole extent, the olecranal portion arising from the tendon of the dorsi-epitrochlearis as well as from the olecranon, and inserted into the annular ligament, and into the fascia over the carpal "heel." The condyloid part is small, and is inserted into the fascia, sheathing the flexor tendon of the functional digit, and through it into the first phalanx. Possibly the muscle described below as flexor sublimis in Tamandua may be of this type, and this is the more probable, as it receives a slip from the olecranon and ulna similar to the second head in Myrmecophaga, but more extensive.

The flexor carpi ulnaris arises, in Myrmecophaga, like the palmaris, by two heads—olecrano-ulnar and condyloid—which continue separate for about half the length of the forearm; there they unite and form a strong tendon, which is inserted into the pisiform bone. The condyloid part gives off a slip, which is attached to the superficial annular ligament at its middle point. This portion is quite distinct from the palmaris. In Manis, this muscle is also double, but the upper larger part arises from both the inner condyle and the olecranon, while the lower arises from the shaft of the ulna along its inner side. Both are inserted together into the pisiform bone.

As the pisiform bone is of very large size in Cyclothurus, so this muscle is proportionally large, and even more complexly divided. It consists of three distinct parts—1st. A condyloid part similar to the condyloid head in

Myrmecophaga; this is inserted by two slips—one into the front and the other into the lower side of the pisiform; 2ndly, an olecrano-ulnar part, like the second part in Myrmecophaga, but much larger, and extending down to the carpal end of the ulna; this muscle is inserted under the second tendon of the foregoing; 3rdly, an ulnar part, like the second part in Manis, but arising rather lower (lower one-third), and from the outer (dorsal) side of the bone. This portion is inserted into the extremity of the pisiform, and is the "Abaisseur du pisiforme" of Cuvier.

Flexor digitorum sublimis is quite distinct in Cyclothurus, arising from the inner condyle, and ending in a single tendon, which splits, to allow the long tendon for the third toe to traverse it, while it ends in the sides of the second phalanx of that toe. Mr. Galton describes a separate perforated flexor for the second toe, which I did not see. In Myrmecophaga, there is no flexor sublimis, nor is there in Manis. In Tamandua, however, there is a flexor sublimis present, modified into a tensor of the annular ligament, arising from the inner condyle and inserted into the deeper annular ligament, and strengthened by a second olecrano-ulnar slip.

The flexor profundus digitorum in Myrmecophaga is a very large muscle with five heads—one, already described, from the tendon of the inner head of the triceps, one from the upper two-thirds of the radius, one from the olecranon, one from the shaft of the ulna, and one from the inner condyle of the humerus. All these heads unite and form a single fleshy belly, which, passing under the deep annular ligament, ends in three tendons—one central, enormous, for the third digit; one smaller, to the fourth toe; one still smaller, to the index, and a very fine slip to the pollex. In Cyclothurus, there are three parts in the common deep flexor muscle—one from the condyle, one from the upper part of the shaft of the radius, and a third from the ulna and from the lower end of the outer edge of the radius: these three unite, and send off two tendons, which are distributed to the third and fourth digits, that for the third being much the larger. The condyloid portion of the palmaris longus is closely related to the surface of this muscle, lying parallel to the flexor sublimis, and it is described by Mr. Galton as forming a part of this muscle. (As this portion is in the exact position of Rapp's Spannmuskel des ligamentum annulare, it is most likely of the same

nature; but the last named is much more closely applied to the flexor profundus, and hence might possibly be a flexor sublimis. I have mentioned it under that head as doubtful, but believe that the condyloid palmaris of Cyclothurus and the tensor of Tamandua are of the same nature.) A strong tendinous vinculum, arising from the cuneiform bone, is attached to the tendon going to the third digit.

In Manis, there is a cartilaginous sesamoid in the palm, into which the tendon of the muscle passes, and from which the digital tendons spring. No similar sesamoid exists in the other Anteaters, nor in Orycterope: its presence in Manis is interesting, as it is characteristic of all the cataphractous Edentates.

There are two lumbricales in Myrmecophaga—one (on the radial side of the large tendon) for the index, and one for the middle finger, both springing from the middle strong tendon, and passing to the dorsal and lateral aponeurosis of the digits. In Manis, there are also two, passing to the minimus and medius respectively. In Cyclothurus there are two—one from the tendon for the third toe, and inserted into the end of the small fourth; and a second from the tendon for the second toe, inserted into the first phalanx of the third toe (outer side).

The short muscles of the hand are very well developed. In Myrmecophaga, there is a small superficial flexor of the fifth digit, arising from the annular ligament on its inner (ulnar) side by a tendon: this small muscle gives off a slip to the ulnar side of the lumbricalis for the medius. No such arrangement was found in any of the others. The minimus also is provided with a pyriform abductor, arising from the pisiform, and a deeper flexor brevis from the same source. In Manis, the minimus is provided with an abductor along its inner (ulnar or postaxial) edge, arising from the carpus and base of the metacarpus, and an adductor, palmar interosseous muscle along its preaxial border. In Cyclothurus, the minimus is absent, but a large mass of muscle arose from the inferior and postaxial side of the long pisiform, which went to the inner side of the manus: this represented, probably, the adductor minimi, but acts on the pisiform, not on the hand.

For the pollex in Cyclothurus there is a smaller muscle developed, which comes over the tendons in the hand, lying on a plane superficial to the an-

nular ligament: it is inserted into the preaxial side of the manus and rudimentary trapezium. In Myrmecophaga, there is a short abductor from the trapezium and annular ligament; a small flexor brevis pollicis, and an adductor pollicis, both arising from the fibrous tissue covering the palmar surface of the carpal bones, rather than from any of the latter individually. In Manis, there is an adductor pollicis from the scapho-lunar bone to the first phalanx of that digit, but no other polliceal muscle.

For the other digits there are the following interosseous muscles developed (in Myrmecophaga) an abductor indicis with an intermetacarpal origin: (bicipital in Manis), and an oblique palmar adductor indicis arising from the unciform process (from magnus in Manis). The medius has a superficial adductor arising from the unciform, and a deep adductor from the front of the metacarpal. There is only one such in Manis, representing the second of these, but with a double metacarpal origin from the 3rd and 4th metacarpals. There is also an abductor medii for the 1st and 2nd metacarpals in both Manis and Myrmecophaga. The annularis has also in both an abductor with an intermetacarpal two-headed origin, and an adductor, arising from the 4th metarcarpal (Myrmecophaga), or from the os magnum (Manis). In Cyclothurus, there was a flat, thick palmaris brevis, absent in the others, arising from the pisiform between the abductors of the thenar and hypothenar edges of the manus, and inserted into the middle of the palm; it was slightly connected to the two lateral muscles. Under the tendons were the following: 1st, abductor indicis, a very slender muscle, with a very small origin from the trapezius, inserted into the preaxial side of the index; 2ndly, adductor indicis, consisting of two slips, one arising from the unciform, the other from the bones of the middle metacarpal bone, both inserted together into the ulnar side of the first phalanx of the index; 3rdly, adductor medii, large, from the unciform to the inner side of the first phalanx of this digit; 4thly, adductor quarti digiti, very small, to the rudiment of the ring finger.

The pronator quadratus is absent in Manis, but in Myrmecophaga it occupies the whole length of the forearm below the radial tuberosity. The range of motion of the forearm bones was very considerable; after the removal of the humerus, the radial head was found capable of rotating

through an arc of 70°. There are several fat masses interposed between the forearm bones at the lower end, and there is no ulnar interarticular fibro-cartilage. The pronator quadratus in Cyclothurus also stretches for the entire extent of the forearm, its fibres having a downward and outward trend; it is also as large in Tamandua.

The supinator radii longus is very large in all. In Myrmecophaga it arises high above the outer condyle, separate from the deltoid and triceps, and is inserted into the lower end of the radius, and the front of the wrist.

In Manis, it arises from a tendinous arch of the external intermuscular septum, which is slung across the notch for the musculo-spiral nerve, and extends up for a distance of one-third of the humerus. The insertion is purely radial, and only into the lower end of the bone. In Cyclothurus, this muscle reaches its maximum of development, and consists of two parts, first a superficial, wide, thick portion from the infra-deltoidean ridge of the humerus, which passes downwards to be inserted into the anterior annular ligament and palmar fascia; this part can act as a simultaneous flexor of the two joints over which it passes. The second deeper part underlies this, and arises, as in Manis, from the strong tendinous sling of the outer intermuscular septum, which bridges over the musculo-spinal nerve. This part is inserted into the lower third of the radius, opposite to the insertion of the end pronator. These two parts were continuous at their origin in Mr. Galton's specimen, more separate in mine, while in Mr. Galton's the deeper part was bilaminar, an arrangement not found in mine—one of the very numerous instances of what Mr. Parker has happily called "the reptilian looseness" of regularity in these Edentates. In Tamandua, this muscle is smaller than in Myrmecophaga. (In the latter, Pouchet seems to have overlooked its existence.)

Supinator brevis in Myrmecophaga lies deeply sunk in the external groove of the radius, in Cyclothurus it also lies deeply overlapped by the abductor pollicis major, and its insertion is, as in the larger Anteater, into the whole length of the preaxial edge of the radius. The posterior interosseous nerve preserves its constant relation to this muscle; in none of these does it arise from the ulna, but in the Manis it is very strong, and has a fabella in its origin, a rare condition which I have once seen in Man.

Extensor carpi radialis is a single muscle in all, attached to the base of the third metacarpal, hence representing the extensor carpi radialis brevior of human anatomy. It is bridged by a strong fine fibrous band in all. The end of the tendon in Cyclothurus is flattened, but not split. Extensor carpi ulnaris is a simple strong muscle, arising from the back of the outer condyle, and inserted into the back of the fifth metacarpal, it is a flat, fleshy band in Cyclothurus, inserted into the rudiment of the fourth metacarpal. Meckel found it going to the fourth and fifth (?) rudiments.

Extensor digitorum communis in Myrmecophaga arises from the condyle, as usual; at a point about one-sixth of its length below this, it gives off a slip, which passes to the back of the fourth finger and acts as an extensor for it. The rest of the muscle becomes tendinous a short distance above the wrist, and then divides into two portions, one ulnar strong flat band, which passes to the medius, and a second slender one on the radial side, which joins the indicial tendon of the extensor indicis, and is distributed chiefly to the index finger, and by a separate ulnar band to the middle finger as well. In Manis, the extensor digitorum communis has a single strong tendon, which chiefly is inserted into the middle digit. In Cyclothurus it is also inserted into the dorsum of the third digit, together with the next muscle.

Extensor digitorum brevis in Cyclothurus arises from the lower end of the ulna and from the dorsal surface of the os magnum and unciform bones, it is inserted along with the last, under the tendon of which it lies. Mr. Galton found a slip of it inserted into the index finger.

In Manis, this muscle is large and distinct, arising from the dorsum of the carpus by flat tendinous fibres; it becomes immediately fleshy, and is inserted into the terminal phalanx of the index by some fibres. These muscles may be degraded representatives of the extensor indicis, as, in the other Anteaters, this muscle is present, and in its ordinary place. In Myrmecophaga the extensor indicis arises from the back of the ulna, as usual, below the short extensors of the thumb; its tendon gives off a slender extensor tendon to the thumb, and then joins the slip already described from the extensor communis, with which it is inserted.

Extensor minimi digiti, arising from the condyle, is inserted into the fourth and fifth digits in Myrmecophaga, and in Manis it is larger pro-

portionally in Cyclothurus, where it is inserted by a wide tendon into the rudiment of the ulnar digit, as well as into the ulnar side of the third; it overlaps the extensor digitorum communis.

Extensor ossis metacarpi pollicis (abductor pollicis major) arises from a ridge on the upper and back part of the ulna, and from the surface of the bone in the radial side of the ridge. It is inserted into the base of the first metacarpal, as well as into the adjacent surface of the trapezium. Most of the tendon was inserted into the first-named place, which was not the case in Pouchet's specimen, in which the majority of the fibres went to the trapezium as in Tamandua. In the Cyclothurus, this muscle is very large, and is inserted into the rudiment of the trapezium entirely. It is similar in Manis.

Extensor secundi internodii pollicis is a very small muscle in Myrmecophaga and Manis, but is distinct in both, not in any of the others.

EXPLANATION OF PLATES.

PLATE XXVI.

Myrmecophaga jubata.

Fig. 1. MUSCLES OF SIDE OF FACE.

a. Tendons of Levator tubii superioris alæque nasi.
b. Malar bone.
c. Lachrymal gland.
d. Temporal.
e. Fibrous zygoma.
f. External meatus.
g. Digastric.
h. Pterygoid.
i. Masseter.
j. Buccinator.
k. Mylo-hyoid.
l. Buccal glands.
m. Buccal fibres.

Fig. 2. DISSECTION OF NECK.

a. Supraspinatus.
b. Descendens noni nerve.
c. Superior laryngeal nerve.
d. Trapezius.
e. Pneumogastric nerve.
f. Spinal accessory.
g. Sterno-mastoid (cut and reflected).
h. Superior occipital nerve.
i. Splenius.
j. Portio dura nerve.
k. External maxillary.
l. Parotid.
m. Temporal.
n. Mucous membrane of mouth (cut and displaced).
o. Tongue.
p. Sublingual gland.
q. Gustatory nerve.
r. Facial artery.
s. Tympano-hyal.
t. External carotid.
u. Hypoglossus.
v. Stylo-pharyngeus.
w. Sterno-glossus.
x. Hypoglossal nerve.
y. Stylo-hyal.
z. Stylo-hyoid muscle.
a.' Cerato-hyal.
b.' Hyo-ceratic muscle.
c.' Hyoid bone.
d.' Thyro-hyoid muscle.
e.' Thyroid cartilage.
f.' Laryngeal pouch.
g.' Crico-thyroid muscle.
h.' Trachea.

Fig. 3. Muscles of back of Sternum.

a. & *d.* Sterno-glossus.
b. & *e.* ,, thyroid.
c. Sternum (viewed from within).
f. Triangularis sterni.

Fig. 4. Muscles of the Sole.

a. Flexor hallucis brevis.
b. Abductor hallucis.
c. Sesamoid.
d. Extensor digitorum communis.
e. Os calcis.
f. Calcaneo-cuboid.
g. Cuboid.

Fig. 5. Muscles of Leg.

a. Extensor digitorum longus.
b. Tibialis anticus.
c. Extensor hallucis longus.
d. Extensor indicis longus.
e. ,, digitorum brevis.
f. Fifth toe slip to last muscle (*e*).

Fig. 6. Muscles of Arm.

a. Palmaris longus.
b. Pronator teres.
c. Supinator longus.
d. Brachialis coracoidalis.
e. Biceps longus.
f. Pectoralis minor.
g. Suprascapular nerve.
h. Supraspinatus.
i. Insertions of serratus magnus.
j. Coraco-brachialis.
k. Subscapularis.
l. Teres major.
m. Latissimus dorsi.
n. Dorso-epitrochlear.
o. Anconeus internus.
p. Supra-condyloid.
q. Flexor carpi ulnaris.

Plate XXVII.

Manis multiscutatus.

Fig. 1. Fore Leg.

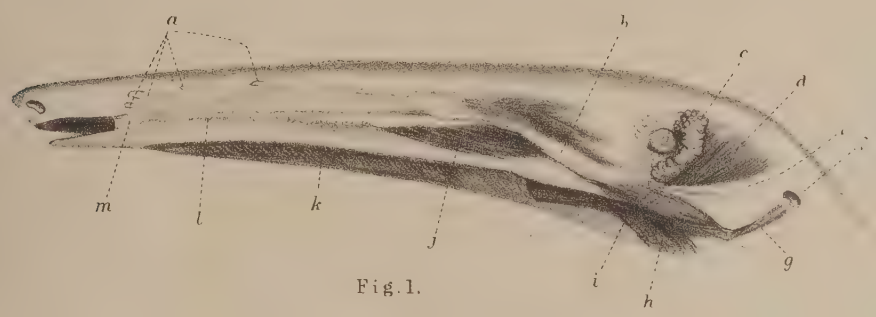
a. Trapezius.
b. Latissimus dorsi.
c. Teres major.
d. Triceps externus.
e. Deltoid.
f. Ulna.
g. Extensor carpi ulnaris.
h. Extensor minimi digiti.
i. Extensor digitorum communis.
j. Trachelo-acromial.
k. Masto-humeralis.
l. Sterno-mastoid.

Fig. 2. Hind Leg.

a. Gastrocnemius externus.
b. Vastus externus.
c. Rectus femoris.
d. Gluteus medius.
e. Pyriformis.
f. Quadratus femoris.
g. Semitendinosus.
h. Biceps Femoris.

Fig. 3. Tongue and Sternum.

a. Buccal membrane.
b. Tongue.
c. Sheath of tongue.
d. Sterno-glossus.
e. Presternum.
f. Sterno-costalis.
g. Sterno-glossus.
h. Xiphisternal cornua.
i. Fusion of sterno-glossi.
j. Right Xiphisternal cornu.
k. Accessory rib, cartilages appended thereto.



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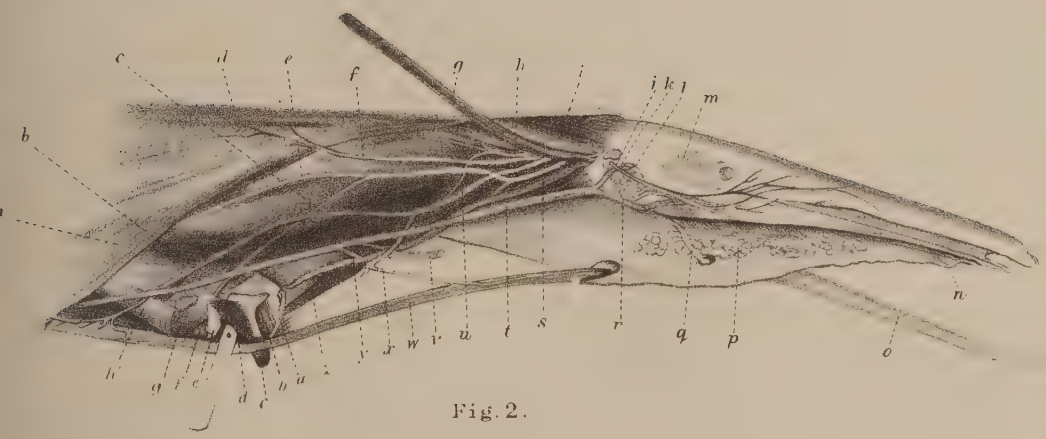
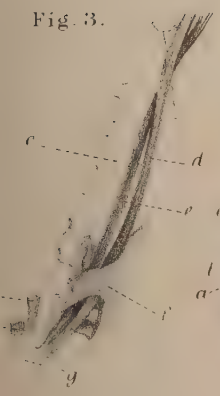


Fig. 2.

Fig. 3.



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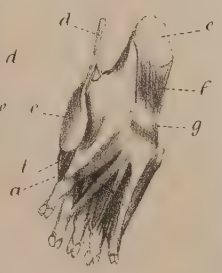

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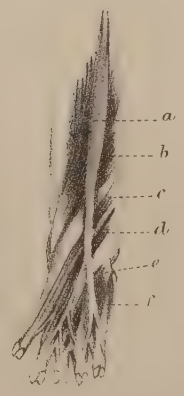
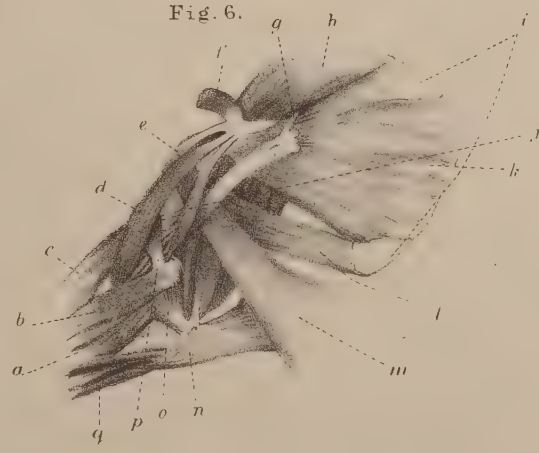


Fig. 6.



del. G. H. Ford lith.

Mintern Bros. imp.

Myrmecophaga jubata.

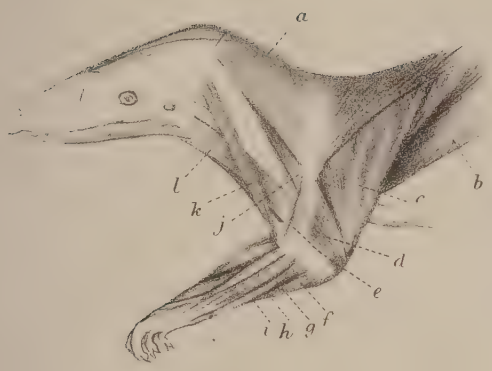


Fig. 1.

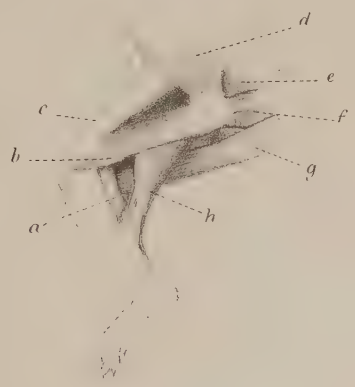


Fig. 2.

Fig. 3.

del. G.H.Ford lith.

Mintern Bros imp

Manis multiscutatus.

XV.—*On the Seychelles Fern Flora.* *By* J. G. Baker, F. L. S., Royal Herbarium, Kew.* [With Plates XXVIII. to XXXI.]

Read June 22, 1874.

The following is a complete list of the species known as having been found in the Seychelles Islands, with an outline of their general distribution. A note of admiration denotes that specimens, gathered by the collector, whose name it follows, have been seen by the author.

Gleichenia.

G. dichotoma, Willd. Hooker et Baker, Syn. Fil., second edition (quoted throughout), p. 15. Ward! Kersten:—Bourbon, Madagascar, Mauritius, Comoro isles, Natal, Guinea, and throughout Tropical Asia and America. [In the uncultivated regions of many of the Islands, especially Mahé, I found this beautiful species very abundant.—E. P. W.]

Cyathea.

C. sechellarum, Mett. Hooker et Baker, Syn. Fil., p. 22. Pervillé! Ward! The only true tree fern of the group. An endemic species. [This species is not uncommon in Mahé and Praslin. There is quite a forest of it on the mountain footpath leading from Port Victoria across Mahé. Some of the stems attain to a height of from twenty to thirty feet; and in one valley in the Island of Praslin, I saw specimens probably from forty to fifty feet in height. In a portion of land at Port Victoria, which had been burnt about

* With notes on some of the species, by E. Perceval Wright, M. D., F. L. S.

twenty years previous to my visit, there were some young tree ferns, upwards of fifteen feet high, indicating that the growth of Tree-fern stems is not so slow as is often imagined.]

Hymenophyllum.

H. ciliatum, Sw. Hooker et Baker, Syn. Fil., p. 63. Ward! Johanna Island, Mauritius, Madagascar, Bourbon, Himalayas, New Zealand, common in Tropical America.

H. polyanthos, Sw. Hooker et Baker, Syn. Fil., p. 60. Horne! Widely diffused through the Tropics, and extending a little beyond them both north and south.

H. elasticum, Bory. Hooker et Baker, Syn. Fil., p. 64. Horne! Mauritius, Bourbon, and Seychelles, where it is abundant. It is questionable if this differs from *H. hirtellum*, Sw.

Trichomanes.

T. rigidum, Sw. Hooker et Baker, Syn. Fil., p. 86. Pervillé! Wright! Natal, Cape Colony, Johanna Island, Madagascar, Mauritius, Bourbon, and throughout Tropical Asia, northward to Japan, southward to New Zealand, and throughout Tropical America.

T. muscoides, Sw. Hooker et Baker, Syn. Fil., p. 75. Horne! Tropical America, Asia, Polynesia, and Africa.

T. digitatum, Sw. Hooker et Baker, Syn. Fil., p. 76. Horne! Mauritius, Bourbon, Java, and Polynesian Islands.

Davallia.

D. repens, Kuhn, Fil. Afr., p. 158. *D. pedata*, Sm. Hooker et Baker, Syn. Fil., p. 89. Wright! Mauritius, Bourbon, through Tropical Asia to Japan and Queensland.

D. denticulata, Mett. Kuhn, Fil. Afr., p. 157. *D. elegans*, Swartz. Hooker et Baker, Syn. Fil., p. 95 (including *D. nitidula* and *Vogelii* as

varieties). Kersten, Natal, Guinea, Angola, Madagascar, Johanna Island, and common in Tropical Asia.

D. Speluncæ, Baker. Hooker et Baker, Syn. Fil., p. 100. Ward! Natal, Madagascar, Bourbon, Congo, Guinea, and through Tropical Asia and America.

D. tenuifolia, Swartz. Hooker et Baker, Syn. Fil., p. 102. Pervillé! Madagascar, Mauritius, Bourbon, Comoros, Natal; common in Tropical Asia and Polynesia.

Lindsaya.

L. Kirkii, Hook., MSS. Hooker et Baker, Syn. Fil., p. 108. *L. Pervillei*, Mett. et Kuhn, Fil. Afr., pp. 17 et 68. Kirk! Bouton! Pervillé! Kersten. Lady Barkly! Rawson! Wright! A very distinct and beautiful species, only known here. [This fine species, named after Dr. Kirk, Vice-Consul at Zanzibar, is not uncommon on the sides of the mountains at Mahé. As it has never been figured, the accompanying drawing by Mr. Fitch (Plate XXVIII.), will not be without interest].

Adiantum.

A. caudatum, L. Hooker et Baker, Syn. Fil., p. 115, including *A. rhizophorum*, Swartz, as a variety. Kersten, Pervillé (both the type and variety). Mauritius, Bourbon, Madagascar, Comoro Isles, Zambesi Land, Guinea, Angola, common in Tropical Asia, rare in Tropical America.

Lonchitis.

L. pubescens, Willd. Hooker et Baker, Syn. Fil., p. 128. Ward! Guinea, Cape Colony, Natal, Mauritius, Bourbon, Comoro Isles; rare in Tropical America, not known in Tropical Asia.

Pellæa.

P. Barklyæ, Baker. Hooker et Baker, Syn. Fil., p. 151. Lady Barkly!

Dr. Kirk! A very distinct species, not known elsewhere (Plate XXIX.)

P. hastata, Link. Hooker et Baker, Syn. Fil., p. 152. *P. elliptica.* Lam. Kuhn, Fil. Afr., p. 80. Kirk! Cape Colony, Natal, Zambesi Land, Mauritius, Bourbon, Johanna Island, Abyssinia, Cape Verdes, Angola; only known in Africa.

PTERIS.

P. quadriaurita, Retz. Hooker et Baker, Syn. Fil., p. 158. Horne! All round the World within the Tropics, and a little beyond them; varies much, especially in size.

P. biaurita, L. Hooker et Baker, Syn. Fil., p. 164, including *P. nemoralis*, Willd. as a variety. Kersten teste Kuhn. Bourbon, Mauritius, Guinea, Johanna Island, Angola, and through Tropical Asia and America.

P. incisa, Thunb. Hooker et Baker, Syn. Fil., p. 172. Pervillé! Ward! Cape, Madagascar, Bourbon, Mauritius, Tristan d'Acunha, Ascension Island, to Van Diemen's Land, Chili, China, and W. Indies.

P. tripartita, Swartz. Hooker et Baker, Syn. Fil., p. 172. *P. marginata*, Bory. Kuhn, Fil. Afr., p. 84. Perville! Wright! Guinea, Mauritius, Bourbon, Comoros, and through Tropical Asia to Erromanga and Queensland.

CERATOPTERIS.

C. thalictroides, Brong. Hooker et Baker, Syn. Fil., p. 174. Horne! Throughout the Tropics in quiet waters.

ASPLENIUM.

A. Nidus, L. Hooker et Baker, Syn. Fil., p. 190. Horne! Mauritius, Johanna, Seychelles, Himalayas, Japan, to Society Islands, New Caledonia, Queensland, Norfolk and Lord Howe's Islands.

A. tenerum, Forst. Hooker et Baker, Syn. Fil., p. 201. Given by Kuhn as a Seychelles species, and gathered lately by Horne. Guinea, Diego Garcia (Lady Barkly!), Ceylon, Malaya, Polynesia.

A. hirtum, Kaulf. Hooker et Baker, Syn. Fil., p. 205. *A. pellucidum*,

Lam. Kuhn, Fil., Afr., p. 110. Kirk! Bourbon, Mauritius, Seychelles, Madagascar, Comoros, Tropical Asia.

A. caudatum, Forst. Hooker et Baker, Syn. Fil., p. 209. Horne! Polynesia, Australia, Comoros, Brazil.

A. resectum, Smith. Hooker et Baker, Syn. Fil., p. 210. Kirk! Guinea, Angola, Mauritius, Bourbon; common in Tropical Asia, and Polynesia.

A. cuneatum, Lam., Hooker et Baker, Syn. Fil., p. 214. Kirk! Natal, Mozambique, Johanna Island, through Tropical Asia and America.

A. bulbiferum, Forst. Hooker et Baker, Syn. Fil., p. 218. Kirk! Natal, Johanna Island, through Tropical Asia to Mexico.

A. sechellarum, Baker. (*A polypodioides*, Mett. Hooker et Baker, Syn. Fil., Edit. 1, p. 238). Arborescent; stem and rachises brown-stramineous, glabrous, muricated; basal scales firm, lanceolate, brown, $\frac{3}{4}$ inches; frond 5 to 6 feet long; subdeltoid, tripinnatifid; pinnæ oblong-lanceolate, $1\frac{1}{2}$ to 2 feet long; pinnules ligulate-lanceolate, sessile or short-stalked, 2 to $3\frac{1}{2}$ inches long, $\frac{3}{4}$ to 1 inch broad, cut down to a narrow wing into close ligulate toothed segments, 1 to $1\frac{1}{2}$ inches broad; texture, subcoriaceous; surfaces bright green, naked; veins, 8 to 12 jugate, simple, with sori running nearly from the mid rib to edge. Seychelles frequent in damp shady woods. Bouton! Horne! 1872. Madagascar, Boivin. Trunk 6 feet high.

Actiniopteris.

A. radiata, Link. Hooker et Baker, Syn. Fil., p. 246. Pervillé! Angola, Zambesi Land, Macalisberg, Abyssinia, Johanna, Mauritius, Bourbon, Tropical Asia.

Aspidium.

A. capense, Willd. Hooker et Baker, Syn. Fil., p. 254. *A. coriaceum*, Swartz. Ward! Cape, Natal, Mauritius, Bourbon, Tristan d'Acunha, Polynesia, New Zealand, Australia, Tropical America, Chili.

Nephrodium.

N. Hornei, Baker. Hooker et Baker, Syn. Fil., p. 500. Stem and rachises naked, glossy stramineous; frond deltoid, 2 to 3 feet each way;

4-pinnatifid; lower pinnæ much the largest, deltoid, produced on lower side; lowest pinnules and segments close, lanceolate, acute, stalked, or the latter winged to base; final lobes close, oblong, blunt, 2 to 3 lines broad, inciso-crenate; texture membranous; surfaces bright green, glabrous; veins pinnate in final lobes; sori minute submarginal, restricted to the lobes; involucre glabrous, sub-persistent. Named after its discoverer, Mr. Horne, Seychelles, Horne, 182!

N. (*Lastrea*) *Wardii*, Baker. n. sp. (Plate XXX.), received from Swinburne Ward, Esq., and also from Lady Barkly, gathered by Mr. Neville. Frondibus deltoideis quadripinnatifidis membranaceis utrinque viridibus glabris, rachibus, omnium graduum griseis glabris politis fragilibus, pinnis lanceolatis distincte petiolatis latere inferiore subreducto infimis maximis, pinnulis lanceolato-deltoideis latere inferiore subreducto basi cuneato-truncato, infimis distincte petiolatis lateralibus aquantibus, segmentis inequilateraliter lanceolato-deltoideis profunde pinnatifidis, dimidio inferiore reducto basi cuneato-truncato, venis pinnatis, venulis ultimis erecto-patentibus, soris prope costas segmentorum subuniseriatis. Root, scales, and stem not seen. Rachises grey, polished, quite glabrous, and scaleless; very fragile. Frond deltoid (in Lady Barkly's specimen, which may not represent the full size of the plant, measuring a foot and a half each way), thrice pinnate, and then once pinnatifid, quite glabrous on both sides, dark green above, rather paler beneath. Pinnæ lanceolate, shortly stalked, the lowest the longest, nearly a foot long by four inches broad; the lower side a little smaller than the upper, and its pinnæ placed higher on the rachis. Pinnules above a dozen pair, below the uppermost simple ones, lanceolate deltoid; the lower side decidedly cuneato-truncate at the base, only the lowest anterior lobe toothed. Veins pinnate in the lowest anterior lobe, forked in the upper ones, with a single sorus at the base of each on the short anterior vein, so that they form a row near the midrib of the segment. Sori of medium size, not in good condition in the specimens. This appears to be such a well-marked species that we venture to describe it, though several of its characters still remain to be worked out from other specimens than those already obtained. In general habit it takes rank with

N. Boryanum and *N. catopteron.* Hooker et Baker, Syn. Fil., p. 284, but may be readily distinguished from both by its fragile, membranous, perfectly glabrous fronds, with lower pinnules and segments not more than twice as long as broad, and decidedly unequal-sided, and cut away at an angle of 45° with the rachis on the lower side at the base.

N. cucullatum, Baker. Hooker et Baker, Syn. Fil., p. 290. Kersten, Wright! Mauritius, Bourbon, Madagascar, Comoro Isles, and through Tropical Asia to Fiji.

N. molle, Desv. Hooker et Baker, Syn. Fil., p. 293. Horne! South America, East Indies, Madeira, to Cape Town; Mascaren Islands, Seychelles, Himalayas to N. S. Wales.

N. pleiotomum, Baker. (Plate XXXI. B.) Hooker et Baker, Syn. Fil., p. 297. Seychelles, Wright! Horne! Madagascar, Boivin.

Nephrolepis, Schott.

N. acuta, Presl. Hooker et Baker, Syn. Fil., p. 301. Pervillé! Kersten. Guinea, Natal, Angola, Mauritius, Comoros, Bourbon, Madagascar, and through Tropical Asia to America.

Oleandra, Car.

O. articulata, Cav. Hooker et Baker, Syn. Fil., p. 302. Ward! Pervillé! Guinea, Angola, Natal, Madagascar, Mauritius, Bourbon. Not known in Asia or elsewhere.

Polypodium.

P. Pervillei, Mett., Kuhn. Fil. Afr., p. 150. (Plate XXXI. A.) Pervillé! Wright! A new species not known elsewhere, described from Mr. Pervillé's specimens by Mettenius, and published lately by Dr. Kuhn, and now regathered by Prof. Wright. It comes next to *P. subevenosum*, Baker, Hooker et Baker, Syn. Fil., p. 320, from Penang, having narrowly strap-shaped simple fronds, not more than a line broad, with simple inconspicuous veins, and round sori in a row close to the midrib. [I found this near Port Victoria, Mahè]

P. serrulatum, Mett. Hooker et Baker, Syn. Fil., p. 323. Horne! West Indies and Mexico to Brazil, Peru, and Juan Fernandez, Sandwich Isles, Madagascar, Mauritius, Seychelles, Guinea Coast.

P. adnascens, Sw. Hooker et Baker, Syn. Fil., p. 349. Given by Kuhn as a Seychelles species. A plant of Mauritius and Rodriguez, but very rare out of Tropical Asia, where it is plentiful, and widely diffused.

P. irioides, Lam. Hooker et Baker, Syn. Fil., p. 360. Horne! North India and Chusan to Fiji, N. S. Wales, Natal, Mascaren Isles, Zambesi Land, Seychelles.

P. Phymatodes, L. Hooker et Baker, Syn. Fil., p. 364. Kersten, Wright! Guinea, Natal, Angola, Zambesi Land, Mauritius, Bourbon, Comoros, Madagascar, and through Tropical Asia and Polynesia.

P. sechellarum, Baker. Hooker et Baker, Syn. Fil., p. 508. Stem tufted, $\frac{1}{2}$–1 in., clothed with fine short spreading brown hairs; frond pendulous, ligulate, 3–6 inches long, 6–7 lines broad, blunt, narrowed suddenly at base, cut down to a narrow wing into oblong blunt lobes, 2–$2\frac{1}{2}$ lines broad; texture rigidly subcoriaceous; surfaces thinly clothed, especially at the edge, with fine soft brown hairs; veins 4–6 jugate in pinnæ, not reaching the edge; sori round, 6–8 to a lobe, midway between costa and margin. Mahè, Seychelles, Horne! Mauritius, Lady Barkly.

Monogramme, Schk.

M. graminea. Schk. Hooker et Baker, Syn. Fil., p. 375. Pervillé, teste Kuhn. Cape Colony, Mauritius, Bourbon, not known in Tropical Asia.

Antrophyum, Kaulf.

A. immersum, Mett. Hooker et Baker, Syn. Fil., p. 393. Horne! Mauritius, Bourbon, Seychelles, Malaya.

Vittaria, Sm.

V. elongata, Sw. Hooker et Baker, Syn. Fil., p. 395. Pervillé! Ward!

Wright! Johanna, Bourbon, Guinea, Mauritius, and through Tropical Asia.

V. scolopendrina, Thwaites. Hooker et Baker, Syn. Fil., p. 396. Pervillé! Kirk! Mozambique, Bourbon, Madagascar, Tropical Asia.

ACROSTICHUM, L.

A. latifolium, Sw. Hooker et Baker, Syn. Fil., p. 403. Ward! Rare in Tropical Asia. Common in Tropical Asia. Not known in Africa.

A. viscosum, Sw. Hooker et Baker, Syn. Fil., p. 406. Ward! Guinea, Angola, Mauritius, Bourbon, Tropical Asia, Tropical America.

A. spathulatum, Bory. Hooker et Baker, Syn. Fil., p. 408. Horne! Cuba and Mexico to Brazil and Peru; Tristan d'Acunha, Ceylon, Seychelles, Mascaren Isles, Natal.

A. sorbifolium, L. Hooker et Baker, Syn. Fil., p. 412. Pervillé! Guinea, Madagascar, Mauritius, Bourbon, Tropical Asia, and America. Mettenius and Kuhn regard the Seychelles plant as an endemic species, and call it *Lomariopsis Pervillei*, Kuhn, Fil. Afr., p. 53.

A. repandum, Blume. Hooker et Baker, Syn. Fil., p. 419. Boivin, Bouton! Wright! Bourbon, and through Tropical Asia to Queensland and New Caledonia. Mettenius and Kuhn regard the Seychelles and Bourbon plant as distinct from the Asiatic one, and call it *Chrysodium bipinnatifidum*, Mett. *Vide* Kuhn's Filices Africani, p. 50.

A. aureum, L. Hooker et Baker, Syn. Fil., p. 423. Horne! Florida and Bermudas to Peru and South Brazil, Polynesia, Bengal, Hong Kong, and Formosa to Queensland, Seychelles, Zambesi, Natal, Guinea Coast.

A. spicatum, L. Hooker et Baker, Syn. Fil., p. 424. Horne! Himalayas (to 6000 feet) to Ceylon, Madagascar, and Mascaren Islands, South China, Philipines, Queensland, Seychelles, Society Islands.

PLATYCERIUM.

P. alcicorne, Desv. Hooker et Baker, Syn. Fil., p. 425. Temperate Australia, and a larger form in the Seychelles and Mascaren Isles.

Schizæa.

S. pennula, Sw. Hooker et Baker, Syn. Fil., p. 430. Horne! West Indies to Rio, New Caledonia, Isle of Pines, Seychelles.

S. digitata, Sw. Hooker et Baker, Syn. Fil. p. 430. Wright! Seychelles, and through Tropical Asia to Fiji.

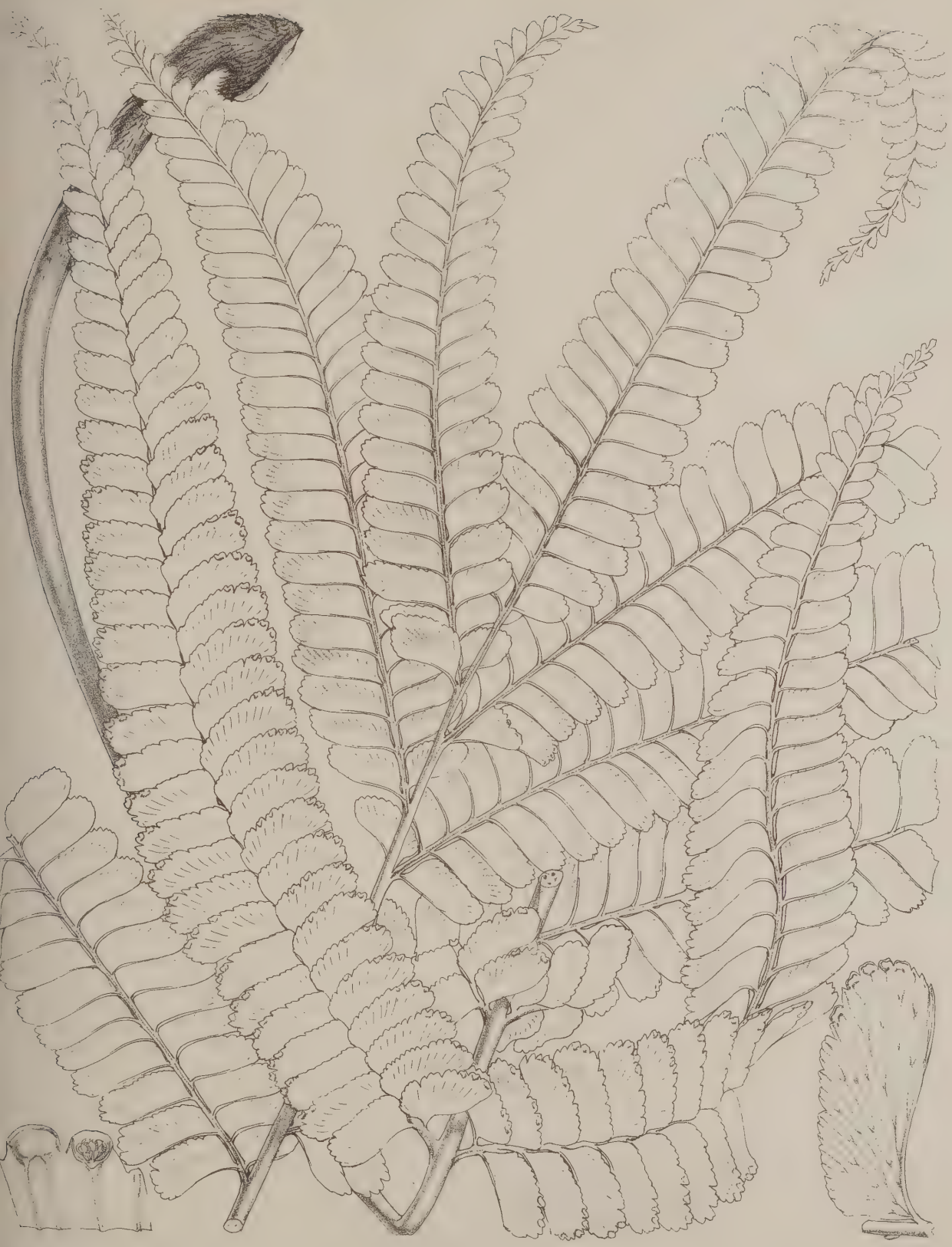
Angiopteris.

A. erecta, Hoffm. Hooker et Baker, Syn. Fil., p. 440. Horne! Himalayas to Japan and Ceylon, Seychelles, Madagascar, New Caledonia, Queensland, and Society Isles.

Ophioglossum.

O. pendulum, L. Hooker et Baker, Syn. Fil., p. 446. Horne! Polynesian Isles to N. S. Wales, Assam, Malaccas, Philippines, Ceylon, Seychelles, and Mascaren Isles.

O. palmatum, L. Hooker et Baker, Syn. Fil., p. 446. Pervillé, *teste* Kuhn. Common in Tropical America, not elsewhere known, except in the Seychelles, and Bourbon.



W H Fitch, del. et lith

J N. Fitch, imp.

Lindsaya Kirkii, *Hook.*

W. H. Fitch, del. et lith.

J. N. Fitch, imp.

Pellœa (Allosorus) Barklyæ, *Baker*.

W.H. Fitch, del et lith.

J.N. Fitch, imp

Nephrodium (Lastrea) Wardii, *Baker.*

Fitch, del. et lith.

J.N.Fitch imp

A. Polypodium Pervellei, *Mett.*

B. Nephrodium (Sagenia) pleiotomum *Baker.*

XVI.—*Researches on the Structure of the Spines of the Diadematidæ (Peters).* *By* H. W. Mackintosh, B.A. [With Plates XXXII.–IV.]

Read January, 1875.

The systematic investigation of the structure of the spines of the regular Echinoidea (Desmosticha, Haeckel), to settle as definitely as possible the extent and nature of variation in the genus, species, or individual, and thus to determine how far, if at all, we may look to them to afford useful characters in classification, is a question which has hardly received the attention that it merits. I was induced to take up the subject some eighteen months ago, by both Dr. Macalister and Dr. Perceval Wright, under whom my studies in biology have been carried on, and at first in a desultory sort of way, but latterly more systematically, have devoted the greater part of my leisure to it.

The order is not a very large one, including, according to Professor A. Agassiz,* only one hundred and twelve species, contained in thirty-nine genera; but the investigation is a somewhat tedious process, as all the specimens are prepared by myself, and I have been anxious to examine as large a number as possible, in order to obtain as wide a basis of observation as I could. Hence I willingly adopted a suggestion offered to me by Dr. Wright, to work them out by families, rather than attempt the whole order at once; and as I have been enabled to procure specimens of all the genera, and eight out of the twelve species of Diadematidæ, it appeared to be the most favourable group to select. I hope to have the honour of laying before the Academy, at future periods, notes of researches into the remaining families of the order, according as occasion offers. I take this opportunity of

* Revision of the Echini, 1872-3. I am indebted to the kindness of Dr. Wright for opportunities of consulting this work, no Dublin library possessing a copy.

acknowledging the great assistance I have received from Dr. Macalister and Dr. Wright, not only in the way of valuable advice and encouragement, but also for many specimens of the various species. I am also indebted to Dr. Carte, Director of the Royal Dublin Society's Museum, for specimens of the spines of Centrostephanus, Diadema, and Echinothrix. Professor Grube of Breslau has also very kindly sent me some spines of the rare *Asthenosoma varium.*

The descriptions which follow are based upon the examination of more than one hundred and fifty sections, taken from various parts of different spines of all the specimens I could get, which, however, were not nearly so numerous as I would wish; for conclusive inferences in an investigation of this kind can only safely be drawn on very extensive data: and hence I would remark at the outset, that the results deduced are by no means to be regarded as decisive, but simply as the consequences following on the examination of the material at my disposal; wider investigations may either confirm or overturn my present views as to the taxonomic value of spine-structure, and hence I will divide the paper into two parts, taking up in the first the descriptions of the various forms of the spines, and dealing briefly in the second part with the inferences which may be drawn from them. The usual terminology will be adopted throughout; thus the cavity of the spine whereby it articulates with the tubercle of the corona is the "socket;" next, there is the "head," which extends up to the "milled ring," which is immediately succeeded by the "shaft" (the "neck," or part next to the milled ring, not being differentiated in the Diadematidæ), and this may be conveniently divided into proximal, median, and distal regions. The spines examined were exclusively those called "primary interambulacral" by Professor Agassiz, and were all taken from authentically named specimens. The mode of preparing the sections is briefly detailed at the end of the descriptions.

Descriptions of the Spines.

Before giving an account of my own work, a few words as to what has been done in this department by others may not be out of place. Although

every book on the microscope contains allusions more or less lengthy to the nature and structure of the spines of Sea Urchins, it is more on account of their beauty, than with any idea of detailing the different forms to be met with, and they are mostly genera belonging either to Cidaridæ or Echinometridæ which are described. The late Professor Quekett, in his "Lectures on Histology" (1852), has figured on page 225 what seem to me to be sections of the spines of *Diadema setosum*, but mentions them as being taken from a hollow spine of an Echinus; Cidaris and Echinus, however, are the only two generic names with which he appears to be familiar, including under the latter such well-known forms as Heterocentrotus (Acrocladia) and Echinometra. Professor Agassiz devotes part of a chapter of his "Revision" to the spines of the whole order, but deals mainly with Cidaridæ, and dismisses the Diadematidæ in about twenty lines. He illustrates the subject by two plates of photographs, but these are very poor; either the sections were much too thick, or the artist was not very *au fait* at his work: whatever be the reason, the illustrations in this part of the work contrast strongly with the magnificent photographs which embellish the other parts of the volume, and are invaluable aids to identification of species. Professor Agassiz remarks that the spines of the Diadematidæ exhibit a combination of the concentric and cellular modes of growth; near the base there are two rings, the inner composed of spokes of small cells (reticulated tissue), irregularly arranged between the large solid outer wedges; this is then succeeded by a ring of irregularly arranged cells, edged by a row of small solid wedges; higher up on the shaft the solid wedges have become connected, and are separated by the spokes of minute cells. He then adds a few observations with regard to the solid wedges, to which I shall have occasion to allude farther on. Whether knowingly or not, Professor Agassiz has taken his description of the basal part of the spine from a section through the milled ring, and therefore there seems to be a great difference between the structure in that part, and in the section taken higher up; whereas, if the section had been from the basal part of the shaft, and not the milled ring, the "outer ring of irregularly arranged cells" would have disappeared, since it is nothing but the milled ring itself. A glance at the

photographs from which the descriptions are taken* renders this quite clear, and shows no greater difference than would be expected from tracing the growth of the solid wedges.

I shall now proceed to give a general description of the structure of the spines as it is shown by transverse and longitudinal sections taken from every part, from socket to tip, and will then take up the systematic definition and description of the spines of each genus and species that I have had the opportunity of examining, giving in each case the number of specimens from which spines have been taken, so as to give an idea of the limits of my researches.

All the spines of the family, with the exception of the fusiform ones of Centrostephanus, and sometimes of Asthenosoma, are of the "verticillate" kind, that is, the longitudinal ridges are divided by annular notches going inwards and downwards, so that a longitudinal section shows a strongly serrated outer border, each tooth of the saw being very finely pointed, and directed towards the tip of the spine (Plates XXXII., fig. 25; XXXIII., fig. 34). It is this verticillation which gives the characteristic appearance to the spines of members of this family, and renders them, perhaps, the most easily recognised group of all the Echinoidea. Another tolerably common peculiarity (*Astropyga radiata* being nearly always exceptional, and *Asthenosoma varium*, and *Echinothrix calamaris* often so) is that they are hollow, the cavity commencing a short distance above the top of the socket, and extending apparently to about a corresponding distance from the tip, although this point is rather difficult to ascertain precisely, as, in consequence of the brittle nature of the spines, the ends are almost invariably broken off.

If a transverse section be taken so as to pass through the socket (Plate XXXI., fig. 9), it will be seen that this part of the spine is composed of very fine reticulated tissue, the "limestone cells,"† as Professor Agassiz

* Sections of the primary interambulacral spine of *Echinothrix calamaris*. I shall have occasion to notice these when speaking of the spines of that genus.

† The term "cell" seems hardly accurate for what are really interspaces or foramina in the calcification; however, as it is a convenient word, and is always used by Professor Agassiz, I shall follow his example, and employ it throughout.

calls them, being exceedingly small, and the solid bars comparatively very thick. This is specially marked towards the circumference, where the cells are so small that, if the section be ground sufficiently thin to render this part thoroughly distinct, the inner parts will have disappeared entirely. The solidity of this region would seem to be due to its being calcified under the pressure caused by the muscle or contractile membrane which moves the spine, since it is specially marked in the long conical spines of Centrostephanus, in which the muscle is very strong, and the milled ring to which it is attached decidedly prominent. In transverse section the solid bars do not show much regularity of arrangement, a slight tendency towards a radiate disposition being only occasionally manifested; but in longitudinal section there is always a lattice-like structure, the solid bars intersecting more or less at right angles, and sometimes (*D. mexicanum*, Plate XXXI., fig. 3) having a slightly sinuous course. The cavity of the socket in the conical spines of Centrostephanus (Plate XXXI., fig. 9) appears to be lined by a continuous limestone sheath—perforated by what look very like tubes, but are probably the projecting ends of some of the solid bars—of which, however, I have not been able to obtain satisfactory indications in longitudinal sections; in the other genera this lining is mainly formed by the confluence of the solid bars, and a distinct sheath is not differentiated. The same spines which exhibit the sheath also show well the structure of the ridges which are developed on the margin of the spine surrounding the socket; these are composed of reticulated tissue, whose solid bars radiate at acute angles from each side of the central line. These ridges are occasionally to be met with in the other species, and are repeated, though on a much larger scale, in the milled ring.

The difference between the central and circumferential parts of the spine in this region becomes more marked as we ascend. A transverse section, taken so as to pass just above the socket, shows the central part occupied by a circular portion of reticular tissue, whose cells are comparatively very large, being smallest in Diadema (Plate XXXI., fig. 1), while outside this there are indications of the first formation of the solid wedges so characteristic of the family, in the shape of stout solid bars joined by strong

transverse pieces, and branching circumferentially to fuse with the fine reticulated tissue which encases the whole head as far as the milled ring. Coming up higher towards the milled ring, we find these characters being developed, the central reticular tissue gets slighter, and often disappears. The solid wedges become more distinct, and the circumferential close tissue diminishes in quantity; a glance at the illustrations (Plates XXXII., fig. 21; XXXIII., fig. 36) at once renders plain the changes which take place. Longitudinal sections, taking in the whole head, and a small part of the shaft, show very well the gradual growth of the different structures met with in latter region of the spine (Plate XXXII., fig. 3); thus we have the basal portion occupied by the reticulation above described, which, at first thinning off, finally expands into the milled ring; then just above the socket are seen a series of solid pillars, the internal ones representing the pedicles of the solid wedges, and the external, separated from them in the figure by a band of network, gradually expanding to form the main part of these wedges, which run throughout its whole length. If a transverse section be taken immediately below the milled ring (Plate XXXII., fig. 26), we have the spine already assuming characters which enable its genus to be predicated; the solid wedges have become quite distinct, but usually bifurcate externally into the reticular sheath which still persists, and the central network has either disappeared entirely (*e.g.*, *D. mexicanum*), is only very slightly developed (*e.g.*, *E. calamaris*, Plate XXXII., fig. 21), in the form of one or two threads in the figure, or is surrounded by a sheath from which the solid wedges come off (*e.g.*, *E. turcarum*, Plate XXXII., fig. 26).

A transverse section taken through the milled ring shows that this is composed of reticular tissue, like that forming the sheath of the head of the spine (Plates XXXI., fig. 2; XXXII., figs. 17, 27; XXXIII., fig. 31); externally the cells are small, but as they approach the centre they become larger, and each segment of the milled ring is attached to the solid wedge belonging to it, by two or more pedicles of limestone, which in those forms whose solid wedges are long in proportion to their base are merely the divided extremities of those wedges; the segments are joined to each other by bands of reticular tissue, and their solid bars have a more or less palmate

or pinnate arrangement. The ring varies a good deal in size, both in spines from different individuals, and in different spines from the same specimen; it is very rarely all on the same plane, and hence it is difficult, and in many cases impossible, to obtain a perfect section through this region, even independently of the difficulty of keeping the milled portion attached to the rest of the spine. Professor Agassiz seems to have misapprehended the true nature of this milled ring; for in his general remarks on the structure of the spines, he says,* "The notches of the milled ring are the continuation of the striæ or ornamentation of the shaft proper, only the calcareous tissue is more compact, and the dividing lines between the longitudinal striæ are so marked as to be readily distinguished by the naked eye as indentations in the circumference of the flaring collar." The examination of longitudinal sections not only of spines of Diadematidæ, but also of many other genera, renders it quite clear to my mind that the milled ring is the continuation, not of the ornamentation on the shaft, but of the calcareous network enveloping the head; for it will be seen in the figures that this tissue, after expanding to form the ring, rapidly diminishes, and disappears *in toto* a very short distance up the shaft, whilst the ornamentation is in reality a solid outgrowth of the solid wedges (Plate XXXI., fig. 3), which begin internal to the calcareous sheath, and gradually diverge to reach the circumference. That this is the true nature of the verticillations, is quite clear to Professor Agassiz, for with regard to the spines of the Diadematidæ, he remarks,† "The wedges form the verticillation so characteristic of Diadematidæ, and in fact constitute the ornamentation of all the spines, whether they form an independent sheath, as in the Cidaridæ, growing in size at the same time, and independently with, the central part of the shaft, or as in the other Echini, form a part of the inner substance of the shaft, and increase at the same time with the central interlamellar substance." I am rather at a loss to reconcile these two statements, unless Professor Agassiz means that the calcareous network which forms the milled ring is a less condensed form of that constituting the solid wedges;

* "Revision of the Echini," page 651. † *Loc. cit.*, page 654.

though from the reading of the passage, I would be inclined to infer the reverse. Transverse sections through the shaft show different types for each genus, but the general plan is easily described. It is characterised in the main by the predominance of solid structures, the central cavity (which in some cases is occupied by a loose reticulation) being surrounded by a solid foraminated ring, from which usually spring the solid wedges, but which sometimes (Centrostephanus, Plates XXXI., figs. 10–13; XXXII., 17–19; *E. turcarum*, Plate XXXII., figs. 27, 28), gives origin to a network from which the wedges are derived; these wedges are connected together either by transverse bars, or by "spokes" of reticular tissue, and cause externally the verticillations mentioned above. The principal deviations from this type are, on the one hand, the fusiform spines of Centrostephanus (Plates XXXI., fig. 14; XXXII., fig. 19), where the solid pieces are reduced to small marginal triangles by the great development of the network, and on the other, the minute spines of *Asthenosoma varium* (Plate XXXIII., figs. 36–41), in which the reticulation is still more extensive, and the wedges are merely slight expansions of the solid bars which run out from the centre; the whole structure reminds one of a miniature Heterocentrotus (Acrocladia), a genus belonging to the next family; however, Asthenosoma is so aberrant in many respects from the ordinary Echinoid type, that a considerable deviation in spine-structure might be expected, even greater than what actually occurs. Longitudinal sections of the hollow shafts, if taken in the ordinary way, of course result in dividing the spine into two portions, and little or no information is gained; but if one half only be ground away, or if the section be taken in a tangent plane to the central cavity, a good deal of light is thrown on the arrangement of parts. The former method is in every case preferable, since even with the dark coloured spines of Diadema, if a strong beam of light be used, all the details can be readily made out.* The spine is lined throughout by a continuous sheath of limestone, perforated by regular oval foramina arranged vertically over one another;

* I have found the Webster condenser with graduating diaphragm, as made by Collins of Great Portland-street, London, a very useful appliance, on account of the great variety of illumination obtainable by it.

they are tolerably uniform in size, and the apertures of each row alternate nearly regularly with those of the row next to it (Plates XXXII., fig. 25; XXXIII., figs. 34, 40, 41); occasionally, however (*C. Rodgersii*), two or even three rows intervene before two occur on the same level. It is this sheath which forms the solid ring seen in the transverse sections, and the foramina are the mouths of tubes which pass through it to open into cells placed between the bases of the solid wedges, or (Centrostephanus and *E. turcarum*) into the innermost row of cells of the network immediately surrounding the sheath; in either case, free communication must be afforded between the central cavity and all the other parts of the spine, and the question presents itself, "Are these tubes the means whereby the nutrient matters necessary for the vitality and growth of the spine are supplied?" This is a point which I have not opportunities to work out, but it would be worth while for those who are in favourable circumstances to investigate it, especially as so little is known definitely as to the mode of growth of these organs. Surrounding the sheath, we have either a band of reticulation (Centrostephanus and *E. turcarum*), or a few vertical rows of large apertures, and outside these the solid wedges extend to the margin, and terminate in the sinuous serration already alluded to. At regular intervals, nearly always corresponding to the bases of the teeth of the serration, are placed three or four rings of transverse bars, connecting together the solid wedges, and helping to strengthen the entire fabric; these are the bands almost always seen in the cross sections.

Such is the general structure of the spines; fuller details, and examples of the remarkable effects caused by injury, as well as remarks on the variation in type of each genus, will come in more appropriately in connexion with the complete descriptions of the spines of the different species to which I now proceed, giving in every case, as accurately as I can, a definition of what seems to be the typical arrangement in each species which I have examined, and supplementing this by thorough descriptions of the structure in different parts, so far as this may be worthy of notice. The type structure is taken from sections of the median region of the shaft.

Diadema (Schynvoet.)

Spines elongate, slender, strongly verticillate, usually dark brown or purple in colour, sometimes with alternating light and dark zones. In transverse section, central hollow never occupied by reticulation, moderate, diameter about equal to length of solid wedges (one-third that of spine); foraminated ring strong, not sinuous; reticulated tissue sparing or absent; solid wedges in the form of elongate isosceles triangles with concave sides, whose base is about one-third of the height.

Sections of the spines of this genus are readily recognised by the above characters, which I have found practically invariable, the only exceptions being specimens from a spine marked Diadema, but which both externally and internally seems to belong to Echinothrix, and has probably come in amongst the others by accident.

The variation in the structure of different parts of the same spine is very slight, being most marked at the distal end, where there is sometimes a tendency exhibited for the solid wedges to become urn-shaped; but even this slight change is unfrequent, the specimen figured (Plate XXXI., fig. 6), being almost the only one of my preparations which shows it. Sections of the proximal region, even when they pass through the milled ring, do not show any deviation worth noticing, from the definition above given, and the only normal departure from it that I have noticed has been in cases where the section is taken near the free end of one of the teeth of the serration, where the bases of the solid wedges, instead of being curved, are prolonged into forms something like a sheaf of corn, or one-half of a dumb-bell; but this diversity is more apparent than real, for there will always be one or more of the solid wedges presenting their usual appearance, and thus showing the true condition (Plate XXXI., fig. 4).

Professor Agassiz, in his recently published "Revision," has limited to a large extent the number of species contained in this genus; and although the evidence at present before me militates to some degree against his decision, still, as it is comparatively very small, and as he has manifestly based his opinions on the examination of a large number of specimens, it would

be premature as well as presumptuous on my part to reject his conclusions, and I therefore content myself with indicating those points wherein I differ from him, leaving it to future investigations to decide the question finally. The structure in the two species is as follows:*—

D. mexicanum, A. Agass. (Two specimens). Solid wedges joined by a single broad transverse band towards the circumference, and by several narrow bands towards the centre (Plate XXXI., figs. 1–4).

D. setosum, Gray (Seven specimens). Solid wedges joined by a single broad band towards the circumference, and by few or no narrow bands (Plate XXXI., figs. 5–8).

The differences between the two are very slight, if indeed they can be said to exist at all, and the above definitions are based on the comparison of a large number of specimens of spines, and contain the only distinctions which seem at all constant; occasionally the solid wedges in *D. mexicanum* have convex instead of concave sides.

A section taken slightly above the top of the socket in *D. mexicanum* (Plate XXXI., fig. 1) shows the central part filled with a network whose cells are almost perfectly circular, one of them being almost exactly in the middle; surrounding this portion are a number of solid bars radiating outwards, and separated by rows of large oval foramina; this is succeeded by a zone of reticulated tissue of a coarse character, which merges externally into the ordinary network which envelopes the whole head. A short distance above this, the central reticulation has almost disappeared, being represented only by a few solid bars; the foraminated ring is distinctly visible, the radiating bars have become more separate and clear, being united only by some transverse bars, or a small piece of very open network, and the close reticulation is limited to the margin of the section; immediately below the milled ring these characters are still more pronounced, and the solid bars are beginning to assume their characteristic striation and cuneate form, having become thickened externally, and bifurcated to support the reticulated zone, which exhibits indications of expanding into

* I reserve the discussion of the question of specific differences for the second part of the paper, and here merely give the facts as they occur.

the milled ring, a section through which (Plate XXXI., fig. 2), only shows the wedges with a wider base; in the broad transverse connecting bands are well seen the halves of the funnel-shaped tubes which pass through them, placing the internal foramina in communication with the elongate spaces between the extremities of the wedges; the radiating arrangement of the solid ribs in the segments of the milled ring are also well shown; a longitudinal section of the head and milled ring (Plate XXXI., fig. 3) exhibits well the gradual growth of the pedicles of the wedges, which commence at the top of the socket in the form of a number of pillars joined by network, which subsequently resolves itself into a series of large foramina and connecting bars, and then by the elongation of the former, and consequent removal to regular distances of the latter, assumes the normal condition as shown in sections taken through the median region of the spine; in the same specimen may also be noticed the gradual deflection outwards of the solid ribs of the reticulated sheath to form the milled ring, which then runs in almost horizontally and disappears, its upper surface being armed by a series of projecting points. Sections through the proximal parts of the spine (Plate XXXI., fig. 4) show very numerous narrow transverse bars, which occasionally bifurcate, indicating their original netted condition; it is here also that the solid wedges more commonly display the convex instead of the concave edge; and with the exception of usually a reduction in the number of the narrow transverse bars, these characters are maintained to the distal extremity. Longitudinal sections, made so as to remove half the spine, show no points of deviation from the general type; the foramina are small and regularly alternating.

In *D. setosum*, the sections through the head and milled ring present practically the same characters. The longitudinal section of this part has no features of special interest. In the transverse sections of the median region of the shaft (Plate XXXI., fig. 5), there are no points to be noticed beyond the differences alluded to in the definitions; longitudinal sections of the same part show that the funnel-shaped tubes mentioned in connexion with *D. mexicanum*, as passing through the transverse connecting bars, are due to the confluence at each extremity of two of these bars placed in close

vertical apposition to each other. Cross sections of the distal extremity (Plate XXXI., fig. 6), show the greatest amount of deviation from the typical structure; for it will be seen there that the central hollow, instead of being regularly circular, has become distorted and angulated, that the foraminated ring is surrounded by and scarcely distinguishable from a belt of network which runs out between the solid pieces, and becomes continuous with the transverse bars, and that the solid wedges themselves, having lost their elongate triangular form, have become urn-shaped by the development of a convex outline to their sides; this variation, however, is not always found, for many specimens adhere to the normal form. In Plate XXXI., fig. 7, is shown a remarkable specimen taken from the distal end of a spine; I have unfortunately no record of it, but judging by analogous appearances in Centrostephanus, presently to be described, I have no doubt as to its being the result of an injury. It will be seen that the central portion, instead of being hollow, is occupied by a reticulation consisting of netted rays surrounding several large foramina; the solid ring which should bound the central hollow has disappeared almost entirely, being only discoverable by careful arrangement of the light, and its place is supplied by a zone of network through which may be observed passing outwards the stems of the solid wedges, which are here represented by very short urn-shaped pieces, of a purplish red colour, some of which have portions of the serrations adherent to them, giving an increased appearance of a symmetry to the specimen. This section is the more interesting as exhibiting a sort of transition both in colour and structure to the injured spines of the next genus Centrostephanus.

On the aboral surface of the corona of many specimens of *D. setosum* may be observed groups of spines of a somewhat different appearance to those found on the rest of the test; the difference chiefly consists in their lighter colour and much greater fineness, since they are scarcely thicker than a horse-hair. The diversity in structure, however, is not very great, for I have succeeded in obtaining both transverse and longitudinal sections, and find that it chiefly obtains in the relative reduction in length of the solid wedges, and the absence of serrations; it is easy, moreover, to find forms

which are almost exactly intermediate between the ordinary and those hair-like spines, both in size and external appearance, but these show no divergence whatever from the normal type. I regret that I am unable to show any transverse sections of these spines; for though I obtained many sufficiently good for description, and several perfect enough to mount, these latter were broken during setting, and the time at my disposal has not permitted me to replace them, as it is a matter of considerable difficulty to grind sections of such minute objects, and they are far too brittle to allow of their being cut.

In the Museum of the Royal Dublin Society, there are two specimens named *D. antillarum*, a species originally described by Phillipi, but which Professor Agassiz, though formerly acknowledging, as the specimens were named by him, now considers as only a variety of *D. setosum*, Gray. The spines differ in external appearance from those of *D. setosum* which I have seen, but this is not a safe criterion; however, on making sections of them, I found that they differed much more from *D. setosum* than this did from *D. mexicanum*. In the structure of the head and milled ring, transverse sections show nothing peculiar, but longitudinal exhibit the early formation of the foraminated sheath, as also the termination of the milled ring on the shaft. Specimens taken from the shaft just above the milled ring also have no very striking differences, though, to any one accustomed to the look of the spines in *D. setosum* and *D. mexicanum*, they suggest a distinction; but sections through the median (Plate XXXI., fig. 8), and to a less degree through the distal regions, are at once distinguishable by the presence of a belt of network surrounding the foraminated ring, which is not so regular here as in the other forms; this network sends out narrow spokes between the solid wedges, which latter are shorter and more curved in outline than in either *D. mexicanum* or *D. setosum*. Longitudinal sections display no features of special interest, the amount of reticulated tissue shown depending partly on the portion of the spine from which the section is taken, but mainly on the plane of section, since if this passes through the middle of the solid pieces, little or no network is seen; but if on the contrary it is

along the spokes of cells, then the spine will appear to be chiefly made up of reticulated tissue.

Centrostephanus (Peters).

Spines dimorphic. One set few in number, only found on the oral surface of the corona, moderately short, fusiform, longitudinally striated, not verticillate, bright purplish red in colour. In transverse section, central hollow never occupied by a reticulation, moderate, diameter about one-third that of spine; foraminated ring thin, not sinuous, or but slightly so; reticulated tissue very abundant, forming a zone whose breadth nearly equals the diameter of the central cavity; solid wedges in the form of short urn-shaped pieces, whose base is about two-thirds of the height. The other set numerous, scattered over the whole surface of the corona, long, conical, strongly verticillate, not longitudinally striated, dark red in colour. In transverse section, central hollow never occupied by a reticulation, large, diameter about two-thirds that of spine; foraminated ring thin, sinuous; reticulated tissue scanty, forming a zone whose breadth is about one-tenth the diameter of the central cavity; solid wedges in the form of urn-shaped pieces, whose base equals about one-half the height.

The existence in this genus of the dimorphic spines, which appear to be constant, as they are mentioned in all the descriptions, renders it easily recognised, while the invariable character of the internal structure of both is no less distinctive.

The genus, according to Professor Agassiz, contains three species; one of these, *C. coronatus*, Verrill, I have never seen, and am not aware that the structure of the spines has been examined; the characters of the other two are as follow:—

C. longispinus, Peters. (One specimen.) Fusiform spines; solid wedges ovate, without constriction near base, separated from each other by narrow pieces of reticulated tissue. Conical spines; solid wedges elongate, slender (Plate XXXI., figs. 9–15).

C. Rodgersii, A. Agassiz. (One specimen.) Fusiform spines; solid wedges urn-shaped, with constriction near base, edges closely apposed. Conical spines; solid wedges short, broad (Plate XXXII., figs. 16–20).

The differences thus specified, though apparently very slight, appear to be pretty constant, at least as far as may be judged from an examination of the spines of the only specimens I could get, and I have found in practice very little difficulty in recognising one from the other (Plate XXXI., fig. 9).

Cross sections through the socket of the conical spines in *C. longispinus*, show well the structure of the ridges surrounding this part, to which muscular fibres are attached, as well as the continuous lamina lining the cavity of the socket; both features, as well as the presence of a large milled ring and strong muscle, are indicative probably of extensive motile power in these spines. In a section taken just above the socket, the central portion is seen to be nearly vacant, a few slight solid rods being the only remnant of the network left; the exceedingly dense nature of the marginal reticulation is also worthy of notice, conferring as it does an almost enamel-like hardness on this portion of the spine, and rendering it susceptible of a high degree of polish; the sinuous nature of the outline, which may be seen in some parts of sections taken through this region, is due to the fact that the section passes through the summits of some of the ridges mentioned above. A section taken about half way between the socket and the milled ring shows the solid pieces in the form of spokes radiating out from the central reticulation; some of them have already become slightly thickened towards their outer ends, but there is as yet no indication of a differentiation into urn and stalk, such as may be noticed in the section through the milled ring where these have assumed their characteristic appearance. Transverse sections through the proximal and middle regions of the shaft (Plate XXXI., fig. 10) differ in no appreciable respect; the spines are chiefly remarkable for their extremely light and elegant structure, as well as for the rich crimson colour which renders thick sections exceedingly handsome objects; it will be observed that the transverse bands are very numerous, apparently with a view to strengthen the fabric, which otherwise would be very apt to

give way; even as it is, there is great difficulty in obtaining anything like a perfect specimen, as an "urn" not joined to its fellows is almost certain to break off. I cannot speak with certainty as to the structure of the distal portion of the spine. In the solitary specimen in Dublin, that in the Museum of Trinity College, the tips of nearly all the conical spines were broken off, and even those that were approximately entire had evidently suffered the same fate during the lifetime of the animal, and had undergone repair—a process which effects very curious changes in structure, which I have been enabled to examine very fully, as Dr. Macalister kindly placed the specimen at my disposal for the purpose. The invariable effect of injury to a conical spine seems to be the production of characters intermediate between the two forms in that portion of the spine which undergoes repair, the portion below the injury of course remaining unaltered. This result, however, only seems to follow if the injury be some distance up the shaft, and of an extensive character; such, for instance, as the entire breaking off of the end, for a slight injury proximally only produces a greater deposition of calcic carbonate at the point injured. Plate XXXI., fig. 12, exemplifies the point in question (being taken from a spine just above the milled ring), the injury having caused an extension outwards of several of the solid wedges, the largest of which has a tube passing inwards, with a slight dilatation at about the level of termination of the unaltered wedges, and terminating in an irregular patch of darker colour; on two of the urns, to one side of this, may be observed a network of what I took at first to be solid bars, but, on careful examination with a quarter-inch objective, found to be tubes penetrating pretty deeply into the substance of these solid wedges; the irregularity of the striation here, which can hardly be shown in a drawing, seems to indicate a corresponding irregularity in the formation or deposition of the calcareous material; and it appears not unlikely that this is due to the attack of some boring animal, or possibly the hypha of a fungus, such as so frequently penetrates shells of Gasteropoda. Externally the only change made was the production of a small swelling, rising from the milled ring, and terminating about one-sixteenth of an inch up the shaft. The same spine received a more extensive injury some dis-

tance up, and this has produced a change similar to what is shown in that part of the specimen represented in Plate XXXI., fig. 13, which has become netted. It will be seen that the central cavity has become irregular, and instead of the normal narrow band of reticulation with large cells surrounding the foraminated ring, there is developed a broad zone of network with very small irregularly arranged cells, and very thick walls. The solid wedges too, have altered their shape considerably, and in many places, instead of showing the well-marked distinction between urn and stem that is usually seen, the latter gradually expands, and the solid piece becomes decidedly club-shaped; sections taken up to the tip of the spine preserve this character unaltered, bearing thus a greater similarity to the fusiform than to the conical spines. It would almost seem as if on receiving a primary injury, the repair was effected so as to give a greater degree of strength to the spine, and better enable it to resist further fracture: and I have never been able to find a specimen in which two fractures had been repaired. It is remarkable, however, as I shall notice further on, that injured fusiform spines become like conical ones during the process of repair. Plate XXXI., fig. 13, is a remarkable specimen, in which the injury had been sustained about one-third of the way up the spine, and the section from which the drawing is taken passes exactly through the injured part; it will be noticed that the normal characters are preserved for about one-third of the circumference, the remaining two-thirds exhibiting features like those just described; the section immediately succeeding that figured has the reticulation developed all round.

I have occasionally met with specimens of conical spines which presented a gradual transition towards the structure obtaining in the fusiform ones, without any appearance of having sustained an injury of any sort. One of these is represented at Plate XXXI., fig. 11, and it will be seen that it approximates very closely to the structure of the fusiform spines, and might be readily enough mistaken for them.

Transverse sections of the proximal region of the fusiform spines of this species (Plate XXXI., fig. 14) show usually a narrow irregular fringe of network on the inside of the foraminated ring, which is not very strongly

marked, and is surrounded by the broad zone of reticulation, through which may be seen to pass outwards a series of solid rays, of nearly uniform thickness, which expand near the outer margin into the short, broad, urn-shaped, solid pieces, which are separated from one another by narrow bands of network, and form by their curved bases the longitudinal striation on the spine; the arrangement of the limestone cells in the reticulated tissue is tolerably regular, rows of two or three intervening between each solid ray, and a very regular series of somewhat quadrate cells existing along the course of each. This structure persists throughout the whole course of the spine, the distal region sometimes differing slightly in a little greater prominence of the solid pieces, and a less distinct differentiation of the foraminated ring; in longitudinal sections, the only feature of note brought out is the very small size of the foramina in the lining sheath.

The effect of injury to these fusiform spines seems to be that repairs are effected by the production of structure like that of the conical ones; the longitudinal section figured (Plate XXXI., fig. 15) shows this very plainly; for above the curved line which indicates the position of the fracture, it will be seen that the tissue is more open, and on one side the margin has become serrated, as is the case in the conical spines, though below the fracture it is smooth as usual in the fusiform ones. It is also remarkable (though this cannot be shown in the drawing), that the colour above and below the injury is different; in the former place having the reddish purple shade characteristic of the conical spines, and in the latter the vermilion tint equally constant in the fusiform; the margins on both sides above the fracture, however, still retain the latter colour, and present a curious contrast to the central part, whilst a small patch which occurs on one side, at and below the level of the fracture, had the purple colour of the conical spines. A transverse section of the same spine, taken above the upper end of the longitudinal section just described, shows a somewhat similar structure to that found in repaired conical spines; the central cavity having become irregular, and partially filled up by a very loose reticulation, the foraminated ring has disappeared altogether on one side, and become very

irregular on the other, and the solid pieces have acquired ovate extremities which project very far out from the circumference.

In a cross section of the head of the conical spines in *C. Rodgersii* (Plate XXXII., fig. 16), the central hollow will be observed to be surrounded by a narrow band of very dense reticulation, from which the foraminated ring is afterwards evolved; the solid pieces are tolerably well marked, and the outer sheath of network is very dense, and of a pretty purple colour. A section through the milled ring (Plate XXXII., fig. 17) shows the spine assuming its characteristic structure, with the solid urns well marked, and having their stems surrounded by a relatively large zone of reticulated tissue; the segments of the milled ring have their central parts composed of a very dense network, which becomes more open towards the marginal and internal regions. The only difference which exists between the structure here, and that found in the proximal and median parts of the shaft is, that the zone of network surrounding the stems of the urns gets somewhat narrower, and occasionally the urns assume a more rounded contour. Longitudinal sections show that the lining sheath has very small and nearly linear foramina, which are often arranged on the same level as, instead of alternate with, those next them. In the distal region (Plate XXXII., fig. 18), the zone of reticulated tissue has almost vanished, being indicated only by narrow transverse bars, and the urns have become very narrow in proportion to their length.

Transverse sections of the milled ring of the fusiform spines of this species contrast forcibly with the corresponding sections of the conical ones, for here the milled ring only presents itself as a narrow band with exceedingly wide reticulations; the solid pieces are in the form of clubs with long handles, the foraminated ring is very irregular, and the central hollow occupied by a loose network. A longitudinal section of the head shows well the rapid thinning out of the sheath of network which surrounds the head, according as it approaches the milled ring.

In the normal structure of the shaft (Plate XXXII., fig. 19), there is little to be noticed beyond what has been laid down in the diagnosis, and the arrangement in the proximal and median regions is practically identical;

in the distal part the network is somewhat more irregular, the solid urns are a little more separate from each other, as well as more projecting, the foraminated ring is rather indistinct, and the central cavity is nearly filled up by reticulation.

Injury to these spines produces an effect unlike anything I have observed in the other species; the specimen figured (Plate XXXII., fig. 20), is a slightly oblique section of an injured spine, and it will be seen that the result has been to produce a second circle of solid pieces; the central part of the spine is not at all altered, the foraminated ring and reticulated tissue being quite normal, and in two parts of the section the solid rays are seen to run out and dilate just as usual. In other portions, however, the network is seen to spread out in a continuous sheet as far as the level of termination of the inner row of solid pieces, when it becomes subdivided into spokes which circumscribe wedges which only differ from the normal form in their greater elongation. In those parts where the inner ring of solid pieces is distinct, it will be seen that a second series of quadrilateral segments have been, as it were, superposed on their outer edges, and the spokes of cells which separate the inner parts of the wedges are prolonged between the external segments. Here and there solid rods may be noticed projecting across the wedges, and evidently derived from the network; and in those parts of the section where the extension of the reticulation is best marked, the colour of the conical spines appears in patches. The whole secondary ring of solid pieces is clearly marked out by a darker shade, for the colouring, which is deep at the inner ends of the solid pieces, becomes light towards the circumference, and then again deepens slightly at the commencement of the secondary outgrowths; a longitudinal section shows that the expansion of the reticulation is due to a branch going off from the main axis to the margin. This specimen is interesting, as being the only example I have as yet met with in the Diadematidæ, of that concentric mode of growth which is so characteristic of the Echinometridæ.

Echinothrix (Peters).

The extreme divergence in type, of the two species of this genus which I have examined, compels me to abandon any attempt to frame a definition which would include both, and exclude the other genera; they have hardly a single feature in common, and hence I am disposed to think they should be placed in different genera. I will consider the point more fully subsequently, and meanwhile proceed to give descriptions of the two species with which I am acquainted; the third, *E. Desorii*, Peters, I have never seen.

E. calamaris. A. Agassiz. (Two specimens.) Spines slender, elongate, strongly verticillate, usually with alternate zones of dark and light colour. In transverse section, central hollow, oval, sometimes occupied by loose reticulated tissue, major axis about half as long as that of spine; foraminated ring moderate, not sinuous; reticulated tissue absent, or but slightly developed; solid wedges in the form of isosceles triangles, whose base is about two-fifths of the height (Plate XXXII., figs. 21–25.)

E. turcarum, Peters. (Two specimens.) Spines moderately stout, elongate, verticillate, dark red in colour. In transverse section, central hollow circular, never occupied by a network, very small, diameter about equal to one-sixth that of spine; foraminated ring indistinct; reticulated tissue abundant, in the form of a zone whose breadth equals half the diameter of the central hollow, and sends out numerous radii, whose length is about twice the diameter of the same cavity; solid pieces in the form of elongate triangles, whose base equals about one-fourth of their height (Plates XXXII., figs. 26–28; XXXIII., 29, 30).

It will be observed, that the difficulty which exists here is the reverse of that which obtains in the two preceding genera. In them the obstacle to be overcome was the close similarity of their structure, which rendered it by no means easy to lay down distinctive characters of the species of each; here, on the contrary, the difficulty is to seize on any points which are common to the two. It would be much more easy to construct a diagnosis of Diadema

which would include *E. calamaris*, than one of *E. turcarum* which would do so; and in fact, but for the existence of the dimorphic spines in Centrostephanus, *E. turcarum* would find a more natural position under it, being allied by the structure of its fusiform spines.

In reference to the spines of this genus, Professor Agassiz remarks, that they are not as hollow as those of Diadema, having more of the solidity of Astropyga; and concerning *E. turcarum*, says, that though usually dark, they are sometimes banded, and that all intermediate conditions between the two may be found.

The light fragile structure which exists in the shaft of the spines of *E. calamaris* is found also in the head, for in the section of that part which is figured (Plate XXXII., fig. 21), it will be noticed that the radiating solid bars are very slight, and the network joining them very delicate, even the marginal reticulation being less condensed than usual, whilst the central cavity is already of a large size. In consequence of this fragility, sections through the milled ring are very difficult to make, and I have only succeeded in obtaining one which would show even half the circumference; in it the thinness of the foraminated ring, and the very small size of the solid wedges, are noteworthy; the internal part of the reticulated tissue also has very large cells, but the segments of the milled ring are tolerably dense. In longitudinal sections of the head, owing to the transparent nature of the spine, the structure of the sheath lining the cavity is well seen; the foramina by which it is pieced are very small, and have a remarkable arrangement, being disposed in a series of V-shaped lines, with the apices directed forwards; the angle becomes more and more obtuse as we go up towards the median region, and when this is reached, a circular arrangement obtains, the foramina being alternate in each row (Plate XXXII., fig. 25).

Transverse sections of the proximal part of the shaft (Plate XXXII., fig. 22), present a considerable degree of resemblance to those of Diadema, but may be readily distinguished by the large oval central cavity, the much finer foraminated ring, and the relatively short solid wedges; the bars which connect these are also very much more numerous than they are on the average in that genus, and at intervals anastomose to form a network.

There is sometimes a slight departure from this in the median region, caused by a rapid reduction in the breadth of the solid pieces just inside the base, so that whilst it retains its breadth, the rest of the wedge tapers, at first very rapidly, but afterwards very gradually towards the centre; the specimen figured (Plate XXXII., fig. 23), is as extreme a case as I could pick out, and even in it the facies of the species is unmistakable; far the most common arrangement, however, is the same as that existing in the proximal part. I cannot say with certainty how far the structure of the distal region would differ from that in the median; the same difficulty occurs here as in Centrostephanus, the tips being either entirely broken off, or else fractured and repaired; the specimen figured (Plate XXXII., fig. 24), is taken from the median region of a spine which has undergone the latter change. It will be noticed that the effect of injury has been to obliterate the foraminated ring, to cause a development of loose reticulated tissue in the central cavity, and to produce a zone of network whose cells are irregularly arranged, round the bases of the solid pieces, which themselves are somewhat altered in shape; but this I think is due to the fact that the serrations are so close in the repaired part, that a section taken across it at random is certain to pass through some of the teeth, for I have observed specimens in which the wedges retained the usual character. It will be seen, on comparing the network in the injured conical spines of Centrostephanus with that in the present specimen, that the resemblance is very strong, and quite warrants the supposition that the cause in both cases is the same, though the external appearance of the spine in *E. calamaris* is often so little changed as only to be noticed on a close examination.

A transverse section through the head of the spine in *E. turcarum*, taken just below the milled ring (Plate XXXII., fig. 26), shows characters by which it may be readily recognised. The central part is occupied by a network with large cells and thick interlacing bars, becoming more numerous in the neighbourhood of the outer foraminated ring, which, though but slightly differentiated, is plainly seen to give origin to the solid wedges which have assumed their characteristic appearance and colour, and are separated from one another by the spokes of cells, just as in sections of the

shaft; outside there is a zone of reticulation of an open character. The inner foraminated ring, however, which forms the lining of the central hollow, and the ring of reticulated tissue surrounding it, do not seem to have been as yet differentiated, for by comparing this section with one taken through the milled ring (Plate XXXII., fig. 27), it is seen that the foraminated ring in the former is truly the outer one, since from it arise the solid pieces, whilst it also differs in breadth from the true inner ring, though this latter difference is not so well marked here as in sections of the proximal part of the shaft (Plate XXXII., fig. 28.) The inner ring, with the zone of network immediately succeeding it, appears to be formed by a condensation of the central reticulation seen in fig. 26—though how this exactly takes place, I have not succeeded in tracing to my satisfaction; for longitudinal sections, though showing very clearly the growth of the solid wedges, throw no light on the growth of the inner ring. This, however, is no doubt due to the thickness of the sections; for, having rather a limited supply of material at my disposal, and having already destroyed several, I feared to risk my last specimen in the attempt to produce a perfect section, more especially as this shows very well the serrated outer edge of the solid pieces, while they are yet surrounded by reticulation; the points project a short distance into the network, and then become lost in its ramifications, but are distinctly seen above the level of the milled ring, where the enveloping tissue has disappeared.

In the section of the proximal region figured (Plate XXXII., fig. 28), it will be seen that six of the solid wedges are penetrated by short tubes something like, but less extensive than, those found in a specimen of the conical spines of *C. longispinus;* no alteration, however, has followed, except in one case where the end of the wedge is bifurcated by a deep notch, which may have been caused by the nearly vertical direction of the tube; for since many of these canals are cut across obliquely, and in one case the top of a curve has been ground away, it would seem as if they had a considerable longitudinal extension, though their radial growth is not very great. Injury to the shaft of the spine does not seem to produce much alteration either externally or internally, in the former case being evidenced

only by a somewhat stronger verticillation, and in the latter by a more marked projection of the solid pieces beyond the level of the network, as is seen by comparing Plate XXXII., fig. 28, with Plate XXXIII., fig. 29, the former being taken from the proximal region of an unaltered spine, and the latter from the median part of one which had received an injury below the level from which the section was taken. In the distal part of the spine there is a considerable divergence in structure from that which obtains in the proximal and median regions, but it is a divergence which leads back to a resemblance of the structure found in the head, and at the same time points in the direction of Centrostephanus. It will be observed (Plate XXXIII., fig. 30), that the solid wedges have acquired convex instead of concave sides, thereby assuming a somewhat urnshaped figure, that the ring from which they arise has become obsolete, and they thus present the appearance of springing from the zone of network which surrounds the foraminated ring. Hence, although at first sight strongly resembling the structure found in the head (with the exception of course of the central reticulation), the resemblance is due to the disappearance of a different part, as it is the inner ring which is not represented in the head, whilst the outer one has become obsolete in the distal region.

Astropyga (Gray).

Spines comparatively short, exceedingly slender, strongly verticillate, dark green or brown in colour throughout, or with alternating zones of both. In transverse section, central hollow nearly always occupied by a reticulation, circular, moderate, diameter about equal to one-third that of spine; foraminated ring thin, not sinuous; reticulated tissue absent (except that which exists in the central cavity); solid pieces cuneiform, base about one-half the height, tapering rapidly, and joined by numerous narrow transverse bars (Plate XXXIII., figs. 31–35).

This genus contains but two species, one of which, *A. radiata* (Gray), I have examined, having obtained a large number of spines from two specimens; the diagnosis of the genus will be also at present that of the species.

Whilst the general facies is readily recognisable, there is a considerable degree of variation in the shape of the external portion of the solid wedges, which, however, is almost entirely due to serrations; for, as will be seen in the longitudinal median section (Plate XXXIII., fig. 34), these are very numerous, and run into one another in such a way that a cross section is almost certain to pass through some part of some one of them, and thus we find the outer ends of the wedges varying from the ordinary triangular form like that in Diadema or *E. calamaris*, to a globular or even quadrilateral figure. The structure of the head is not unlike (though on a very reduced scale) that of *E. calamaris*, in the arrangement of its solid radii, but the central hollow is much smaller in proportion, and the network occupying it persists to a much greater degree; the zone of marginal reticulation also has a much closer texture, owing to the smaller size of the cells of which it is made up. A section through the milled ring (Plate XXXIII., fig. 31) at once attracts notice, on account of the very slight development of the solid pieces, for they are little more than bars of uniform thickness throughout, though in all the other members of the family except *E. calamaris*, they have assumed their fully developed character at this point; the extremely open character of the surrounding reticulation is also worthy of remark. In sections of the proximal part of the shaft it is rare to meet with any specimen which does not present at least some trace of a central reticulation, though this may be only in the form of one or two projecting solid bars.

The two sections figured (Plate XXXIII., figs. 32 and 33), are selected to illustrate the extremes of divergence in structure which I have noticed in this part, both as regards the shape of the solid wedges, and also the quantity of the central reticulation, which in one (fig. 33) is only represented by two solid bars with expanded bases, whilst the solid pieces have nearly globular heads, though one of them at least has a rectilinear termination, and another nearly diametrically opposite to it shows very well the way in which one of the teeth of the serration gives rise to this square appearance; in the other section (fig. 32) the central network is of a very compact character, and the solid wedges have very elongate heads, clearly

showing that the plane of division has passed through the broader portions of the serrations, and the margins of the wedges have one or two prominences indicating the origins of some of the solid transverse bars, which are very numerous in both specimens, and are the features which confer the individuality on the structure of the spines. The same characters are to be met with, and about the same degree of variability in form, in the median region, a section being occasionally found entirely devoid of any reticulated tissue; but such specimens are rare, and even in the distal part (Plate XXXIII., fig. 35) the network still presents itself, existing in the specimen figured, as a solid bar running across one end of the central cavity, so that the spines may be regarded as typically possessing a central reticulation, though in the diagnosis I have preferred defining them as they may be found, rather than as they should be. In the longitudinal median section (Plate XXXIII., fig. 34) this network has been washed away for the most part, though by careful focusing it may be seen in places; the closeness of the transverse bars is worthy of notice, as also the relative sparseness of the foramina in the sheath lining the cavity of the spine.

Asthenosoma (Grube).

Spines very short, moderately slender, slightly verticillate, or sometimes longitudinally striate without verticillations, greenish white, with dark zones. In transverse section presenting two types of structure; one form belonging to the striated spines has the central hollow small, diameter varying with position of section, foraminated ring entirely absent, reticulated tissue very abundant, solid pieces very indistinct, in the form, when discernible, of radiating bars slightly thickened at the circumferential extremity. The second type belonging to the verticillate spines has the central hollow variable in diameter, and sometimes entirely obliterated by network, foraminated ring absent, or but slightly differentiated, reticulated tissue variable in quantity, solid pieces in the form of radiating bars of uniform size, till they reach the circumference, where they dilate into paddle- or spade-shaped figures (Plate XXXIII., figs. 36–41.)

This genus, with its two species, is one of the most anomalous, if not the most anomalous of all the Desmosticha, possessed as it is of a flexible

corona, and thus differing from our ideas of what an Echinoid should be. From this diversity of test-structure I was led to anticipate a similar irregularity in the structure of the spines, and was therefore very pleased to obtain from Professor Grube specimens of those organs from the rarest of the two species, *A. varium* (Grube); but as these had to come by post from Breslau, they suffered considerably in the journey, notwithstanding Professor Grube's care. Still, some fragments were in a condition to be cut; and though my examination is not nearly so complete as I would wish, it is vastly better than nothing, and I fancy that I have obtained at any rate the principal modifications in structure, as the spines were manifestly selected by Professor Grube with the view of giving me the greatest variety existing. I do not know anything definite about the structure of the spines in the other species, *A. hystrix* (Wyville Thomson), as I have not seen any description of them anywhere, and therefore I cannot say whether the above definition is diagnostic of the genus, or merely of the species.*

Professor Grube in his description alludes to some spines which are larger than the usual form, are hollow and slightly curved, with funnel-shaped extremities; these spines are those which I have characterised as longitudinally striated, and the description given is taken from sections of a portion of the basal part, about a quarter of an inch in length, which was all that I could find, the rest having been reduced to fragments.

From the descriptions which follow, as well as from the figures, it will be seen that the variability in structure here is greater than in any other of the Diadematidæ, and though presenting a general resemblance to the type form, yet has no very decided affinities to any.

The greatest amount of resemblance to the other genera, as well as the least amount of variation in different spines, is to be found in the head of the verticillate spines (Plate XXXIII., fig. 36), which approaches *E. calamaris* in plan, though of course much smaller; the section shows a small amount of reticulated tissue in its central cavity, and a narrow band here and there adhering to the inner surface of the foraminated ring, which is

* Professor Agassiz in his "Revision" figures a section of one of the spines, but the amplification is too slight to enable anything accurate to be made out. It presents, however, a general resemblance to the definition given above of the striated spines.

very thin but quite distinct, and has a tolerably regular ring of transverse bars outside it, joining together the solid pieces, which are in the form of spokes of uniform thickness running out to the marginal reticulation, which is of an open character. This agreement in the structure of the head with that found in the other genera is only what we might expect, since there is a far less variation here than in sections of the shafts, the different characters not having had, as it were, time to develop themselves. I have not been able to get a section through the milled ring, but the longitudinal section of the head does not present any character of importance in this respect; it shows, however, the great relative length of the head, the comparatively high level at which the solid pieces begin to develop, and the variable size of the foramina in the lining sheath, as well as the absence of serrations on the shaft. I regret that I was not able to examine the structure in this region of the striated spines, as, from the anomalous arrangement obtaining in the shaft, it is probable that corresponding differences would exist in the other part.

Sections of the fragment of the shaft of one of these striated spines present the remarkable appearance represented at Plate XXXIII., fig. 37. It is taken as near the base as possible, and it will be seen that the outline is irregular, having a lateral protuberance on one side, that the central foramen is by no means properly defined, the foraminated sheath being absent, and the reticulated tissue projecting irregularly into it; the solid pieces are practically obsolete, for though there is an indication of a radiate arrangement, this is caused by the disposition of the bars of the network, rather than by any differentiation of the wedges; the circumference of the spine is encircled by a narrow band of homogeneous calcification, which shows no projections of any sort, and therefore this portion of the spine must have been perfectly smooth. Sections taken nearer to the median region have also an irregular outline, and the central hollow still further reduced in size, leading to the conclusion that it would disappear altogether a little higher up—an inference rendered more probable by the absence of the foraminated sheath—but there are indications of the existence of solid pieces on one side at least, in the form of a number of radii thick-

ened at their outer ends, and taking rise in the central network; occasionally I have noticed these projecting from the circumference, so as to give rise to the striation of the spine; on the opposite side of the section these solid ribs are not at all clear, but at the margin the reticulated tissue may be seen to send out irregular prolongations, which to a limited extent mark off triangular pieces in the solid sheath, which here, as in the other specimen figured, surrounds the circumference.

In the verticillate spines there is a very considerable degree of diversity in the different sections, though they all retain the same general appearance. There is nearly every possible variation in the size of the central hollow; in some it is completely filled up, and not even the foraminated ring remains to separate central from median reticulation; in others this cavity is visible, but has one or two solid bars stretching across it, showing plainly enough that the network is there; in the section figured (Plate XXXIII., fig. 38), there is an attempt at a ring round the central cavity, but it is a very bad attempt, and nothing like the true foraminated ring of the other genera, and indeed this structure does not seem to be even developed, for though the central cavity may be well marked out, there is no foraminated ring round it, for that which does encircle it is only formed by the coalescence of the inner bars of the network, and there are no holes in it, as is readily seen by using a quarter-inch objective; in yet another specimen (Plate XXXIII., fig. 39), the central hollow is twice as wide as in fig. 38, but even in it, though there does not seem to be any trace of a network occupying it, there is no indication of any solid ring, properly so called. The only place where I have found any representation of the true ring is in the longitudinal median section (Plate XXXIII., 40), where it may be seen plainly enough at one (the distal) end, as the lining sheath, but the foramina are irregular both in size and disposition, and if the sheath be traced downwards towards the proximal part, it will be seen rapidly to merge into what might be called a reticulated, rather than a foraminated sheath. The same section shows well the slight serration which exists along the border, contrasting forcibly in this respect with most of the other forms.

In the midst of all this variability, however, the facies of the spine is preserved. This is exhibited mainly in the shape and disposition of the solid pieces, for the reticulated tissue is not remarkable for anything but the irregular size of its cells, which are smaller towards the circumference. The solid pieces always exist in the form of spokes running out from the outer border of the central or median network, and on reaching the circumference, dilate into wedges, which in their typical form (*i.e.*, if taken at the base of the serration) resemble nearly equilateral triangles, with a slightly convex base, and decidedly concave sides; but if the section pass through the teeth of the serration, this shape is exchanged for one which is more like the blade of some of the paddles used by Polynesian islanders, than anything else I have seen, consisting of a rounded free extremity which becomes slightly narrower, and then expands on each side into a narrow process with either a sharp or a rounded end, and then having thus reached its broadest part rapidly narrows to the solid radiating piece. Towards the distal end of the spine, the chief difference from the structure just described seems to be, that the central hollow becomes large and well defined, being scarcely ever occupied by a network, and that the terminal solid pieces become more prominent, and thus increase the distinctness of the verticillations. The spines in the aboral or abactinal part of the corona seem to consist of little else than a tube of limestone, with small oval regularly arranged foramina. Owing to their extreme fineness, I have not as yet succeeded in getting a transverse section of any of them, but the specimen figured (Plate XXXIII., fig. 41), which is a fragment rendered transparent by boiling in Canada balsam, shows no indication of any other kind of structure.

Method of Preparing the Sections.

The spines are first boiled in Canada balsam until the tissue has become thoroughly permeated by it, and the balsam itself has been rendered perfectly hard; this not only makes the spines transparent and gets rid of the air, but also prevents the interstices becoming filled up by particles of dirt,

as well as giving them sufficient consistency to bear further manipulation. This is specially needed in the case of spines like the conical ones of Centrostephanus, when it becomes next to an impossibility to mount the sections without breaking off one or more of the "urns." The spine having been thus prepared, slices of one-sixteenth to one-eighth of an inch in thickness are taken off with a fine saw such as is used in cutting fretwork designs, and, a drop of balsam having been previously hardened by boiling on a glass slide, the slice is attached to it by whichever side happens to be flattest. The upper surface is then ground and polished on a Water of Ayr stone, and, having been first washed with water and thoroughly dried, is then washed with a brush dipped in chloroform; the section is then turned over so as to place the polished side downwards, and the upper surface is treated in the same way, the grinding being continued till the section is reduced to the required thinness; a final washing with chloroform is then resorted to, and the specimen mounted under a thin glass cover with a drop of balsam. I have found chloroform preferable to turpentine, as it is evaporated so much more readily, besides dissolving off the superficial soiled layer of balsam more quickly. I have not found any practical advantage in tying a number of spines together, or in putting a number of sections on the same slide and reducing them simultaneously; if the spines be too slender to bear cutting with the fret saw (*e.g.*, the spines of Astropyga, Asthenosoma, or Diadema), I have found that the easiest way was to lay the spine, previously saturated with hard balsam, on a slide, and cut off small pieces by the sudden pressure of a penknife.

GENERAL REMARKS.

The first point which naturally suggests itself for consideration is the question of how far spine structure may be depended on for taxonomic characters. The evidence, so far as I have been able to collect it, not only in the Diadematidæ, but also in the Cidaridæ, Arbaciïdæ, Echinometridæ, and Echinidæ, is altogether in favour of the supposition that there is in each genus a central type form, or in the case of dimorphic spines, two type forms, of

which there are more or less discernible specific variations, and that this generic type is as constant as any other single generic character. It has been often stated that the structure of a spine varies according to the part of the shaft from which it is taken; but I think that the foregoing remarks and the figures show plainly that this variation, even at its maximum, is very slight, and so far as my experience of the genera of other families goes, the same thing seems to be true of them. One difficulty, however, occurs in the genus Echinothrix, which certainly presents in the two species which I have examined two forms of structure which seem to be generically distinct, and hence I have been led to doubt somewhat the correctness of Professor Agassiz's step in doing away with Dr. Gray's genus Garelia, which he himself had formerly retained as a sub-genus to include *E. turcarum*, but in his Revision he remarks that he has seen a greater difference in the spines between *E. Desorii* and *E. calamaris*, than between the latter and *E. turcarum*. This however seems to refer only to external appearance, which is by no means a safe indication of internal structure; and if one may judge by his general description of spine structure in the Diadematidæ, Professor Agassiz has only examined spines from one species, and that species is *E. calamaris*, though if I were to judge by the two sections photographed, I would say they belonged to *E. turcarum*, for they agree closely with its structure, and are therefore very different from the two specimens of the former species which I have examined; and hence I am disposed to think that the true Echinothrix type of structure is to be found in *E. turcarum*, and that the specimens of *E. calamaris* which I have come across are divergences from the true type. This is rendered the more probable from the fact that Dr. Gray's genus Garelia is co-extensive with Dr. Peters' Echinothrix, including as it does specimens of *E. calamaris* and *E. Desorii*, as well as of *E. turcarum*—that is, if Professor Agassiz's views are correct.

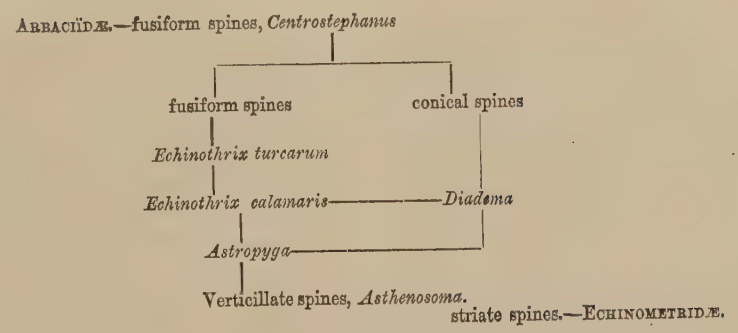
As regards specific differences, it is not so easy to speak decidedly. Leaving Echinothrix out of account, I have had the opportunity of contrasting species from two of the genera, Diadema and Centrostephanus, and in the former the difference in structure between *D. mexicanum* and *D. setosum* is very slight indeed, and fully confirms the doubt which evidently

exists in Professor Agassiz's mind as to whether they can be considered distinct; for after mentioning the stouter and less finely verticillate condition in *D. mexicanum* as possible indications of specific differences, where so few exist, he says, "the pedicellariæ do not help us in the comparison," nor, I would add, do the spines afford us much aid either. I have already alluded to the much greater differences in spine-structure which obtain between *D. antillarum* (Phil.), and *D. setosum* (Gray), though whether this persists throughout all the individuals formerly included under *D. antillarum* or not, I will not venture to predict; it may be worth noticing, however, in this connexion, that the sections figured in Quekett's "Histology" (p. 225), already alluded to, seem to be taken from *D. setosum* and *D. antillarum*, the latter especially being very fairly represented, though the former is not very accurate. Not having examined the other species of Astropyga, nor of Asthenosoma, I cannot state what differences may obtain in them; further observations on this subject are much wanted, and till they are forthcoming it cannot be laid down with certainty what value in classification may be placed on spine-structure, though I think I have shown that they are at least as characteristic as the spicules, which Mr. C. Stewart's investigations* demonstrate to be at least generically, if not specifically, peculiar.

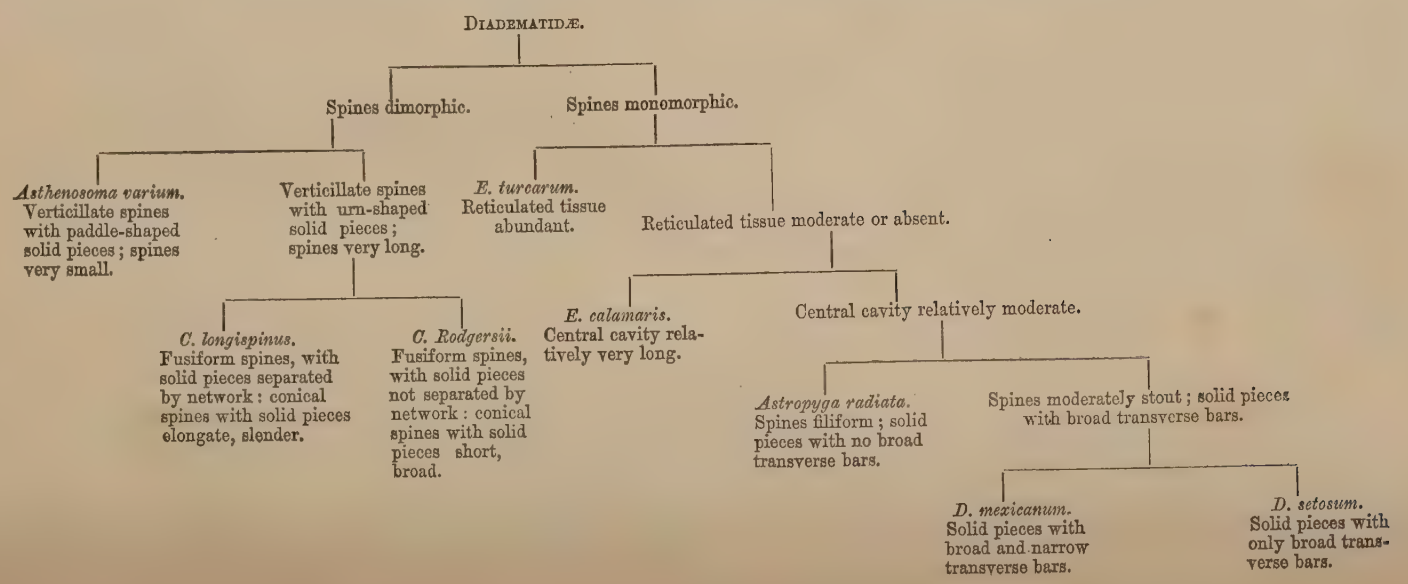
I append a scheme showing the affinities, both generic and familiar, as indicated by spine-structure, as well as an analysis of the family constructed on the same data—not, of course, with the idea of superseding the usual method of characterising by arcs of pores, structure of actinal and abactinal systems, &c., but rather to indicate what a convenient "key" may be formed in this way, in order to determine the position of a specimen.

* On the Spicules of the Regular Echinoidea. Transactions of the Linnæan Society, vol. xxv., p. 365. In my summary of the literature of this subject on page 521, I had forgotten to mention Mr. Stewart's paper, "On the Minute Structure of Certain Hard Parts of the Genus Cidaris. "Quarterly Journal of Microscopical Science," vol. xi., n. s., p. 51, Plate IV.

AFFINITIES OF DIADEMATIDÆ.



ANALYSIS OF GENERA AND SPECIES OF DIADEMATIDÆ.



As regards the mode of growth of the spines of Echinoidea, little or nothing definite is known. Dr. Carpenter * thinks that the membrane covering the socket is continued over the rest of the spine, and that new formations may be presumed to take place in its substance, though he says this membrane cannot be clearly traced to any distance from the base. I have never found it in the dried specimen extending beyond the lower surface of the milled ring, and doubt very much if it goes up the shaft at all. Quekett † remarked, that though he was satisfied of the fact that the corona is covered by an organic membrane, which is commonly said to cover the spines also, yet he had never been able to detect any traces of it on the latter, notwithstanding that he had acted on the fresh spines with acid, which however shows the existence of organic matter very small in quantity except at the proximal part.

The striæ which are so marked a feature in the solid wedges, not only in Diadematidæ, but also in all the other genera which I have examined, and which surround an excentric spot (like the hilum of a starch granule) placed usually at the pointed or internal end of the wedge, are, I suppose, indications of periodic deposition of calcic carbonate, &c., though how the spine increases in size, more especially in diameter, is a point which I have not yet determined; the absence of anything like concentric arrangement, such as is so prominently seen in Echinometridæ, points out an entirely different method of development; and I doubt if the explanation which has been put forward to account for the absence of rings in our native Echini—that the spines are formed and cast off annually—will apply here, for spines of various sizes may be observed to coëxist on the tests of Centrostephanus at least, if not on most of the others.

Another point of interest is, as to what is contained in the cavity of the spine of the living specimen.

The peculiar appearance of the foraminated sheath, its invariable occurrence, and equally constant structure, seem to indicate a special function, and to point out that it is a specialization of the characteristic Echinoderm tissue. That it is a specialization cannot be doubted, for I

* The Microscope and its Revelations, 4th Edition, p. 558. Dr. Carpenter has since abandoned this view, and believes that he has proved the existence of an axis of protoplasm in the spine. "Monthly Microscopical Journal," vol. iii., 1870, p. 225.

† *Loc. cit.*, p. 228.

have not seen anything like it elsewhere, either in Asteroidea, or in Echinoidea; the nearest approach, perhaps, is to be had in the ovarian spicules of *Goniocidaris geranioides*, figured by Mr. C. Stewart,* but this is only a remote semblance; but that it is only a specialization of the usual tissue, and not a different kind, is, I think, equally clear from the fact that in Asthenosoma we have the foraminated sheath gradually merging into the ordinary network. Before I had seen Dr. Carpenter's paper (*loc. cit.*), the idea suggested itself to my mind that the cavity was occupied by an axis of protoplasm, or, perhaps, by a cord of cells like the ectoderm of Cœlenterata, and that through these foramina prolongations of the protoplasm, or processes of the cells enter into, and thence ramify through the other parts of the spine. Dr. E. Perceval Wright suggested to me the possibility of these spines containing cnidæ, basing the inference on the extremely severe effects caused by portions of the spines of the living animals breaking off in the skin of the hand, and giving rise to intense pain and swelling of the arm, which in his case lasted for some time—results which seemed too great to be merely the consequence of the mechanical irritation caused by the intrusion of foreign particles; for though Professor Leith Adams tells me he has known very severe pain caused by treading on a common Echinus, it is a very different thing putting the naked foot on a bristling sea-urchin, and getting the spines driven up into it, and merely touching the tips of the spines with one's fingers, which produced the effects mentioned by Professor Wright; moreover, since cnidæ exist undoubtedly in Synaptæ, it seems by no means improbable that they may be found in Echinoidea too, and certainly none of the order are better adapted to contain cnidæ than are the hollow spined Diadematidæ. Still, it is possible that the irritation mentioned might be caused by a large quantity of animal matter, such as would be the case if the cavity of the spine were filled with either sarcode or ectodermoid cells.†

In conclusion, I cannot but express my regret that the foregoing remarks are so deficient in many points of interest; still, I trust, that as a contribution to our knowledge of the spine-structure of a group hitherto almost unknown, this paper may not be altogether valueless.

* *Loc. cit.*

† The stinging property of the spines of this family has been known from the time of Rumphius.

EXPLANATION OF THE PLATES.

Plate XXXI.

Diadema mexicanum.

Fig. 1. Transverse section of head of spine.
Fig. 2. Transverse section of milled ring.
Fig. 3. Longitudinal section of head.
Fig. 4. Transverse section of proximal region of shaft.

Diadema setosum.

Fig. 5. Transverse section of median region of shaft.
Fig. 6. Transverse section of distal region of shaft.
Fig. 7. Transverse section of distal region of shaft (injured spine).
Fig. 8. Transverse section of median region of shaft (*D. antillarum*, Phil.)

Centrostephanus longispinus.

Fig. 9. Transverse section of socket of conical spine.
Fig. 10. Transverse section of median region of conical spine.
Fig. 11. Transverse section of distal region of conical spine.
Figs. 12, 13. Transverse section of conical spine (injured).
Fig. 14. Transverse section of proximal region of fusiform spine.
Fig. 15. Longitudinal section of fusiform spine (injured).

Plate XXXII.

Centrostephanus Rodgersii.

Fig. 16. Transverse section of head of conical spine.
Fig. 17. Transverse section of milled ring of conical spine.
Fig. 18. Transverse section of distal region of conical spine.
Fig. 19. Transverse section of proximal region of fusiform spine.
Fig. 20. Transverse section of fusiform spine (injured).

Echinothrix calamaris.

Fig. 21. Transverse section of head.
Fig. 22. Transverse section of proximal region.
Fig. 23. Transverse section of median region.
Fig. 24. Transverse section of injured spine.
Fig. 25. Longitudinal section of median region.

Echinothrix turcarum.

Fig. 26. Transverse section of head.
Fig. 27. Transverse section of milled ring.
Fig. 28. Transverse section of proximal region.

Plate XXXIII.

Echinothrix turcarum.

Fig. 29. Transverse section of injured spine.
Fig. 30. Transverse section of distal region.

Astropyga radiata.

Fig. 31. Transverse section of milled ring.
Figs. 32, 33. Transverse section of proximal region.
Fig. 34. Longitudinal section of median region.
Fig. 35. Transverse section of distal region.

Asthenosoma varium.

Fig. 36. Transverse section of head.
Fig. 37. Transverse section of proximal region.
Fig. 38, 39. Transverse section of median region.
Fig. 40. Longitudinal section of median region.
Fig. 41. Portion of median region of abactinal spine.

The drawings have been all executed under the superintendence of Mr. G. H. Ford. The fact of the artist working at a distance, which rendered personal supervision impossible, will account for the absence of a few minor details which would otherwise have appeared. In every other respect I think they will be found to leave nothing to be desired.

[Since the above paper was communicated to the Academy, Professor Dr. Peters of Berlin has kindly sent me, through my friend, Professor E. P. Wright, spines from the original specimen of *C. longispinus*, and it now appears that the form which I have here alluded to under that name is nothing more than a variety of *C. Rodgersii*. In my next communication I hope to describe the structure of these spines, as well as those of *E. Desorii*, which Professor Dr. Peters has also sent me.—*June* 15, 1875.]

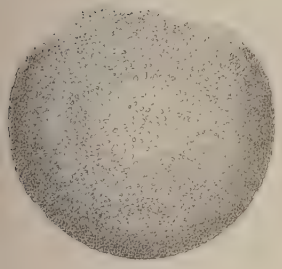
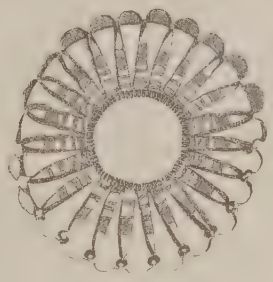


Fig. 1.

Fig 2



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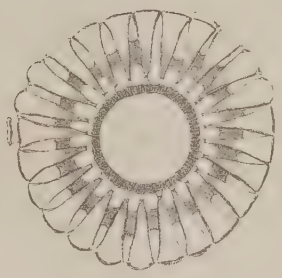

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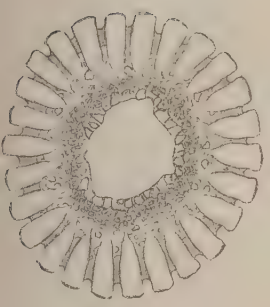

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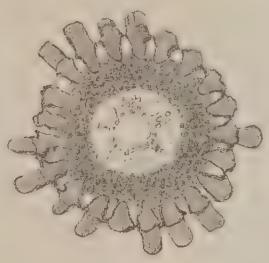
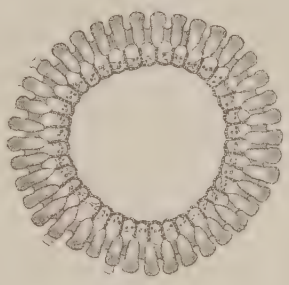


Fig 7.

Fig 8.



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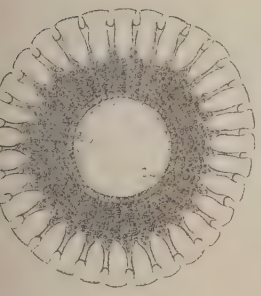

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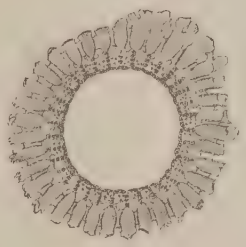

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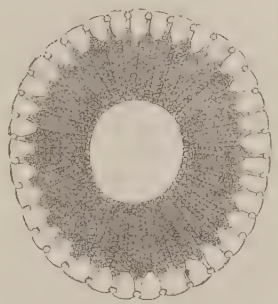

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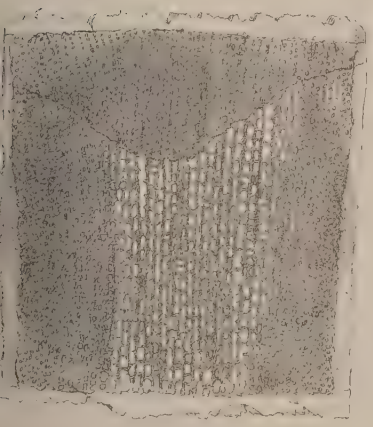

Fig 3.

Fig. 9

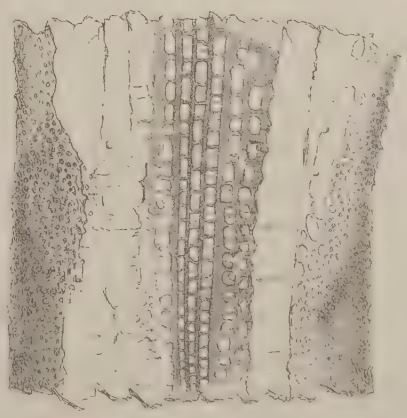

Fig. 15

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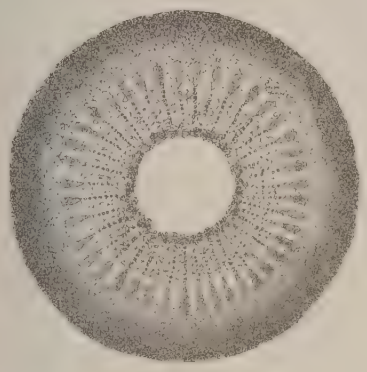

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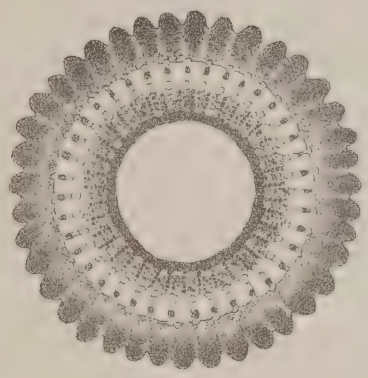

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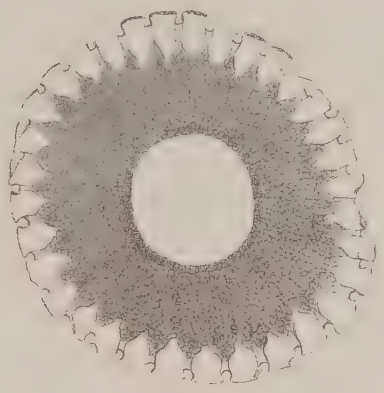

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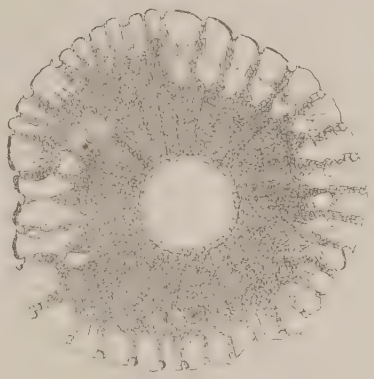

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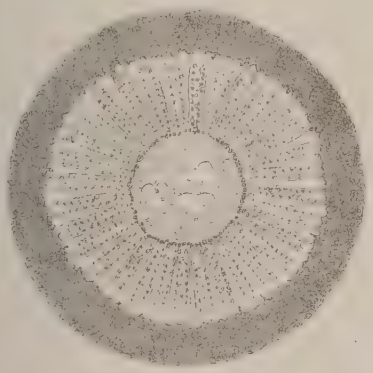

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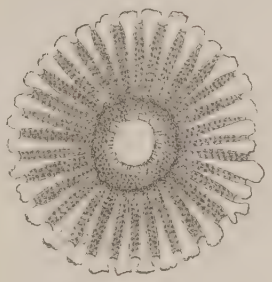

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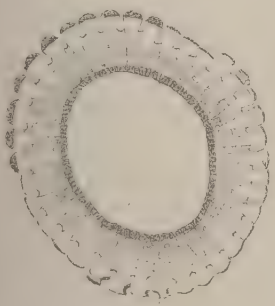

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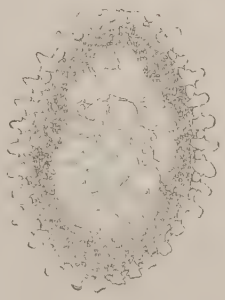

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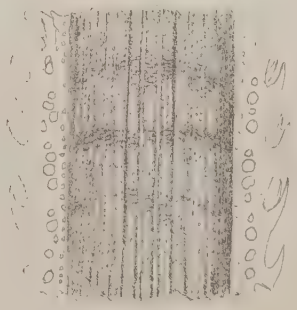

Fig. 25

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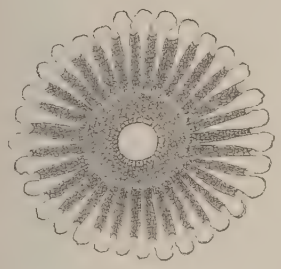

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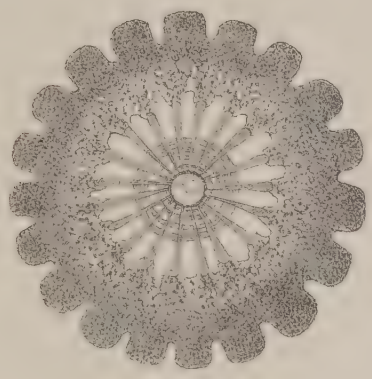

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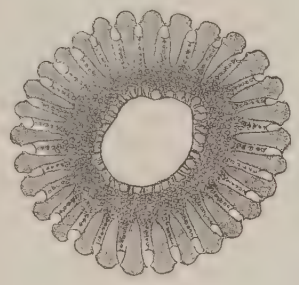

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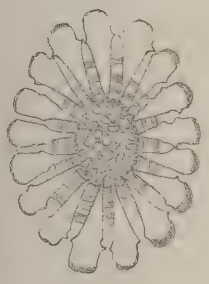

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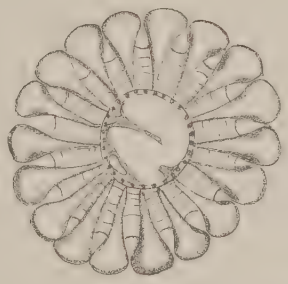

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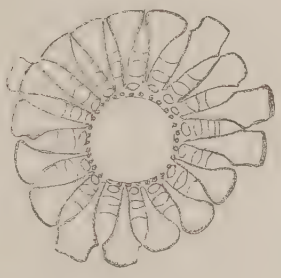

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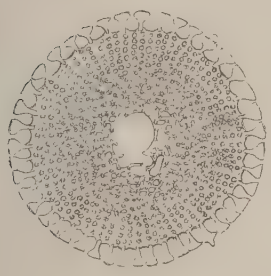

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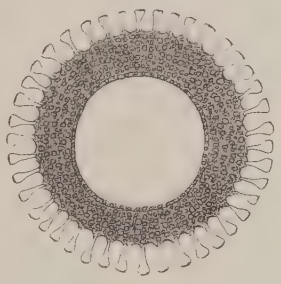

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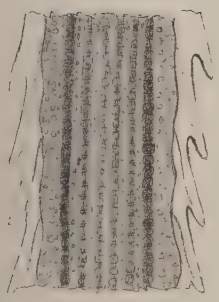

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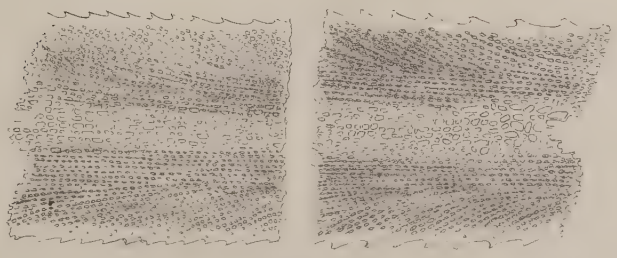

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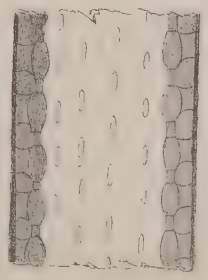

Fig. 41.

G.H. Ford E. Smith.

Mintern Bros. imp.

Diadematidæ.

XVII.—*On Nine-point Contact of Cubic Curves.* *By* A. S. Hart, LL. D., S. F. T. C. D.

Read April 26, 1875.

In addition to double points, or nodes, on cubic curves, there are, as is known, other singular points at which the curve has contact of the highest possible order with other lines; of these points, the most familiar are the nine (real or imaginary) points of inflexion at which the curve has three-point contact with right lines. In addition to these, there are the twenty-seven points of contact of tangents drawn from the points of inflexion, at which the curve has six-point contact with conics, and seventy-two other points at each of which it has nine-point contact with an indefinite number of other cubics.

These latter points (which appear to me to possess a peculiar interest in the theory of cubic curves) have been discussed by Dr. Salmon in a memoir* read before the Royal Society, on the 17th June, 1858, in which he not only determined the number of the points, but also gave in a very simple form the equation by which they are to be found. The equation of the curve being given in the canonical form.

In the present paper I propose to give the complete solution of this equation, and also to give its general form in terms of the invariants and covariants of the cubic; but first I would call attention to the geometrical position of the points, and show that in general six of them are real, and the rest imaginary.

First, then, it is known that the points in question are those which

* Proceedings of the Royal Society of London, Vol. ix., p. 333.

coincide with their third tangentials, and therefore that they must lie on the serpentine branch of the curve, as the tangential of every point lies on that branch; now the serpentine branch is divided into three parts, A B, B C, C A, by the three points of inflexion A, B, C, and it is easy to see that two of the nine-point contacts occur in each of these parts. For if the first point moves from A towards B, its tangential will move in a retrograde direction from A through C to B, where it will meet the first point; and while this first tangential moves from A to C, the second tangential will move from A through B to C, and as the first completes its course to B, the second will go on from C through A to B, where it will overtake the original point; in like manner, while the second tangential proceeds in a direct course from A through B, C, A, and B a second time, the third tangential will move in a retrograde direction through C to B, and thence twice round the serpentine branch, and therefore it must meet the original point twice between A and B. In like manner, there will be two points of concourse between B and C, and two others between C and A, and these are evidently the only real points of nine-point contact besides the points of inflexion; but as there are also six imaginary points of inflexion, and as these and the real points lie in threes on twelve right lines, it follows that there are twelve sets of points of nine-point contact corresponding to these twelve lines, making seventy-two points in all, of which the first six only are real.

The equation by which Dr. Salmon determines these points on the curve whose equation is $x^3 + y^3 + z^3 + 6mxyz = 0$ is

$$\{(x^3 - y^3)^2 + (y^3 - z^3)^2 + (z^3 - x^3)^2\}\ \{x^6y^3 + y^6z^3 + z^6x^3 - 3x^3y^3z^3\}\ \{x^3y^6 + y^3z^6 + z^3x^6 - 3x^3y^3z^3\} = 0.$$

Performing the multiplications, and adopting Dr. Salmon's notation for the invariants and covariants of the cubic, this equation may be reduced to the form (A)

$$\Theta^4 + 54S\Theta^2H^4 - 27T\Theta H^6 - 243S^2H^8 = 0.$$

It was, in fact, in this form that I had obtained the equation before I was aware of Dr. Salmon's investigation, and it led immediately to the obvious

conclusion, that, being a homogeneous equation between two variables Θ and H^2, it must be capable of being resolved into factors, each of which is of the sixth degree in the co-ordinates x, y, z, an important result, which is universally true for every determination of singular points on a cubic by an algebraic relation between the covariants. The factors in the present case are

$$(\Theta + 9m^2H^2)\,(\Theta - 3\,(m-1)^2\,H^2)\,(\Theta^2 + 3\,(1-2m-2m^2)\,H^2 + 9\,(1+m+m^2)\,H^4).$$

The first of which is identical with Dr. Salmon's factor

$$(x^3 - y^3)^2 + (y^3 - z^3)^2 + (z^3 - x^3)^2,$$

but the other factors not being of the same degree as his, it follows that the equation is capable of further reduction, and in fact these factors are easily reduced to the six cubic factors

$$(x^2y + y^2z + z^2x) \quad (x^2y + \theta y^2z + \theta^2z^2x) \quad (x^2y + \theta^2y^2z + \theta z^2x)$$

$$(xy^2 + yz^2 + zx^2) \quad (xy^2 + \theta yz^2 + \theta^2zx^2) \quad (xy^2 + \theta^2yz^2 + \theta zx^2)$$

(θ being an imaginary value of $\sqrt[3]{1}$). In like manner the first factor is reduced to the two cubics

$$(x^3 + \theta y^3 + \theta^2z^3) \quad (x^3 + \theta^2y^3 + \theta z^3),$$

and combining this with the equation of the curve, it can be further reduced to the linear factors (a being taken $= \sqrt[3]{(1 + 8m^3)}$)

$$(\sqrt[3]{(1-a)}\,x + \sqrt[3]{(1-\theta a)}\,y + \sqrt[3]{(1-\theta^2 a)}\,z)\;(\sqrt[3]{(1-a)}\,x + \theta\sqrt[3]{(1-\theta a)}\,y$$

$$+ \theta^2\sqrt[3]{(1-\theta^2 a)}\,z)\,(\sqrt[3]{(1-a)}\,x + \theta^2\sqrt[3]{(1-\theta a)}\,y + \theta\sqrt[3]{(1-\theta^2 a)}\,z)$$

$$(\sqrt[3]{(1-a)}\,x + \sqrt[3]{(1-\theta^2 a)}\,y + \sqrt[3]{(1-\theta a)}\,z)\;(\sqrt[3]{(1-a)}\,x + \theta\sqrt[3]{(1-\theta^2 a)}\,y$$

$$+ \theta^2\sqrt[3]{(1-\theta a)}\,z)\;(\sqrt[3]{(1-\theta a)}\,x + \theta^2\sqrt[3]{(1-\theta^2 a)}\,y + \theta\sqrt[3]{(1-\theta a)}\,z),$$

all of which are of course imaginary.

The remaining cubic factors may in like manner be reduced to linear factors by substituting for the coefficient m the trigonometrical expression $\frac{1+\sqrt{3}\cot A}{4}$, and denoting by α^3, β^3, γ^3 the three values of $\sin\frac{A}{3}$. The factors will then be

$$\left(\frac{\alpha}{\beta}x+\frac{\beta}{\gamma}y+\frac{\gamma}{\alpha}z\right)\left(\frac{\beta}{\gamma}x+\frac{\gamma}{\alpha}y+\frac{\alpha}{\beta}z\right)\left(\frac{\gamma}{\alpha}x+\frac{\alpha}{\beta}y+\frac{\beta}{\gamma}z\right)\left(\frac{\beta}{\alpha}x+\frac{\gamma}{\beta}y+\frac{\alpha}{\gamma}z\right)$$

$$\left(\frac{\gamma}{\beta}x+\frac{\alpha}{\gamma}y+\frac{\beta}{\alpha}z\right)\left(\frac{\alpha}{\gamma}x+\frac{\beta}{\alpha}y+\frac{\gamma}{\beta}z\right)\left(\frac{\alpha}{\beta}x+\frac{\beta}{\gamma}y+\theta\frac{\gamma}{\alpha}z\right)\left(\frac{\beta}{\gamma}x+\frac{\gamma}{\alpha}y+\theta\frac{\alpha}{\beta}z\right)$$

$$\left(\frac{\gamma}{\alpha}x+\frac{\alpha}{\beta}y+\theta\frac{\beta}{\gamma}z\right)\left(\frac{\beta}{\alpha}x+\frac{\gamma}{\beta}y+\theta\frac{\alpha}{\gamma}z\right)\left(\frac{\gamma}{\beta}x+\frac{\alpha}{\gamma}y+\theta\frac{\beta}{\alpha}z\right)\left(\frac{\alpha}{\gamma}x+\frac{\rho}{\alpha}y+\theta\frac{\gamma}{\beta}z\right)$$

$$\left(\frac{\alpha}{\beta}x+\frac{\beta}{\gamma}y+\theta^2\frac{\gamma}{\alpha}z\right)\left(\frac{\beta}{\gamma}x+\frac{\gamma}{\alpha}y+\theta^2\frac{\alpha}{\beta}z\right)\left(\frac{\gamma}{\alpha}x+\frac{\alpha}{\beta}y+\theta^2\frac{\beta}{\gamma}z\right)$$

$$\left(\frac{\beta}{\alpha}x+\frac{\gamma}{\beta}y+\theta^2\frac{\alpha}{\gamma}z\right)\left(\frac{\gamma}{\beta}x+\frac{\alpha}{\gamma}y+\theta^2\frac{\beta}{\alpha}z\right)\left(\frac{\alpha}{\gamma}x+\frac{\rho}{\alpha}y+\theta^2\frac{\gamma}{\alpha}z\right)$$

The line represented by the first of these factors meets the curve in the points

$$\left(\frac{\alpha}{\gamma},\ \frac{\beta}{\alpha},\ \frac{\gamma}{\beta}\right),\quad \left(\frac{\beta}{\alpha},\ \theta\frac{\gamma}{\beta},\ \theta^2\frac{\alpha}{\gamma}\right),\quad \left(\frac{\beta}{\alpha},\ \theta^2\frac{\gamma}{\beta},\ \theta\frac{\alpha}{\gamma}\right),$$

and so of the others; the real points are therefore

$$\left(\frac{\alpha}{\gamma},\ \frac{\beta}{\alpha},\ \frac{\gamma}{\beta}\right),\quad \left(\frac{\beta}{\alpha},\ \frac{\gamma}{\beta},\ \frac{\alpha}{\gamma}\right),\quad \left(\frac{\gamma}{\beta},\ \frac{\alpha}{\gamma},\ \frac{\beta}{\alpha}\right),\quad \left(\frac{\alpha}{\beta},\ \frac{\beta}{\gamma},\ \frac{\gamma}{\alpha}\right),\quad \left(\frac{\gamma}{\alpha},\ \frac{\alpha}{\beta},\ \frac{\beta}{\gamma}\right),\quad \left(\frac{\beta}{\gamma},\ \frac{\gamma}{\alpha},\ \frac{\alpha}{\beta}\right),$$

the first of which is the tangential of the second, the second of the third, and the third of the first, and similarly with regard to the fourth, fifth, and sixth points.

Again, it will be seen that through each of these points there pass not only the real line given by the preceding equations, which cuts the cubic in

two imaginary points, but also two imaginary lines, each of which cuts the cubic in two other of the imaginary points. Thus, through the first of the points given above there will pass the two imaginary lines

$$\frac{\gamma}{\alpha}x+\theta\frac{\alpha}{\beta}y+\theta^2\frac{\beta}{\gamma}z=0,\quad \frac{\gamma}{\alpha}x+\theta^2\frac{\alpha}{\beta}y+\theta\frac{\beta}{\gamma}z=0,$$

which meet the curve respectively in the points

$$\left(\frac{\alpha}{\gamma},\ \theta\frac{\beta}{\alpha},\ \theta^2\frac{\gamma}{\beta}\right),\ \left(\frac{\gamma}{\beta},\ \theta^2\frac{\alpha}{\gamma},\ \theta\frac{\beta}{\alpha}\right);\quad \left(\frac{\alpha}{\gamma},\ \theta^2\frac{\beta}{\alpha},\ \theta\frac{\gamma}{\beta}\right),\ \left(\frac{\gamma}{\beta},\ \theta\frac{\alpha}{\gamma},\ \theta^2\frac{\beta}{\alpha}\right);$$

and so of the others.

In like manner the line $\sqrt[3]{(1-a)}\,x+\sqrt[3]{(1-\theta a)}\,y+\sqrt[3]{(1-\theta^2 a)}\,z=0$ meets the curve in the imaginary points

$$\left(\frac{2m+1-\theta^2 a}{\sqrt[3]{(1-\theta^2 a)}},\ \frac{2m+1-a}{\sqrt[3]{(1-a)}},\ \frac{2m+1-\theta a}{\sqrt[3]{(1-\theta a)}}\right),\ \left(\frac{2m+1-\theta a}{\sqrt[3]{(1-\theta a)}},\ \frac{2m+1-\theta^2 a}{\sqrt[3]{(1-\theta^2 a)}},\right.$$

$$\left.\frac{2m+1-a}{\sqrt[3]{(1-a)}}\right);\ \left(\frac{2m+1-a}{\sqrt[3]{(1-a)}},\ \theta\frac{2m+1-\theta a}{\sqrt[3]{(1-\theta a)}},\ \theta^2\frac{2m+1-\theta^2 a}{\sqrt[3]{(1-\theta^2 a)}}\right);$$

and through the first of these imaginary points will also pass the two lines

$$\sqrt[3]{(1-\theta a)}\,x+\sqrt[3]{(1-\theta^2 a)}\,y+\sqrt[3]{(1-a)}\,z=0,$$

and

$$\sqrt[3]{(1-\theta^2 a)}\,x+\theta\sqrt[3]{(1-a)}\,y\times\theta^2\sqrt[3]{(1-\theta a)}\,z=0.$$

The four factors into which I have resolved the original equation appear to correspond to the four groups, of three right lines in each, which contain in each group the nine points of inflexion, and we have now seen that the eighteen points of nine-point contact determined by each of these factors lie three by three on eighteen right lines, three of the lines passing through each of the points.

Returning to the two groups of real points, the equations of the tangents at the three points

$$\left(\frac{\alpha}{\gamma}, \frac{\beta}{\alpha}, \frac{\gamma}{\beta}\right), \quad \left(\frac{\beta}{\alpha}, \frac{\gamma}{\beta}, \frac{\alpha}{\gamma}\right), \quad \left(\frac{\gamma}{\beta}, \frac{\alpha}{\gamma}, \frac{\beta}{\alpha}\right),$$

are

$$\frac{\beta}{\gamma}(\gamma^3-\alpha^3)\,x+\frac{\gamma}{\alpha}(\alpha^3-\beta^3)\,y+\frac{\alpha}{\beta}(\beta^3-\gamma^3)\,z=0=X$$

$$\frac{\gamma}{\alpha}(\alpha^3-\beta^3)\,x+\frac{\alpha}{\beta}(\beta^3-\gamma^3)\,y+\frac{\beta}{\gamma}(\gamma^3-\alpha^3)\,z=0=Y\,;$$

$$\frac{\alpha}{\beta}(\beta^3-\gamma^3)\,x+\frac{\beta}{\gamma}(\gamma^3-\alpha^3)\,y+\frac{\gamma}{\alpha}(\alpha^3-\beta^3)\,z=0=Z,$$

and the equation of the curve referred to these three lines as axes of co-ordinates is

$$2\alpha\beta\gamma\,(X^2Y+Y^2Z+Z^2X)+(3\sin A-\sqrt{3}\cos A)\,XYZ=0.$$

Writing this equation in the form

$$x^2y+y^2z+z^2x+2pxyz=0=U.$$

The invariants and covariants will be $S=-p^4-3p$

$$T=-8p^6-36p^3-27. \quad H=x^3+y^3+z^3-p^2(x^2y+y^2z+z^2x)-(2p^3+3)\,xyz$$

$$\Theta=3p^2H^2+(6p^4+9p)\,HU+(9p^6+36p^3+54)\,U^2-27\,(x^5z+y^5x+z^5y)$$
$$-54p\,(x^4z^2+y^4x^2+z^4y^2)-36p^2\,(x^3y^3+y^3z^3+z^3x^3)$$
$$-xyz\,\{162pU+135\,(x^2z+y^2x+z^2y)+216p^2xyz\}.$$

Therefore when $U=0$, the equation (A) will become

$$\{\Theta-3p^2H^2\}\,\{(\Theta-3p^2H^2)^3+12p^2H^2(\Theta-3p^2H^2)^2$$
$$-162pH^4(\Theta-3p^2H^2)+729H^6\}=0. \qquad (B)$$

The first factor in this equation may be reduced to

$$(3x^2z+3y^2x+3z^2y-4p^2xyz)\,(x^3+y^3+z^3+2p\,(x^2z+y^2x+z^2y)+6xyz),$$

which can be further reduced to the linear factors

$$(ax-y)\,(ay-z)\,(az-x)\,(Ax+By+Cz)\,(Bx+Cy+Az)\,(Cx+Ay+Bz),$$

the coefficients of which are determined by the equations

$$a^3+2pa^2+\tfrac{4}{3}p^2a=1$$

$$A^3-2p\,A^2B-\sqrt[3]{(27+8p^3)}\,AB^2-B^3=0,$$

with similar equations for $B:C$ and $C:A$. As each of these equations must have one real root, it follows that the first factor of the equation (B) may be replaced by the equations of six real right lines, each of which will cut the curve in one of the six real points already determined, and will also pass through two of the imaginary points.

XVIII.—*Experiments on the Movements of Water in Plants.* (Part II.) *By* William Ramsay McNab, M. D., Edin., Professor of Botany, Royal College of Science for Ireland.

Read May 10, 1875.

Last autumn, during a visit to the Royal Agricultural College, Cirencester, I had an opportunity of experimenting still further on the movements of fluid in plants. The results were not altogether satisfactory, on account of the bad weather which prevailed during the latter part of summer and the beginning of the autumn. Two sets of experiments were, however, made. In the one series a mixed solution was given to the plant, and the different substances tested for by means of the spectroscope. In the other series the actual quantity of water present in the stem was determined with some care, with the view of finding out how much transpirable water there might be in the stem at a given time. In all, fifteen separate experiments were made and recorded, and I shall here detail them in order. In the former part of this Report* the experiments were lettered A and B, and numbered. The present series I shall letter C.

I. ADDITIONAL SPECTROSCOPIC EXPERIMENTS.

Several experiments were made with different substances, either simply or mixed, but in many cases no results were obtained, and I have, therefore, not considered it necessary to mention more than three experiments. In former experiments I had used Lithium Citrate and Thallium Citrate, but obtained the most satisfactory results with Lithium. New experiments

* *Vide* Transactions, Vol. xxv., page 343.

were made with Thallium Citrate and with a salt of Indium, kindly prepared for me by Professor Church, of Cirencester, to whom my best thanks are again due for his valuable assistance and advice. Indium gave a splendid band in the spectroscope, when some of the solution of the salt was placed on a platinum wire, and the wire heated red-hot; but repeated attempts to test for it in the internodes of the Cherry Laurel, after placing the cut ends of the shoot in the solution for some time, all resulted in failure.

The next experiments were made with a salt of Caesium, supplied by Messrs. Hopkin and Williams, of London. Professor Church kindly made a Citrate of Caesium for me, which I used, in one case with success, in another with no result. The high price of the substance is an objection to its employment, and as the results were not very encouraging I ceased to use it. In the successful experiment with Caesium it was administered along with the Citrates of Thallium and Lithium,* all three being afterwards found in the internodes by means of the spectroscope.

Experiment C 1. Cherry Laurel. Sept. 4, 1874.

A branch 13·1 inches in length was placed in a solution containing the Citrates of Caesium, Thallium, and Lithium, and allowed to absorb the solution for one hour. The experiment was begun at 9.44, A. M., and ended at 10.44. The morning was dull, there being no sun, and the temperature 15·2 Cent. Towards the end of the experiment the sky was brighter, but the temperature had not increased, neither was the sun visible. On removing the shoot from the solution it was cut up into ten pieces, the section being made below each node, the leaf being also detached. There were thus ten internodes and ten leaves. After drying in the water-oven, the whole were examined by means of the spectroscope, and gave the following results:—

* A five per cent. solution was employed.

Internode.	Length of Internode.	Length of Leaf.	Lithium in Leaf.	Lithium in Stem.	Th.	Caes.
1	0·5 inch.	4·5 inches.	3·55 inches.	0·2 inch.		
2	1·2 ,,	4·5 ,,	3·6 ,,	1·2 ,,		
3	1·3 ,,	4·85 ,,	4.25 ,,	1·3 ,,		
4	1·35 ,,	4·8 ,,	4·4 ,,	1·35 ,,		
5	1·55 ,,	4·5 ,,	3·75 ,,	1·55 ,,	0·3 inch.	
6	1·4 ,,	4·55 ,,	4·0 ,,	1·4 ,,	1·4 ,,	
7	1·5 ,,	3·9 ,,	3·5 ,,	1·5 ,,	1·5 ,,	
8	1·5 ,,	4·6 ,,	4·3 ,,	1·5 ,,	1·5 ,,	0·5 inch.
9	1·25 ,,	4·6 ,,	4·3 ,,	1·25 ,,	1·25 ,,	1·25 ,,
10	1·55 ,,	4·5 ,,	4·2 ,,	1·55 ,,	1·55 ,,	1·55 ,,
	13·1			12·8	7·5	3·3

The internodes are here numbered in succession from above downwards, No. 1 being the youngest at the apex of the shoot. No. 10 the oldest at the base of the shoot experimented on. The second column gives the length in inches of each internode, the third giving the length in inches of the attached leaf. In the fourth column the distance to which the Lithium could be traced in each leaf is given. The fifth, sixth, and seventh columns show the distance to which the Lithium, Thallium, and Caesium were traced. The Lithium had gone into every leaf, and within 0·3 of an inch of the apex of the shoot, or, in other words, it was only in the terminal bud, and a small part of the stem below, that no Lithium could be detected. Thallium was observed in the fifth internode, but only at the base, and could not be detected in any of the leaves. The Caesium appeared in the eighth internode, which it had ascended for one-third of its length. As in the case of the Thallium, so with the Caesium, no trace of it could be obtained in the leaves or petioles. In a shoot of Cherry Laurel 13·1 inches in length, Lithium had ascended 12·8 inches; Thallium, 7·5; and Caesium, 3·3 inches in one hour, the day being dull and the temperature only 15·2 Cent.

Experiment C 2. Cherry Laurel. Sept. 5, 1874.

A branch of Cherry Laurel, measuring 5·5 inches, and bearing two leaves, the upper 5·1 inches long, the lower 4·2 inches, was placed in a solution of Caesium Citrate for an hour, and allowed to absorb. The sun was bright during the experiment, and the temperature 15·0 Cent. After drying and examining the shoot with the spectroscope, no Caesium could be detected, except at the base where the stem had been in the solution. Here we have an experiment made apparently under favourable conditions, and yet no traces of Caesium could be obtained in stem or leaf.

Experiment C 3. Cherry Laurel. Sept. 14, 1874.

In this experiment a very large shoot was taken, four feet in length. The parts of the stem for 1874, 1873, and 1872, were entire and uninjured. The length of the growth for 1874 was 16·2 inches; for 1873, 18·5 inches; and for 1872, 12·55 inches; the total length being thus exactly 47·25 inches. The whole was allowed to absorb solution of Lithium Citrate for one hour. The experiment began at 10·10 A.M., the sun shining throughout the experiment, but not brightly, the temperature 23° Cent. The object of this experiment was to determine whether the fluid would move more rapidly through the wood than through the cortical portion of the stem. If the movement of the Lithium be due merely to diffusion, it ought to diffuse rapidly through the watery cortex, but if the movement of the Lithium be due to the special rapid current which exists in the wood portion of the fibro-vascular bundles, then the ascent of fluid in the cells of the cortex would bear no relation to the ascent in the wood.

On examining the stem, no Lithium was found in the part of the stem formed in 1874 and 1873, but the Lithium could be detected throughout the whole of the portion for 1872, 12·55 inches in length. A careful examination of the cortex and stem down to the bast was made, and no Lithium could be observed above the portion which had been immersed in the fluid,

namely, a distance of 1·3 inches. No diffusion whatever had taken place through the cells of the cortex. This is an exceedingly important fact, and it shows that even with a large quantity of water in the cells of the cortex, diffusion is slow. Recently C. Erhart * has made experiments on diffusion by injecting various fluids subcutaneously, and all his experiments give a slow rate of diffusion even with Lithium.

II. QUANTITY OF WATER IN STEM OF CHERRY LAUREL.

In describing these experiments, I shall first give the details of each one separately, and shall afterwards give the general results.

Experiment C 4. Cherry Laurel. Sept. 12, 1874.

Two small lateral branches, developed during 1874, and springing from the shoot produced in 1873, were first examined. They weighed together 2·6325 grammes. The cortical tissues were then removed, the bast portion of the fibro-vascular bundle being included with the cortex, the separation being made at the cambium layer. The wood and pith were then weighed, the weight of the cortical portion being determined by difference. The wood weighed 1·01 gramme, the cortex would therefore weigh 1·6225 gramme. Taking the total weight to be represented by 100, the wood was 38·4 per cent., the cortex 61·6 per cent. of the whole. The wood was next placed in the water-oven, and dried at a temperature not exceeding 100° Cent., until it ceased to lose weight. The last weighings gave the weight of the dry wood as 0·534 gramme, the loss being thus equal to 0·476 gramme, or 18·08 per cent. of the total weight of the branches taken.

Experiment C 5. Cherry Laurel. Sept. 12, 1874.

One lateral shoot of 1874 growing from the wood of 1873 was taken, cut into four pieces, and weighed; the total weight being 3·975 grammes.

* C. Erhart. Ueber subcutane Injection bei Pflanzen. See "Botanischer Jahresbericht," Vol. I. (1873), p. 347.

The pieces were then placed in the water-oven and dried; when the weight became constant they were found to weigh 1·583 gramme, or to have lost 2·392 grammes of water. The total water in the lateral shoot of 1874, taking wood and cortex together, thus amounted to 60·17 per cent. of weight of shoot taken.

Experiment C 4. Cherry Laurel. Sept. 12, 1874.

The terminal shoot of 1874 was taken and weighed, the weight being 4·091 grammes. The cortical tissues were then separated at the cambium layer, and the wood with the pith weighed. The weight after the removal of the cortical tissues was found to be 2·736 grammes. The cortical tissues, as determined by difference, would therefore be 1·355 gramme. The proportion of wood to cortex was very different from that in the lateral shoots of 1874, being, if the whole be taken as 100, wood, 66·9 per cent., cortex, 33·1 per cent. The numbers are in fact almost reversed, the lateral shoot producing much more cortex than the main shoot, which chiefly develops the wood portion of the fibro-vascular bundles. The wood was dried in the water-oven, and when it ceased to lose weight was found to weigh 0·9075 gramme; the loss was therefore equal to 1·817 gramme, or 44·69 per cent. of total weight of shoot. This specimen must have been an exceptional one, because, in making two other experiments in order to verify the results obtained, the water was found to be only one-half the amount got in this experiment.

Experiment C 7. Cherry Laurel. Sept. 12, 1874.

A terminal shoot of 1874, weighing 4·491 grammes, was taken and dried in the water-oven until it ceased to lose weight. The final weighing gave 1·84 gramme, the loss being thus 2·651 grammes, or 59·02 per cent. of total weight of branch.

Experiment C 8. Cherry Laurel. Sept. 12, 1874.

A portion of the main axis for 1873 was taken for the next experiment. It weighed 4·515 grammes. The cortical tissues were then removed as before, and the wood and pith were found to weigh 2·3575 grammes. The weight of the cortex as determined by difference would thus be 2·1575 grammes, and the relative proportions, taking the whole shoot as 100, would be, wood, 52·2, cortex, 47·8. The wood was now placed in the water-oven and dried until constant, the final weighings giving 1·3175 gramme as weight of dry wood. The loss was thus 1·04 gramme, or 23·03 per cent. of total weight of shoot.

Experiment C 9. Cherry Laurel. Sept. 12, 1874.

A portion of the main shoot for 1873, weighing 6·3855 grammes, was taken and dried in the water-oven; when the branch ceased to lose weight, it was found to weigh 2·959 grammes. The loss was thus 3·4265 grammes, or 53·66 per cent. of total weight of the shoot.

If we take these six experiments and compare them together, we get an idea of the relative quantities of water in the cortex and wood of the different shoots. First, let us examine the lateral shoots of 1874, experiments C 4 and C 5. By experiment C 5, we learn that the total quantity of water present in the young lateral shoots was equal to 60·2 per cent. of weight of shoot, thus giving 39·8 as the percentage of dry substance of wood and cortex. By experiment C 4, the actual weight of the wood was determined, and the actual weight of water in the wood; and we can easily determine by difference the quantity of water in the cortex, as well as the weight of the cortex. Taking the whole stem in its fresh state as 100, we get the following percentages:—

Dry wood, ...	20·3
Water in wood, ...	18·1
Dry cortex (by difference), . . .	19·5
Water in cortex (by difference), .	42·1
Total,	100·0

From this we learn that there is twice as much water present in the cortical portion of the stem that there is in the wood and pith.

In the examination of the main axis for 1873, the same general results were obtained. The total quantity of water present, as determined by experiment C 9, was 53·7 per cent., the dry matter being thus 46·3 per cent. The whole stem, taken in its fresh state at 100, gave the following percentages:—

Dry wood, ...	29·2
Water in wood, ...	23·0
Dry cortex (by difference), . . .	17·2
Water in cortex (by difference), .	30·6
Total,	100·0

In experiments C 6, and C 7, the results were not what had been anticipated. Some error may have been made, but of that I am doubtful, and can only account for the results by supposing that the shoot was an exceptional one. The quantity of water present was twice the dry weight of the wood. Experiment C 7 gave the total weight of water in the shoot at 59·0 per cent., the dry substance thus being 41·0 per cent. The other percentages, calculated as above, were:—

Dry wood, ...	22·2
Water in wood, ...	44·7
Dry cortex (by difference), . . .	18·8
Water in cortex (by difference), .	14·3
Total,	100·0

This result led me to make two other sets of experiments, in order to test the accuracy of the above. I found, however, that this was quite an exceptional case, the other two experiments giving results quite in accordance with the percentages expected.

In the six experiments numbered C 10 to C 15, both wood and cortex were carefully examined. The portions of cortex were placed in test-tubes, and thus all loss other than water was prevented during drying and weighing.

Experiment C 10. Cherry Laurel. Sept. 14, 1874.

A terminal shoot for 1874, which weighed 1·963 gramme, was taken. The cortex was carefully removed. The wood and pith weighed 0·8055 gramme, while the cortex weighed 1·1575 gramme. Both were then carefully weighed until they ceased to lose weight, the dry wood weighing 0·399 gramme, the dry cortex, 0·3525 gramme. The wood, therefore, lost 0·4065 gramme, and the cortex, 0·805 gramme of water. Putting the above figures in percentage form, we have the following results:—

Dry wood, ...	21·0
Water in wood, ...	20·0
Dry cortex, ...	18·0
Water in cortex, ...	41·0
Total,	100·0

Experiment C 11. Cherry Laurel. Sept. 14, 1874.

Terminal shoot for 1874 taken, and weighed. The total weight was 1·949 gramme. The wood weighing 0·8745, and the cortex, 1·0745 gramme, respectively. Both were then carefully dried until the weight became constant. The dry weight of the wood was 0·445 gramme, the dry weight of the cortex, 0·336 gramme; the loss of water being in the wood equal to 0·4295 gramme, in the cortex, 0·7385 gramme. Taking the percentage results as in the last experiment, we have these results, viz.:—

Dry wood, ...	22·9
Water in wood, ...	22
Dry cortex, ...	17·2
Water in cortex, ...	37·9
Total,	100·0

Experiment C 12. Cherry Laurel. Sept. 14, 1874.

The main shoot for 1873 was now examined, with the following results:—Branch weighed 3·849 grammes, the wood weighing 2·0915 grammes, and the cortex, 1·7575 gramme. When dried till the weight was constant, the wood was reduced to 1·1295 gramme, and the cortex to 0·6185 gramme; the loss of water being thus for wood, 0·962 gramme, and for the cortex, 1·139 gramme. In percentage form, we have these results:—

Dry wood,	29·3
Water in wood,	25
Dry cortex,	16·1
Water in cortex,	29·6
Total,	100·0

Experiment C 13. Cherry Laurel. Sept. 14, 1874.

In this experiment, a shoot of 1873, similar to that used in C 12, was taken. It weighed 2·877 grammes; the wood weighing 1·46 gramme, and the cortex, 1·417 gramme. After drying, the weight of the wood was 0.8 gramme, of the cortex, 0·502 gramme; the loss of water being from the wood, 0·66 gramme, for the cortex, 0·915 gramme. Or, taking the percentages as in former experiment:—

Dry wood,	27·8
Water in wood,	22·9
Dry cortex,	17·5
Water in cortex,	31·8
Total,	100·0

Experiment C 14. Cherry Laurel. Sept. 14, 1874.

A lateral shoot for 1874, produced from the main shoot of 1873, weighed 1·39675 gramme; the wood weighing 0·5505 gramme, the cortex,

0·84625 gramme. When dried until constant, the wood weighed 0·2745 gramme, and the cortex, 0.2785 gramme; the loss of water being 0·276 gramme from the wood, and 0·5677 gramme for the cortex. Or, taking the percentages, thus:—

Dry wood,	19·6
Water in wood,	19·8
Dry cortex,	20·0
Water in cortex,	40·6
Total,	100·0

Experiment C 15. Cherry Laurel. Sept. 14, 1874.

A shoot similar to that used in Experiment C 14 was taken. It weighed 0·8375 gramme; the wood being 0·303 gramme, the cortex, 0·5345 gramme. When dried, the wood weighed 0·148 gramme, and the cortex, 0·1775 gramme; the loss of water being in wood, 0·155 gramme, in cortex, 0·357 gramme. Or, taking percentages, thus:—

Dry wood,	17·7
Water in wood,	18·5
Dry cortex,	21·2
Water in cortex,	42·6
Total,	100·0

GENERAL RESULTS.

If we take the mean of all these experiments, omitting C 6 as being exceptional, we get the following general results:—

	Terminal Shoots, 1874. C 10 and C 11.	Main Shoots, 1873. C 8, C 12, and C 13.	Lateral Shoots, 1874. C 4, C 14, and C 15.
Wood, dry, . . .	22	29	19
Cortex, dry, . . .	18	17	20
Water in wood, . .	21	26	19
Water in cortex, .	39	31	42
Total water, . . .	60	54	60
Total dry substance,	40	46	40

From this table, giving the mean of eight experiments, we learn that the percentage of dry substance and water in terminal and lateral shoots is the same, although there is less wood and more cortex in lateral than in terminal shoots—while the cortex contains twice as much water, in proportion to its dry weight, as the wood. Comparing the terminal and lateral shoots of 1874 with the main shoots of 1873, we observe that the dry weight of the wood has increased, while the dry weight of the cortex has diminished; the relative proportions of water, however, remain almost unchanged, there being still twice as much water in the cortex, in proportion to its dry weight, as there is in the wood. Then, lastly, if we compare the dry weight of the wood of the main shoot of 1873 with that of the main shoot of 1874, we observe that the addition of an annual ring of wood has not added very largely to the dry weight, the increase only being from twenty-two to twenty-nine.

We observe that the water in the wood varies in quantity from nineteen to twenty-six per cent., while the water in the cortex varies from thirty-one to forty-two per cent.; and that, notwithstanding the larger quantity of water present, Lithium, as demonstrated by former experiments detailed by me in this Report, moves very slowly in the cortex, while it may move forty inches per hour in the wood.

Then, lastly, the quantity of transpirable water must be very small compared with the total water in the shoot. Much of the water in the wood must enter into the composition of the cell-wall, and could not be removed without causing the death of the part. If half the water present in the wood at any given time be contained in the walls of the cells, there will only remain from ten to thirteen per cent. of water which could move through the stem to be transpired by the leaves. Thus, although my original estimate of six or seven per cent. of transpirable water be too low, I do not think the quantity can exceed twelve or fourteen per cent. of the total weight of the shoot.

XIX.—*On the Binary Stars σ Coronae, τ Ophiuchi, γ Leonis, ζ Aquarii, 36 Andromedae, and ι Leonis.* By Dr. W. Doberck, Astronomer at Colonel Cooper's Observatory, Markree, Co. Sligo.

Read June 28, 1875.

In presenting the following calculations to the Royal Irish Academy, I beg to give some account of the works which I have perused, with the view of collecting observations of those binary stars, of which I have considered a determination of the orbit to be possible, and of interest. The library of the Markree Observatory, of which I have been the director for the past year, contains nearly all the older useful astronomical books, and the present proprietor has, in the course of the last year, shown that he is both able and willing to furnish it with the more recent publications. In the meantime the Rev. Dr. Robinson, at Armagh, and Professor Ball, at Dunsink, have kindly allowed me to search in their libraries for the latest measures of double stars, but I found but few which I had not already noticed.

The observations made by the Herschels, Dawes, Fletcher, Miller, Jacob, and Powell, are to be found in the memoirs or monthly notices of the Royal Astronomical Society. Kaiser's and Maedler's first measures are contained in the "Astronomische Nachrichten;" also the measures taken by Engelmann, Winnecke and Dembowski. I have not been able to consult Maedler's "Untersuchungen ueber die Fixsternsysteme," but have found many observations of the double stars, with which I have been occupied, in the "Dorpat observations." I have also found observations in the following works:—

W. Struve: Mensurae Micrometricae Stellarum duplicium et multiplicium per magnum Fraunhoferi tubum annis a 1824 ad 1837 in specula

Dorpatensi institutae (adjecta est Synopsis Observationum de Stellis compositis Dorpati annis 1814 ad 1824, per minora instrumenta perfectarum). Petropoli, 1837.

Encke: Astronomische Beobachtungen auf der Koeniglichen Sternwarte zu Berlin. Zweiter Band. 1844.

Sir J. Herschel: Results of Astronomical Observations made during the years 1834–38, at the Cape of Good Hope, being a completion of a telescopic survey of the whole surface of the visible Heavens, commenced in 1825. London, 1847.

Captain Smyth: Cycle of Celestial Objects. Vol. II. The Bedford Catalogue. London, 1844.

Vice-Admiral Smyth: The Cycle of Celestial Objects continued at the Hartwell Observatory to 1859. London, 1860.

G. Bishop: Astronomical Observations taken at the Observatory, South Villa, Inner Circle, Regent's Park, London, during the years 1839–51. London, 1852.

G. Barcley: Leyton Astronomical Observations. Part III. London, 1873.

Kaiser: Annalen der Sternwarte in Leiden. III. Band.

Secchi: Misure e catalogo di 1321 stelle doppie osserv. al Coll. Romano. Roma, 1859–60.

Robert Main: Observations made at the Radcliffe Observatory. Oxford,

E. Luther: Annalen der Sternwarte in Koenigsberg.

σ Coronae Borealis.

Herschel discovered in 1781 that σ Coronae was double, and in 1802 he recognised its binary character. The components are of the 6th and 7th magnitudes, both white; but the principal star is yellowish, and the companion is bluish. The motion is direct. The distance was in the commencement small, but afterwards it increased rapidly, thus rendering the observations much easier. A redetermination of the elements, in which these later observations were taken into account, seemed to me likely to decide

upon the question of its period, upon which astronomers hitherto did not agree, as can be seen from the following table :—

T	Node.	λ	γ	Period.	a	e	Authority.
1835.60	138° 0′	7°18′	41°15′	286·6	3″·679	0·6112	J. Herschel.
1826.60	25 7	64 38	29 29	608·45	3 ·918	0·6998	Maedler.
1826.48	21 3	69 24	25 39	736·88	5 ·194	0·7256	Hind.
1829.70	3 8	96 53	45 6	240·00	2 ·94	0·3887	E. B. Powell.
1831.17	1 57	101 57	46 47	195·12	2 ·717	0·3088	Jacob.

Employing, as in my former memoir, Herschel's method, I drew the interpolating curve as nearly as possible through the points indicated by the observations, and thus obtained the angles and distances exhibited in Table I. By the aid of these I derived the elements shown in Table II.

TABLE I.

THE ANGLES OF POSITION AND DISTANCES IN THE PROJECTED ELLIPSE, DEDUCED FOR THE CORRESPONDING EPOCHS BY THE INTERPOLATING CURVE.

No.	Epoch.	Angle of Position.	Distance.
1	1780	346°·8	1·546
2	1790	352 ·8	1·054
3	1800	5 ·9	0·765
4	1810	27 ·7	0·605
5	1820	60 ·2	0·520
6	1825	80 ·1	0·484
7	1830	104 ·0	0·454
8	1835	128 ·0	0·480
9	1840	147 ·0	0·566
10	1845	161 ·3	0·655
11	1850	171 ·6	0·733
12	1855	180 ·0	0·847
13	1860	186 ·4	0·953
14	1865	191 ·5	1·018
15	1870	196 ·0	1·096
16	1875	199 ·0	1·147

TABLE II.

ELEMENTS OF σ CORONAE BOREALIS.

T	1828.91.
Node,	6° 43′
λ	89 17
γ	29 40
Period,	843^{yrs}.2
a	6·″001
e	0· 7502

Table III. shows the comparison of the calculated angles and distances, with those given by observations; I feel assured that I should not have been able to represent the observations with any period so short as three hundred years.

TABLE III.

COMPARISON OF THE CALCULATED ANGLES OF POSITION AND DISTANCES OF σ CORONAE WITH THOSE GIVEN BY THE OBSERVATIONS.

Observer.	No.	Epoch.	θ_o	θ_c	$\theta_o - \theta_c$	ρ_o	ρ_c	$\rho_o - \rho_c$
SIR W. HERSCHEL,	1	1781·79	347° 32′	349° 41′	− 2° 9′	—	—	—
,,	2	1802·74	11 24	12 0	− 0 36	—	—	—
SIR J. HERSCHEL &	3	1821·30	65 15	59 31	+ 5 44	—	—	—
SOUTH, ...	4	23·47	72 56	68 41	+ 4 15	1″·45	1″·39	+ 0″·06
SIR J. SOUTH, . .	5	1825·44	77 31	78 3	− 0 32	1·48	1·36	+ 0·12
Sir J. HERSCHEL, .	6	1828·50	92 6	93 37	− 1 31	—	—	—
,,	7	30·28	105 5	102 50	+ 2 15	1·22	1·31	− 0·09
,,	8	31·36	108 46	108 24	+ 0 22	1·38	1·32	+ 0·06
,,	9	32·52	113 33	114 9	− 0 36	1·07	1·34	− 0·27
,,	10	33·26	119 55	117 48	+ 2 7	1·33	1·36	− 0·03
SMYTH, ...	11	1830·76	107 36	105 19	+ 2 17	1·3	1·31	− 0·01
,,	12	32·37	114 54	113 24	+ 1 30	1·4	1·34	+ 0·06
,,	13	33·58	120 42	119 17	+ 1 25	1·2	1·36	− 0·16
,,	14	35·50	130 54	128 1	+ 2 53	1·4	1·42	− 0·02
,,	15	39·67	145 6	144 28	+ 0 38	1·6	1·60	0·00

TABLE III.—(*continued*).

Observer.	No.	Epoch.	θ_o	θ_c	$\theta_o - \theta_c$	ρ_o	ρ_c	$\rho_o - \rho_c$
SMYTH, ...	16	1843·35	155°54′	155°55′	− 0° 1′	1″·8	1″·78	+ 0″·02
,,	17	46·60	162 24	164 8	− 1 44	2·0	1·95	+ 0·05
,,	18	52·25	176 48	175 26	+ 1 22	2·2	2·25	− 0·05
DAWES, ...	19	1830·52	107 17	104 4	+ 3 13	—	—	—
,,	20	31·34	111 32	108 17	+ 3 15	1·57	1·32	+ 0·25
,,	21	32·55	115 24	114 19	+ 1 5	—	—	—
,,	22	33·36	120 37	118 13	+ 2 24	1·30	1·36	− 0·06
,,	23	34·55	125 36	123 47	+ 1 49	—	—	—
,,	24	37·47	136 51	136 16	+ 0 35	—	—	—
,,	25	39·53	144 20	143 58	+ 0 22	1·60	1·59	+ 0·01
,,	26	40·57	147 51	147 30	+ 0 21	1·66	1·64	+ 0·02
,,	27	41·48	150 19	150 24	− 0 5	1·66	1·69	− 0·03
,,	28	42·37	153 16	153 7	+ 0 9	—	—	—
,,	29	43·47	156 30	156 14	+ 0 16	1·77	1·79	− 0·02
,,	30	47·44	166 0	166 3	− 0 3	1·885	2·00	− 0·115
,,	31	48·53	168 38	168 22	+ 0 16	1·995	2·05	− 0·055
,,	32	49·45	170 6	170 14	− 0 8	2·09	2·11	− 0·02
,,	33	51·42	173 49	173 57	− 0 8	2·265	2·21	+ 0·055
,,	34	53·63	177 52	177 46	+ 0 6	2·39	2·33	+ 0·06
,,	35	54·56	178 28	179 16	− 0 48	2·26	2·37	− 0·11
HIND, ...	36	1846·32	162 46	163 30	− 0 44	—	—	—
FLETCHER, . . .	37	1851·22	174 22	173 35	+ 0 47	2·32	2·20	+ 0·12
MILLER, ...	38	1852·31	176 27	175 32	+ 0 55	2·38	2·26	+ 0·12
JACOB, ...	39	1846·21	161 58	163 14	− 1 16	2·25	1·93	+ 0·32
,,	40	53·14	177 54	176 58	+ 0 56	2·18	2·30	− 0·12
,,	41	54·05	177 52	178 28	− 0 36	2·22	2·35	− 0·13
TALMAGE, . . .	42	1865·72	189 8	193 33	− 4 25	—	—	—
,,	43	66·43	189 14	194 17	− 5 3	3·73	2·98	+ 0·75
,,	44	68·93	194 28	196 33	− 2 5	3·61	3·10	+ 0·51
,,	45	71·86	197 20	198 48	− 1 28	3·32	3·23	+ 0·09
COPELAND, . . .	46	1872·28	196 32	199 6	− 2 34	—	—	—
,,	47	73·40	196 44	200 1	− 3 17	—	—	—
,,	48	74·33	198 49	200 50	− 2 1	—	—	—
POWELL, ...	49	1853·35	175 12	177 19	− 2 7	—	—	—
MAIN, ...	50	1867·37	192 4	195 9	− 3 5	3·00	3·02	− 0·02
W. STRUVE, . .	51	1819·62	48 0	53 29	− 5 29	—	—	—
,,	52	27·02	89 21	85 56	+ 3 25	1·31	1·31	0·00
,,	53	28·20	96 30	92 3	+ 4 27	—	—	—
,,	54	30·11	104 54	101 57	+ 2 57	1·22	1·30	− 0·08
,,	55	32·99	118 48	116 29	+ 2 19	1·30	1·35	− 0·05
,,	56	35·50	130 28	128 1	+ 2 27	1·31	1·42	− 0·11
,,	57	36·59	134 44	132 41	+ 2 3	1·435	1·46	− 0·025
,,	58	37·55	139 58	136 36	+ 3 22	1·42	1·50	− 0·08
,,	59	38·45	143 22	140 3	+ 3 19	1·48	1·54	− 0·06
GALLE, ...	60	1839·52	147 50	143 56	+ 3 54	1·55	1·59	− 0·04
KAISER, ...	61	1841·66	148 49	151 0	− 2 11	1·57	1·69	− 0·12
,,	62	43·68	156 20	156 50	− 0 30	1·66	1·80	− 0·14

TABLE III.—(*continued*).

Observer.	No.	Epoch.	θ_o	θ_c	$\theta_o - \theta_c$	ρ_o	ρ_c	$\rho_c - \rho_o$
KAISER, ...	63	1866·68	193°56′	194°30′	− 0°34′	2″·86	2″·98	− 0″·12
MAEDLER, . . .	64	1836·47	138 30	132 11	+ 6 19	—	—	—
,,	65	41·56	152 17	150 40	+ 1 37	1·60	1·46	+ 0·14
,,	66	42·31	156 22	152 56	+ 3 26	1·81	1·69	+ 0·12
,,	67	42·73	157 35	154 10	+ 3 25	1·87	1·75	+ 0·12
,,	68	46·46	165 5	163 49	+ 1 16	2·07	1·94	+ 0·13
,,	69	47·44	166 35	166 3	+ 0 32	2·16	2·00	+ 0·16
,,	70	48·41	168 22	168 7	+ 0 15	2·40	2·05	+ 0·35
,,	71	50·70	173 1	172 37	+ 0 24	2·235	2·17	+ 0·065
,,	72	51·25	174 32	173 38	+ 0 54	2·34	2·20	+ 0·14
,,	73	51·76	176 14	174 33	+ 1 41	2·44	2·22	+ 0·22
,,	74	52·31	176 27	175 32	+ 0 55	2·38	2·26	+ 0·12
,,	75	52·60	177 32	176 2	+ 1 30	2·395	2·27	+ 0·125
,,	76	53·38	177 42	177 22	+ 0 20	2·46	2·31	+ 0·15
,,	77	53·77	178 45	178 1	+ 0 44	2·65	2·33	+ 0·32
,,	78	54·70	179 26	179 30	− 0 4	2·515	2·38	+ 0·135
,,	79	55·78	181 50	181 9	+ 0 41	2·64	2·44	+ 0·20
,,	80	57·39	183 18	183 28	− 0 10	2·46	2·52	− 0·06
,,	81	58·54	183 34	185 3	− 1 29	2·64	2·58	+ 0·06
WINNECKE, . . .	82	1855·54	181 33	180 47	+ 0 46	2·49	2·42	+ 0·07
,,	83	56·39	182 47	182 3	+ 0 44	2·52	2·47	+ 0·05
ENGELMANN, . .	84	1864·45	190 31	192 9	− 1 38	3·115	2·88	+ 0·235
,,	85	65·39	192 42	193 13	− 0 31	2·96	2·92	+ 0·04
SECCHI, ...	86	1855·59	180 5	180 51	− 0 46	2·32	2·42	− 0·10
,,	87	56·43	182 23	182 7	+ 0 16	2·46	2·47	− 0·01
,,	88	57·62	183 34	183 48	− 0 14	2·43	2·53	− 0·10
,,	89	65·81	192 27	193 40	− 1 13	2·98	2·95	+ 0·03
DEMBOWSKI, . .	90	1854·86	179 51	179 44	+ 0 7	2·37	2·39	− 0·02
,,	91	56·42	181 46	182 6	− 0 20	2·69	2·47	+ 0·22
,,	92	58·29	183 12	184 43	− 1 31	2·665	2·57	+ 0·095
,,	93	63·09	190 4	190 34	− 0 30	2·76	2·81	− 0·05
,,	94	64·95	191 14	192 44	− 1 30	2·79	2·90	− 0·11
,,	95	66·92	193 14	194 45	− 1 31	2·89	3·00	− 0·11
,,	96	68·88	195 44	196 30	− 0 46	2·99	3·09	− 0·10
,,	97	70·95	197 4	198 7	− 1 3	3·09	3·19	− 0·10
,,	98	72·96	198 5	199 39	− 1 34	3·12	3·28	− 0·16

τ OPHIUCHI.

When Sir William Herschel discovered the binary star τ Ophiuchi in 1783, he considered it the closest of all his double stars. It affords, as Dawes remarks, a surprising proof of the excellence of his instruments and the keenness of his discernment. It was suspected with a power of 460,

and 932 confirmed it to be double; and one-half, if not three-quarters, of the companion seemed to be behind the large star. ν Ophiuchi was then perfectly free from the wedge-formed appearance which τ Ophiuchi presented with the lower power. Sir J. Herschel examined it in 1830, but did not find any trace of duplicity. Struve saw it single until 1827, in which year he perceived an elongation in the direction 326° or 146°, which he partly ascribed to the effects of the refraction on this (especially for the latitude of his observatory) very low star, and partly to the reappearance of the companion. He noticed it as oblong in 1835, and since then it gradually opened, and has now become of much easier measurement. Dawes remarks that the position has become *very oblique*, which may have occasioned large personal errors. The components are of the 5th and 5½th magnitudes, both yellow.

At first only a smaller number of observations was at my disposal, through which I laid the interpolating curve, as shown in Table IV., and obtained the first approximative elements, Table V., which, however, further inquiries showed to be of but little exactness.

TABLE IV.

THE ANGLES OF POSITION AND THE DISTANCES IN THE PROJECTED ELLIPSE OF τ OPHIUCHI, DEDUCED FOR THE CORRESPONDING EPOCHS BY THE INTERPOLATING CURVE.

No.	Epoch.	Angle of Position.	Distance.
1	1783.27	331°·6	0·716
2	1800	15 ·0	0·546
3	1810	55 ·0	0·477
4	1820	106 ·0	0·473
5	1830	163 ·3	0·393
6	1835	192 ·0	0·451
7	1840	212 ·5	0·569
8	1845	224 ·9	0·684
9	1850	233 ·5	0·864
10	1855	239 ·1	1·032
11	1860	242 ·9	1·299
12	1865	245 ·3	1·474
13	1870	247 ·3	1·720
14	1875	248 ·8	1·618

TABLE V.

First Approximative Elements of τ Ophiuchi.

T	1820.63.
Node,	69° 31′
λ	28 35
γ	53 5
e	0·5818
a	1″·111
Period,	185^{yrs}. 2

Guided by these elements I proceeded to draw the interpolating curve anew, whereby I was able to take all observations into account (Table VI.), and thus I derived the elements exhibited in Table VII. At first I thought it better to deduce the annual mean motion, only using the observations made after 1835, as the star was so extremely unfavourable for measurement before that time. Table VIII. exhibits the comparison of these elements with some of the best measures.

TABLE VI.

The Angles of Position of τ Ophiuchi and the Distances after the Corrected Interpolating Curve.

No.	Epoch.	Angle of Position.	Distance.
1	1780	328°·3	1·085
2	1785	334 ·8	0·876
3	1795	352 ·9	0·624
4	1805	25 ·6	0·505
5	1815	72 ·2	0·436
6	1825	131 ·4	0·396
7	1830	163 ·0	0·486
8	1835	192 ·8	0·529
9	1840	215 ·2	0·560
10	1845	227 ·5	0·757
11	1850	234 ·5	0·908
12	1855	239 ·6	1·102
13	1860	243 ·2	1·260
14	1865	246 ·1	1·508
15	1870	247 ·7	1·988

TABLE VII.

SECOND SYSTEM OF ELEMENTS OF τ OPHIUCHI.

T	1814.0.
Node,	66° 1′
λ	36 26
γ	46 8
e	0·6055
n	+1°·521

TABLE VIII.

COMPARISON OF SOME OBSERVATIONS OF τ OPHIUCHI, WITH THE SECOND ELEMENTS.

Observer.	No.	Epoch.	$\theta_o - \theta_c$	Observer.	No.	Epoch.	$\theta_o - \theta_c$
HERSCHEL, . .	1	1783	−14°·0	DAWES, . . .	12	1854	+ 0°·1
W. STRUVE, .	2	1835	−22 ·7	KAISER, . . .	13	1865	+ 4 ·3
,,	3	1836	−17 ·6	MAIN, . . .	14	1870	+ 2 ·5
SMYTH, . . .	4	1838	− 6 ·8	DEMBOWSKI, .	15	1856	+ 0 ·9
,,	5	42	+ 0 ·6	,,	16	58	+ 2 ·0
,,	6	55	+ 0 ·2	,,	17	63	+ 0 ·9
DAWES, . . .	7	1840	− 2 ·5	,,	18	65	+ 0 ·5
,,	8	41	+ 0 ·4	,,	19	67	+ 0 ·1
,,	9	42	+ 0 ·4	,,	20	69	+ 0 ·0
,,	10	43	+ 1 ·3	,,	21	71	− 0 ·4
,,	11	48	− 0 ·3				

I then varied the annual mean motion, so as to represent Herschel's observation, and consequently corrected the longitude of the node. The result is shown in Table IX. Struve's distances were excluded in the derivation of the semiaxis major. Indeed the re-discovery in itself is so amply crediting the Dorpat refractor and the eyes of the observer, that it cannot surprise us that the measures taken under such exceedingly unfavourable conditions do not agree with the later measures, which could be taken far more easily. Table X. exhibits the comparison of the definitive elements with the observations.

TABLE IX.

DEFINITIVE ELEMENTS OF τ OPHIUCHI.

T	1818.50.
Node,	67° 1′
λ	36 26
γ	46 8
e	0·6055
P	217yrs. 87
a	1″·193

TABLE X.

COMPARISON OF THE OBSERVED ANGLES OF POSITION AND DISTANCES WITH THOSE DERIVED FROM THE THIRD SYSTEM OF ELEMENTS.

Observer.	No.	Epoch.	θ_o	θ_c	$\theta_o - \theta_c$	ρ_o	ρ_c	$\rho_o - \rho_c$
SIR W. HERSCHEL,	1	1783·27	331°36′	332°35′	− 0°59′	obl.	—	—
W. STRUVE, . .	2	1825·67	single	,,	—	—	—	—
,,	3	27·28	146 0	170 40	−24 40	obl.	—	—
,,	4	35·68	192 54	209 49	−16 55	0″·35	0″·68	−0″·33
,,	5	36·62	199 54	212 20	−12 26	0 ·44	0 ·71	−0 ·27
SIR J. HERSCHEL,	6	1830 ..	single	,,	—	—	—	—
MAEDLER, . . .	7	1835 ..	256 42	207 58	+48 44	—	—	—
,,	8	41·53	217 21	222 27	− 5 6	0 ·75	0 ·89	−0 ·14
,,	9	42·57	225 36	224 11	+ 1 25	0 ·77	0 ·91	−0 ·14
,,	10	52·66	238 34	236 8	+ 2 26	1 ·27	1 ·18	+0 ·09
,,	11	53·79	232 17	237 10	− 4 53	1 ·17	1 ·20	−0 ·03
,,	12	54·71	238 11	237 59	+ 0 12	1 ·09	1 ·23	−0 ·14
SMYTH, ...	13	1838·58	214 0	216 44	− 2 44	0 ·5	0 ·79	−0 ·29
,,	14	42·52	227 0	224 5	+ 2 55	0 ·9	0 ·93	−0 ·03
,,	15	55·34	238 48	238 32	+ 0 16	1 ·1	1 ·24	−0 ·14
DAWES, ...	16	1840·68	221 32	220 58	+ 0 34	0 ·88	0 ·85	+0 ·03
,,	17	41·66	225 42	222 42	+ 3 0	0 ·79	0 ·89	−0 ·10
,,	18	42·64	226 54	224 18	+ 2 36	—	—	—
,,	19	43·61	228 58	225 45	+ 3 13	0 ·95	0 ·95	0 ·00
,,	20	48·66	232 45	232 11	+ 0 34	1 ·01	1 ·08	−0 ·07
,,	21	54·57	238 3	237 57	+ 0 6	1 ·22	1 ·23	−0 ·01
DEMBOWSKI, . .	22	1856·22	240 8	239 15	+ 0 53	1 ·2	1 ·26	−0 ·06
,,	23	58·16	241 35	240 46	+ 0 49	1 ·2	1 ·30	−0 ·10
,,	24	63·05	244 34	244 15	+ 0 19	1 ·4	1 ·39	+0 ·01

TABLE X.—(*continued*).

Observer.	No.	Epoch.	θ_o	θ_c	$\theta_o - \theta_c$	ρ_o	ρ_c	$\rho_o - \rho_c$
DEMBOWSKI, . .	25	1865·47	245° 37′	245° 48′	− 0° 11′	1″·4	1″·44	− 0″·04
,,	26	67·06	246 6	246 46	− 0 40	1 ·44	1 ·47	− 0 ·03
,,	27	69·05	247 3	247 55	− 0 52	1 ·41	1 ·49	− 0 ·08
,,	28	71·00	247 35	249 2	− 1 27	1 ·48	1 ·53	− 0 ·05
MITCHELL, . . .	29	1846·51	229 24	229 39	− 0 15	0 ·78	1 ·02	− 0 ·24
FLETCHER, . . .	30	1851·66	239 22	235 12	+ 4 10	1 ·0	1 ·16	− 0 ·16
JACOB, ...	31	1852·65	239 31	236 8	+ 3 23	1 ·10	1 ·18	− 0 ·08
WINNECKE, . . .	32	1856·62	242 39	239 34	+ 3 5	1 ·29	1 ·27	+ 0 ·02
SECCHI, ...	33	1855·53	236 54	238 40	− 1 46	1 ·38	1 ·25	+ 0 ·13
,,	34	66·72	247 33	246 34	+ 0 59	1 ·60	1 ·46	+ 0 ·14
KAISER, ...	35	1865·52	249 23	245 50	+ 3 33	1 ·50	1 ·44	+ 0 ·06
MAIN, ...	36	1870·50	250 13	248 46	+ 1 27	1 ·63	1 ·51	+ 0 ·12
DUNÉR, ...	37	1869·64	248 11	248 18	− 0 7	1 ·41	1 ·50	− 0 ·09
,,	38	70·71	250 42	248 53	+ 1 49	1 ·26	1 ·52	− 0 ·26
,,	39	71·66	251 3	249 25	+ 1 38	1 ·31	1 ·53	− 0 ·22

After that all the calculations were finished, O. Struve's measures of this double star were sent to me from Pulkowa. The comparison of these observations checks the accuracy of the result obtained above, and the agreement is likewise quite satisfactory for the large Pulkowa refractor, especially considering this object's unfavourable position for measure in the high latitude of St. Petersburgh.

TABLE XI.

COMPARISON OF ELEMENTS OF τ OPHIUCHI WITH PULKOWA OBSERVATIONS.

Observer.	No.	Epoch.	θ_o	θ_c	$\theta_o - \theta_c$	ρ_o	ρ_c	$\rho_o - \rho_c$
O. STRUVE,	40	1840·51	223°·1	220°·7	+ 2°·4	0″·94	0″·85	+ 0″·09
,,	41	41·45	231 ·3	222 ·5	+ 8 ·8	0 ·93	0 ·88	+ 0 ·05
,,	42	41·64	225 ·2	222 ·6	+ 2 ·6	—	—	—
,,	43	41·65	227 ·8	222 ·7	+ 5 ·1	0 ·80	0 ·89	− 0 ·09
,,	44	45·65	232 ·4	228 ·6	+ 3 ·8	0 ·87	1 ·00	− 0 ·13
,,	45	46·69	231 ·6	229 ·9	+ 1 ·7	1 ·00	1 ·03	− 0 ·03
,,	46	46·69	229 ·9	229 ·9	0 ·0	0 ·93	1 ·03	− 0 ·10
,,	47	47·82	233 ·9	231 ·2	+ 2 ·7	0 ·97	1 ·06	− 0 ·09
,,	48	51·67	238 ·2	235 ·2	+ 3 ·0	1 ·19	1 ·16	+ 0 ·03
,,	49	52·64	237 ·4	236 ·1	+ 1 ·3	1 ·29	1 ·18	+ 0 ·11
,,	50	52·67	241 ·9	236 ·2	+ 5 ·7	1 ·18	1 ·19	− 0 ·01
,,	51	54·70	236 ·1	238 ·0	− 1 ·9	1 ·20	1 ·23	− 0 ·03
,,	52	55·66	240 ·4	238 ·8	+ 1 ·6	1 ·29	1 ·25	+ 0 ·04
,,	53	55·68	240 ·3	238 ·8	+ 1 ·5	1 ·32	1 ·25	+ 0 ·07
,,	54	57·67	239 ·9	240 ·4	− 0 ·5	1 ·47	1 ·29	+ 0 ·18
,,	55	57·67	240 ·5	240 ·4	+ 0 ·1	1 ·42	1 ·29	+ 0 ·13
,,	56	58·71	240 ·9	241 ·2	− 0 ·3	1 ·47	1 ·31	+ 0 ·16
,,	57	59·63	242 ·7	241 ·9	+ 0 ·8	1 ·64	1 ·33	+ 0 ·31
,,	58	61·63	242 ·9	243 ·3	− 0 ·4	1 ·43	1 ·38	+ 0 ·05
,,	59	65·72	244 ·1	246 ·0	− 1 ·9	1 ·51	1 ·44	+ 0 ·07
,,	60	66·62	243 ·3	246 ·5	− 3 ·2	1 ·75	1 ·46	+ 0 ·29
,,	61	72·58	248 ·1	249 ·9	− 1 ·8	1 ·69	1 ·54	+ 0 ·15
,,	62	74·67	251 ·1	251 ·0	+ 0 ·1	1 ·63	1 ·57	+ 0 ·06

γ LEONIS.

Algieba (γ Leonis = Σ 1424) was first measured by Sir William Herschel, when its motion was much faster than afterwards. The system has a large proper motion. Although this star has not moved through thirty degrees during the last century, still I found that a determination of the orbit would be of interest, as the measures of the angles of position might be presumed to be of a superior accuracy owing to the large distance between the components, and their size, circumstances rendering the measures so easy that, for instance, Maedler commonly observed it in full sunshine. With the naked eye this star is nearly as bright as Regulus. The principal star is deep yellow and between the second and third magnitude; the companion is pale yellow, and between the third and fourth magnitude.

I laid the observed angles down as usual, and drew the interpolating

curve, and found that the positions of the companion could be represented by a straight line, whose equation in rectangular co-ordinates is:

$$Y = X\,tg\ 123°36' - 1·370,$$

where the X axis is directed towards the east, the Y axis against the north. I, however, found the motion in this line so irregular, that I considered this result as a proof that no straight line would represent the observations. As no distances were measured before 1822, I had to adopt the distances before that time as obtained by the tangents; but I laid down the distances observed after that time as ordinates with the time as abscisse, and drew a curve between them, by which, however, startling differences appeared. But I knew that the distances were more influenced by systematical errors than the angles expressed in the same measure, and that, moreover, many discussions of observations of double stars have proved that the observers usually commence with measuring much too large distances, a circumstance which even has occurred in the use of the large refractor in Pulkowa. Besides, I do not consider it safe to reject the observed distances altogether by the determination of the orbit of so wide objects as Algieba. Table XII. exhibits the numbers, the last column contains the interpolated distances.

TABLE XII.

Angles of Position and Distances employed in the Determination of the Elements of Algieba.

No.	Epoch.	Interpolated Angle of Position.	Distance after the Projection.	Distance Interpolated.
1	1780	81°·5	1·124	—
2	1790	88 ·1	1·346	—
3	1800	93 ·0	1·532	—
4	1810	96 ·8	1·658	1·658
5	1820	100 ·2	1·785	1·749
6	1830	103 ·1	2·259	1·841
7	1840	105 ·4	2·384	1·940
8	1850	107 ·4	2·635	2·056
9	1860	109 ·1	3·101	2·173
10	1870	110 ·6	4·153	2·305

Table XIII. exhibits the elements to which I arrived, allowing greatest weight to the distances as obtained by the direct observation. Table XIV. shows the accuracy of these elements.

TABLE XIII.

Elements of Algieba.

T	1741·11
Node,	111°50′
λ	194°22′
γ	43°49′
e	0·7390
P	402yrs. 62
a	2″·00

TABLE XIV.

Comparison of the Observed Angles of Position and Distances with those derived from the Elements of Algieba.

Observer.	No.	Epoch.	θ_o	θ_c	$\theta_o - \theta_c$	ρ_o	ρ_c	$\rho_o - \rho_c$
Sir W. Herschel,	1	1782·71	83°30′	83°59′	− 0°29′	—	—	—
,,	2	1801·72	94 42	94 5	+ 0 37	—	—	—
J. H. & S., . . .	3	1822·24	98 24	100 50	− 2 26	3″·24	2″·51	+ 0″·73
Sir J. South, . .	4	1825·30	101 17	101 37	− 0 20	2 ·72	2 ·56	+ 0 ·16
Sir J. Herschel, .	5	1828·34	101 4	102 23	− 1 19	—	—	—
,,	6	29·88	104 47	102 46	+ 2 1	—	—	—
,,	7	30·28	102 17	102 52	− 0 35	3 ·02	2 ·65	+ 0 ·37
Smyth, ...	8	1831·36	103 12	103 8	+ 0 4	2 ·6	2 ·67	− 0 ·07
,,	9	33·20	102 30	103 37	− 1 7	2 ·8	2 ·70	+ 0 ·10
,,	10	36·42	104 54	104 24	+ 0 30	2 ·5	2 ·75	− 0 ·25
,,	11	39·23	106 0	105 4	+ 0 56	2 ·6	2 ·79	− 0 ·19
,,	12	43·18	107 12	105 58	+ 1 14	2 ·8	2 ·85	− 0 ·05
Dawes, ...	13	1830·39	101 49	103 53	− 2 4	2 ·54	2 ·65	− 0 ·11
,,	14	31·33	102 52	102 8	+ 0 44	2 ·52	2 ·67	− 0 ·15
,,	15	32·31	102 59	103 23	− 0 24	2 ·65	2 ·68	− 0 ·03
,,	16	33·18	104 41	103 36	+ 1 5	2 ·64	2 ·70	− 0 ·06
,,	17	41·26	106 13	105 32	+ 0 41	2 ·83	2 ·82	+ 0 ·01
,,	18	42·33	106 33	105 46	+ 0 47	2 ·72	2 ·83	− 0 ·11

TABLE XIV.—(*continued*).

Observer.	No.	Epoch.	θ_o	θ_c	$\theta_o - \theta_c$	ρ_o	ρ_c	$\rho_o - \rho_c$
DAWES, ...	19	1843·23	107°23′	105°59′	+ 1°24′	2″·82	2″·85	− 0″·03
HIND, ...	20	1845·89	107 22	106 33	+ 0 49	—	—	—
FLETCHER, . . .	21	1850·91	108 4	107 34	+ 0 30	2 ·84	2 ·95	− 0 ·11
,,	22	53·21	108 26	108 2	+ 0 24	3 ·00	2 ·98	+ 0 ·02
MILLER, ...	23	1852·12	110 9	107 48	+ 2 21	2 ·81	2 ·97	− 0 ·16
ROMBERG, . . .	24	1863·21	109 35	109 53	− 0 18	3 ·25	3 ·09	+ 0 ·16
TALMAGE, . . .	25	1866·26	109 10	110 27	− 1 17	3 ·38	3 ·12	+ 0 ·26
,,	26	67·70	110 0	110 43	− 0 43	3 ·17	3 ·13	+ 0 ·04
,,	27	70·32	110 23	111 9	− 0 46	3 ·98	3 ·16	+ 0 ·82
,,	28	71·39	108 35	111 20	− 2 45	4 ·53	3 ·17	+ 1 ·36
SECCHI, ...	29	1855·38	108 18	108 27	− 0 9	2 ·93	3 ·00	− 0 ·07
,,	30	56·95	110 28	108 44	+ 1 44	2 ·98	3 ·02	− 0 ·04
,,	31	58·87	108 7	109 6	− 0 59	3 ·05	3 ·04	+ 0 ·01
,,	32	65·04	110 17	110 13	+ 0 4	3 ·18	3 ·11	+ 0 ·07
DEMBOWSKI, . .	33	1854·76	108 52	108 19	+ 0 33	2 ·97	3 ·00	− 0 ·03
,,	34	63·28	109 17	109 54	− 0 37	2 ·86	3 ·09	− 0 ·23
,,	35	66·90	110 22	110 34	− 0 12	2 ·99	3 ·13	− 0 ·14
,,	36	70·30	110 35	111 9	− 0 34	3 ·10	3 ·16	− 0 ·06
W. STRUVE, . .	37	1821·12	99 52	100 33	− 0 41	—	—	—
,,	38	28·14	102 2	102 20	− 0 18	2 ·46	2 ·61	− 0 ·15
,,	39	31·34	103 16	103 8	+ 0 8	2 ·48	2 ·67	− 0 ·19
,,	40	32·75	103 28	103 29	− 0 1	2 ·50	2 ·69	− 0 ·19
,,	41	35·16	104 56	104 6	+ 0 50	2 ·56	2 ·73	− 0 ·17
,,	42	40·83	105 42	105 27	+ 0 15	2 ·78	2 ·81	− 0 ·03
MAEDLER, . . .	43	1840·8	105 20	105 26	− 0 6	3 ·40	2 ·81	+ 0 ·59
,,	44	41·2	105 5	105 32	− 0 27	2 ·785	2 ·82	− 0 ·04
,,	45	42·2	105 57	105 45	+ 0 12	2 ·78	2 ·83	− 0 ·05
,,	46	46·3	107 9	106 38	+ 0 31	2 ·78	2 ·89	− 0 ·11
,,	47	47·4	107 30	106 52	+ 0 38	2 ·63	2 ·91	− 0 ·28
,,	48	48·4	107 44	107 4	+ 0 40	2 ·65	2 ·92	− 0 ·27
,,	49	49·3	107 51	107 15	+ 0 36	—	—	—
,,	50	50·5	107 48	107 29	+ 0 19	2 ·80	2 ·95	− 0 ·15
,,	51	51·3	108 3	107 39	+ 0 24	2 ·74	2 ·96	− 0 ·22
,,	52	53·8	108 55	108 9	+ 0 46	2 ·81	2 ·99	− 0 ·18
,,	53	54·5	108 4	108 15	− 0 11	2 ·78	2 ·99	− 0 ·21
,,	54	55·3	107 44	108 26	− 0 42	2 ·69	3 ·00	− 0 ·31
,,	55	56·4	108 51	108 38	+ 0 13	2 ·78	3 ·02	− 0 ·24
,,	56	57·4	108 42	108 49	− 0 7	2 ·68	3 ·03	− 0 ·35
,,	57	58·4	108 44	109 0	− 0 16	2 ·94	3 ·04	− 0 ·10
,,	58	59·4	108 58	109 11	− 0 13	2 ·92	3 ·05	− 0 ·13
ENCKE, ...	59	1837·22	100 36	104 36	− 4 0	3 ·26	2 ·76	+ 0 ·50
GALLE, ...	60	1838·34	104 13	104 52	− 0 39	3 ·54	2 ·78	+ 0 ·76
,,	61	39·36	105 47	105 6	+ 0 41	2 ·90	2 ·79	+ 0 ·11
KAISER, ...	62	1840·15	107 36	105 17	+ 2 19	2 ·89	2 ·80	+ 0 ·09
,,	63	41·35	105 12	105 33	− 0 21	2 ·96	2 ·82	+ 0 ·14
,,	64	42·37	107 6	105 47	+ 1 19	2 ·72	2 ·83	− 0 ·11
,,	65	43·33	109 42	105 59	+ 3 43	2 ·97	2 ·85	+ 0 ·12
BESSEL, ...	66	1830·80	102 13	103 0	− 0 47	2 ·62	2 ·66	− 0 ·04
SCHLÜTER, . . .	67	1842·50	105 11	105 49	− 0 38	2 ·955	2 ·84	+ 0 ·11

Table XIV.—(*continued*).

Observer.	No.	Epoch.	θ_o	θ_c	$\theta_o - \theta_c$	ρ_o	ρ_c	$\rho_o - \rho_c$
Wichmann, . .	68	1846·33	105° 55′	106° 38′	− 0° 43′	3″·38	2″·89	+ 0″·49
Auwers, . . .	69	1861·32	109 37	109 33	+ 0 4	3 ·35	3 ·07	+ 0 ·28
Luther, . . .	70	1863·21	108 9	109 54	− 1 45	3 ·58	3 ·09	+ 0 ·49
Schmidt, . . .	71	1855·8	—	—	—	3 ·65	3 ·01	+ 0 ·64
Winnecke, . . .	72	1855·29	111 6	108 26	+ 2 40	3 ·07	3 ·00	+ 0 ·07
,,	73	56·29	111 36	108 38	+ 2 58	2 ·87	3 ·02	− 0 ·15
Engelmann, . .	74	1864·31	112 56	110 5	+ 2 51	3 ·39	3 ·10	+ 0 ·29
,,	75	65·42	111 29	110 18	+ 1 11	3 ·245	3 ·11	+ 0 ·13
Dunér, ...	76	1869·39	111 10	111 0	+ 0 10	2 ·98	3 ·16	− 0 ·18
,,	77	70·38	111 55	111 10	+ 0 45	3 ·10	3 ·16	− 0 ·06
,,	78	71·39	112 17	111 20	+ 0 57	2 ·99	3 ·17	− 0 ·18
,,	79	72.44	112 25	111 30	+ 0 55	3 ·14	3 ·18	− 0 ·04
,,	80	74·09	113 12	111 47	+ 0 25	3 ·06	3 ·20	− 0 ·14
,,	81	75·46	113 28	112 1	+ 1 27	3 ·10	3 ·21	− 0 ·11

ζ Aquarii.

The components of ζ Aquarii are both white, and of the 4th and $4\frac{1}{2}$th magnitude. It was discovered by Sir William Herschel, in 1781, when the components were about $4\frac{1}{2}''$ apart, and since then the distance decreased very slightly, but the value of the semi-axis major, as obtained separately from the measures taken by the same observer, differ so widely and irregularly, that it is probable another century must elapse before this element can be obtained with any accuracy. The present value 7″·64 may differ by some tenths of a second from the truth. Of course, the elements can but be accepted as the first approximation, owing to the small part of the circumference hitherto described by the components. Table XV. exhibits the data given by the projection, Table XVI., the elements, and Table XVII., the comparison with the observations. It was not possible to draw the interpolating curve exactly through the observations; they were lying in waves on both sides of it, which circumstance would seem to indicate some perturbing body. But I afterwards remembered, that Mr. Burnham had detected a faint star in the neighbourhood.

TABLE XV.

THE ANGLES OF POSITION, AND THE DISTANCES IN THE PROJECTED ELLIPSE, DEDUCED FOR THE CORRESPONDING EPOCHS BY THE INTERPOLATING CURVE.

No.	Epoch.	Angle of Position.	Distance.
1	1780	18°·9	1·829
2	1790	15 ·2	1·642
3	1800	11 ·3	1·445
4	1810	6 ·9	1·474
5	1820	1 ·9	1·443
6	1830	356 ·9	1·404
7	1840	351 ·8	1·388
8	1850	346 ·8	1·371
9	1860	341 ·3	1·357
10	1870	335 ·4	1·296

TABLE XVI.

ELEMENTS OF ζ AQUARII.

T	1924·15
Node	140° 51′
λ	134° 40′
γ	44° 42′
e	0·6518
P	1578$^{yrs.}$·33
a	7″·64

TABLE XVII.

COMPARISON OF THE ANGLES OF POSITION AND DISTANCES CALCULATED FROM THE ELEMENTS OF ζ AQUARII, WITH OBSERVATIONS.

Observer.	No.	Epoch.	θ_o	θ_c	$\theta_o-\theta_c$	ρ_o	ρ_c	$\rho_o-\rho_c$.
W. HERSCHEL,	1	1781·37	18° 23′	17° 43′	+ 0° 40′	4″·56	4″·38	+ 0″·18
,,	2	1802·01	11 58	9 34	+ 2 24	—	—	—
J. H.& S.,	3	1822·27	359 29	0 38	− 1 9	4 ·99	3 ·93	+ 1 ·06
Sir J. SOUTH,	4	1825·73	1 1	358 58	+ 2 3	4 ·01	3 ·89	+ 0 ·12
Sir J. HERSCHEL,	5	1828·55	356 13	357 38	− 1 25	5 ·21	3 ·86	+ 1 ·35
,,	6	29·60	356 21	357 7	− 0 46	5 ·22	3 ·84	+ 1 ·38
,,	7	29·72	356 51	357 2	− 0 11	3 ·75	3 ·84	− 0 ·09
,,	8	36·25	352 0	353 50	− 1 50	3 ·91	3 ·76	+ 0 ·15
DAWES,	9	1830·55	356 12	356 39	− 0 27	3 ·95	3 ·84	+ 0 ·11
,,	10	32·87	355 16	355 30	− 0 14	3 ·88	3 ·81	+ 0 ·07
,,	11	33·94	354 11	354 59	− 0 48	3 ·93	3 ·80	+ 0 ·13
,,	12	39·77	350 5	352 5	− 2 0	3 ·73	3 ·73	0 ·00
,,	13	41·86	348 26	350 59	− 2 33	3 ·47	3 ·71	− 0 ·24
,,	14	42·82	348 46	350 29	− 1 43	3 ·45	3 ·70	− 0 ·25
HIND,	15	1843·79	348 14	349 59	− 1 45	3 ·53	3 ·69	− 0 ·16
,,	16	45·63	348 37	349 2	− 0 25	3 ·57	3 ·66	− 0 ·09
,,	17	46·58	348 8	348 51	− 0 43	—	—	—
SMYTH,	18	1831·83	356 0	356 0	0 0	3 ·6	3 ·82	− 0 ·22
,,	19	32·71	355 18	355 35	− 0 17	4 ·0	3 ·81	+ 0 ·19
,,	20	34·90	353 48	354 31	− 0 43	3 ·8	3 ·78	+ 0 ·02
,,	21	38·04	352 24	352 57	− 0 33	3 ·5	3 ·75	− 0 ·25
,,	22	42·59	348 54	350 37	− 1 43	2 ·7	3 ·70	− 1 ·00
,,	23	48·02	347 42	347 46	− 0 4	3 ·3	3 ·64	− 0 ·34
,,	24	52·81	346 54	345 8	+ 1 46	3 ·2	3 ·58	− 0 ·38
JACOB,	25	1845·87	348 6	348 54	− 0 48	3 ·20	3 ·66	− 0 ·46
,,	26	46·80	347 37	348 24	− 0 47	3 ·94	3 ·65	+ 0 ·29
FLETCHER,	27	1850·88	348 6	346 11	+ 1 55	3 ·35	3 ·60	− 0 ·25
,,	28	52·91	347 1	345 4	+ 1 57	3 ·21	3 ·58	− 0 ·37
MILLER,	29	1851·72	346 40	345 43	+ 0 57	3 ·34	3 ·59	− 0 ·25
,,	30	52·94	345 43	345 3	+ 0 40	3 ·53	3 ·58	− 0 ·05
POWELL,	31	1853·77	345 48	344 35	+ 1 13	—	—	—
,,	32	55·79	342 17	343 25	− 1 8	3 ·66	3 ·54	+ 0 ·12
ROMBERG,	33	1863·18	339 24	339 2	+ 0 22	3 ·20	3 ·43	− 0 ·23
TALMAGE,	34	1865·72	338 49	337 26	+ 1 23	3 ·27	3 ·40	− 0 ·13
,,	35	66·74	339 52	336 48	+ 3 4	4 ·24	3 ·39	+ 0 ·85
,,	36	68·76	339 11	335 32	+ 3 39	4 ·42	3 ·35	+ 1 ·07
,,	37	70·63	333 20	334 18	− 0 58	—	—	—
SECCHI,	38	1855·77	346 26	343 26	+ 3 0	3 ·40	3 ·54	− 0 ·14
,,	39	66·71	337 50	336 47	+ 1 3	3 ·39	3 ·39	0 ·00
DEMBOWSKI,	40	1854·88	344 53	343 57	+ 0 56	3 ·75	3 ·55	+ 0 ·20
,,	41	56·64	343 2	342 55	+ 0 7	3 ·64	3 ·53	+ 0 ·11
,,	42	58·15	341 31	342 3	− 0 32	3 ·57	3 ·52	+ 0 ·05
,,	43	63·14	339 2	339 4	− 0 2	3 ·52	3 ·43	+ 0 ·09

TABLE XVII.—(*continued*).

Observer.	No.	Epoch.	θ_o	θ_c	$\theta_o-\theta_c$	ρ_o	ρ_c	$\rho_o-\rho_c$
DEMBOWSKI, . .	44	1867·18	337° 4′	336° 31′	+ 0° 33′	3″·35	3″·38	− 0″·03
,,	45	69·88	336 28	334 49	+ 1 39	3 ·35	3 ·34	+ 0 ·01
STRUVE, . . .	46	1821·99	358 12	0 45	− 2 33	3 ·76	3 ·93	− 0 ·17
,,	47	25·73	359 48	358 58	+ 0 50	3 ·60	3 ·89	− 0 ·29
,,	48	32·81	355 17	355 32	− 0 15	3 ·46	3 ·81	− 0 ·35
,,	49	36·05	352 43	353 57	− 1 14	3 ·39	3 ·77	− 0 ·38
MAEDLER, . . .	50	1841·44	352 23	351 13	+ 1 10	3 ·86	3 ·72	+ 0 ·14
,,	51	42·71	350 29	350 33	− 0 4	3 .51	3 ·70	− 0 ·19
,,	52	44·50	349 27	349 37	− 0 10	3 ·85	3 ·68	+ 0 ·17
,,	53	50·04	347 1	346 39	+ 0 22	—	—	—
,,	54	52·81	345 51	345 8	+ 0 43	3 ·58	3 ·58	0 ·00
,,	55	54·15	345 16	344 23	+ 0 53	3 ·61	3 ·56	+ 0 ·05
,,	56	55·82	344 12	343 25	+ 0 47	3 ·57	3 ·54	+ 0 ·03
,,	57	57·41	344 24	342 29	+ 1 55	3 ·77	3 ·53	+ 0 ·24
,,	58	58·67	342 34	341 45	+ 0 49	3 ·53	3 ·51	+ 0 ·02
ENCKE, ...	59	1836·52	352 0	353 43	− 1 43	4 ·05	3 ·76	+ 0 ·29
,,	60	37·61	350 33	353 10	− 2 37	3 ·78	3 ·75	+ 0 ·03
GALLE, ...	61	1838·67	350 19	352 38	− 2 19	3 ·85	3 ·74	+ 0 ·11
KAISER, ...	62	1840·01	353 42	351 57	+ 1 45	3 ·49	3 ·73	− 0 ·24
,,	63	41·82	349 48	351 0	− 1 12	3 ·29	3 ·71	− 0 ·42
,,	64	43·86	350 54	349 57	+ 0 57	3 ·33	3 ·69	− 0 ·36
BESSEL, ...	65	1830·98	355 44	356 26	− 0 42	3 ·52	3 ·83	− 0 ·31
,,	66	34·77	354 18	354 35	− 0 17	3 ·69	3 ·78	− 0 ·09
SCHLÜTER, . . .	67	1841·96	350 50	350 56	− 0 6	3 ·61	3 ·71	− 0 ·10
LUTHER, . . .	68	1856·19	349 20	343 51	+ 5 29	4 ·01	3 ·54	+ 0 ·47
,,	69	1862·85	342 7	339 14	+ 2 53	3 ·32	3 ·43	− 0 ·11
AUWERS, . . .	70	1861·09	341 34	340 18	+ 1 16	—	—	—
,,	—	61·82	—	—	—	3 ·58	3 ·45	+ 0 ·13
BRÜNNOW, . . .	71	1868·76	336 37	335 33	+ 1 4	3 ·42	3 ·35	+ 0 ·07
DUNÉR, ...	72	1867·69	337 12	336 12	+ 1 0	3 ·34	3 ·37	− 0 ·03
,,	73	68·84	336 36	335 29	+ 1 7	3 ·22	3 ·35	− 0 ·13
,,	74	69·48	337 24	335 5	+ 2 19	3 ·16	3 ·35	− 0 ·19
,,	75	70·99	337 12	334 4	+ 3 8	3 ·35	3 ·33	+ 0 ·02

36 ANDROMEDAE.

36 Andromedae is a very close Binary Star, first measured by Sir John Herschel, in 1830. The components are both yellow, and between the sixth and seventh magnitudes. Smyth used a central paper disc to improve the separating power, which he recommends. Both the angles and the distances have since increased continually. I drew the projected ellipse be-

tween the points laid down, partly with the distances obtained by drawing tangents, partly with the distances as obtained by a curve laid through the directly observed distances. Mäedler's distances are certainly too large, a circumstance which cannot surprise us, as Dawes has found Mäedler's distances, in case of ζ Herculis, systematically much too large, the difference amounting on some occasions to half a second. Table XVIII. shows the data obtained by the projection, Table XIX., the elements deduced therefrom, and Table XX., the comparison with the observations.

TABLE XVIII.

THE ANGLES OF POSITION, AND THE DISTANCES IN THE PROJECTED ELLIPSE, DEDUCED FOR THE CORRESPONDING EPOCHS, BY THE INTERPOLATING CURVE.

No.	Epoch.	Angle of Position.	Distance. Calculated.	Distance. Observed.
1	1830	305° ·3	0·80	0·85
2	1835	312 ·7	0·81	0·95
3	1840	320 ·1	0·84	1·06
4	1845	326 ·9	0·89	1·15
5	1850	332 ·6	0·94	1·20
6	1355	337 ·9	1·00	1·26
7	1860	342 ·4	1·11	1·28
8	1865	345 ·9	1·24	1·35
9	1870	349 ·0	1·32	1·39

TABLE XIX.

ELEMENTS OF 36 ANDROMEDAE.

T	1798·80
Node,	57° 54′
λ	142 19
γ	41 39
e	0·6537
P	349$^{\text{yrs.}}$ 1
a	1″·54

TABLE XX.

Comparison of the Angles of Position and Distances, calculated from the Elements of 36 Andromedae, with Observations.

Observer.	No.	Epoch.	θ_o	θ_c	$\theta_o - \theta_c$	ρ_o	ρ_c	$\rho_o - \rho_c$
J. Herschel, . .	1	1830·78	307° 4′	306°26′	+0°38′	0″·90	0″·89	+0″·01
,,	2	31·79	308 40	308 14	+0 26	0·77	0·90	−0·13
Smyth, ...	3	1835·92	315 42	314 48	+0 54	1·1	0·95	+0·15
,,	4	39·77	318 30	320 17	−1 47	1·1	1·01	+0·09
,,	5	43·12	322 54	324 34	−1 40	1·0	1·05	−0·05
,,	6	52·83	335 48	335 10	+0 38	1·3	1·19	+0·11
Dawes, ...	7	1839·82	317 45	320 21	−2 36	1·08	1·01	+0·07
,,	8	40·84	319 10	321 43	−2 33	1·11	1·02	+0·09
,,	9	41·55	320 23	322 37	−2 14	1·05	1·03	+0·02
,,	10	42·94	322 50	324 20	−1 30	1·01	1·05	−0·04
,,	11	43·88	323 35	325 29	−1 54	1·12	1·06	+0·06
Fletcher, . . .	12	1851·93	336 23	334 20	+2 3	1·12	1·18	−0·06
Talmage, . . .	13	1865·73	347 2	346 2	+1 0	1·08	1·38	−0·30
,,	14	66·83	344 12	346 51	−2 39	1·38	1·39	−0·01
,,	15	69·72	349 28	348 49	+0 39	—	—	—
,,	16	72·86	347 25	350 49	−3 24	1·24	1·48	−0·24
Struve, ...	17	1832·14	307 48	308 46	−0 58	0·85	0·90	−0·05
,,	18	36·90	320 28	316 13	+4 15	0·94	0·97	−0·03
Kaiser, ...	19	1842·69	323 24	324 2	−0 38	0·99	1·04	−0·05
Maedler, . . .	20	1846·99	329 0	329 9	−0 9	1·26	1·11	+0·15
,,	21	47·81	330 11	330 3	+0 8	1·19	1·12	+0·07
,,	22	48·05	329 4	330 18	−1 14	1·28	1·13	+0·15
,,	23	51·00	333 48	333 25	+0 23	1·40	1·17	+0·23
,,	24	51·78	334 52	334 11	+0 41	1·315	1·18	+0·135
,,	25	52·60	335 19	334 57	+0 22	1·35	1·19	+0·16
,,	26	53·08	337 5	335 24	+1 41	1·43	1·20	+0·23
,,	27	53·87	336 35	336 11	+0 24	1·28	1·21	+0·07
,,	28	54·98	336 55	337 13	−0 18	1·42	1·22	+0·20
,,	29	55·44	336 26	337 38	−1 12	1·34	1·23	+0·11
,,	30	56·98	339 59	339 3	+0 56	1·35	1·25	+0·10
,,	31	57·95	340 23	339 52	+0 31	1·41	1·26	+0·15
,,	32	58·04	340 8	339 58	+0 10	1·34	1·27	+0·07
,,	33	58·89	346 59	340 42	+6 17	1·52	1·28	+0·24
,,	34	59·08	346 15	340 51	+5 24	1·13	1·28	−0·15
Winnecke, . . .	35	1856·09	344 13	338 15	+5 58	1·30	1·24	+0·06
Engelmann, . .	36	1865·51	348 27	345 52	+2 35	1·625	1·37	+0·255
Secchi, ...	37	1866·05	349 31	346 17	+3 14	1·31	1·38	−0·07
Dembowski, . .	38	1856·08	335 52	338 13	−2 21	1·20	1·23	−0·03
,,	39	63·24	344 7	344 11	−0 4	1·13	1·34	−0·21
,,	40	65·25	345 26	345 41	−0 15	1·22	1·37	−0·15
,,	41	67·26	347 17	347 8	+0 9	1·22	1·40	−0·18
,,	42	70·38	350 49	349 15	+1 34	1·31	1·44	−0·13
Brünnow, . . .	43	1868·76	350 45	348 10	+2 35	1·57	1·42	+0·15

ι LEONIS.

ι Leonis was discovered by Struve, in Dorpat, in 1827. The components are of the fourth and eighth magnitude, A yellow, B purple or blue. He inferred afterwards a retrograde motion, and represented the position by the following formula, which he obtained from his own measures by the method of least squares :—

$$\theta = 92^\circ{\cdot}38 - 0^\circ{\cdot}834\,(t - 1832{\cdot}01).$$

I have obtained by the graphical method the formula :—

$$\theta = 93^\circ{\cdot}75 - 0^\circ{\cdot}544\,(t - 1830).$$

The distances however differed regularly from the average value $2''{\cdot}4$, and I have represented their increase by :—

$$\rho = 2''{\cdot}20 + 0''{\cdot}01\,(t - 1830).$$

Application of the method of least squares would not be practicable, as the errors are not *accidental*, when all the observations are taken into account. Even the treatment of one observer's measures by this method might seem unjustifiable, at least the determination of probable errors of the Elements of a Binary Star will be found quite imaginary. Constant corrections to be applied to the measures of different observers could, I presume, only be determined by graphical methods, but the material is not sufficient, and it is still less for the determination of the probable errors of these corrections. The last expressions represent the observations as exhibited in Table XXI.

TABLE XXI.

Comparison of the Angles of Position and the Distances obtained from the Formulae for ι Leonis, with Observations.

Observer.	No.	Epoch.	θ_o	θ_c	$\theta_o - \theta_c$	ρ_o	ρ_c	$\rho_o - \rho_c$
W. Struve, .	1	1827·28	97° 0′	95° 14′	+ 1° 46′	2″·30	2″·17	+ 0″·13
,,	2	28·35	97 6	94 40	+ 2 26	2 ·29	2 ·18	+ 0 ·11
,,	3	29·37	93 48	94 8	− 0 20	1 ·89	2 ·19	− 0 ·30
,,	4	31·37	91 24	93 0	− 1 36	2 ·07	2 ·21	− 0 ·14
,,	5	32·39	93 6	92 26	+ 0 40	2 ·14	2 ·22	− 0 ·08
,,	6	33·34	90 28	91 55	− 1 27	2 ·17	2 ·23	− 0 ·06
,,	7	35·33	90 20	90 51	− 0 31	2 ·41	2 ·25	+ 0 ·16
Maedler, .	8	1846·31	82 49	84 53	− 2 4	2 ·31	2 ·36	− 0 ·05
,,	9	47·36	81 17	84 19	− 3 2	2 ·35	2 ·37	− 0 ·02
,,	10	50·62	79 57	82 33	− 2 36	2 ·27	2 ·41	− 0 ·14
,,	11	51·28	80 3	82 10	− 2 7	2 ·47	2 ·41	+ 0 ·06
,,	12	59·33	76 19	77 50	− 1 31	2 ·57	2 ·49	+ 0 ·08
Kaiser, . .	13	1842·59	88 12	86 56	+ 1 16	2 ·30	2 ·33	− 0 ·03
Winnecke, .	14	1855·34	81 22	79 59	+ 1 23	2 ·75	2 ·45	+ 0 ·30
Engelmann, .	15	1865·70	73 50	74 21	− 0 31	2 ·92	2 ·56	+ 0 ·36
Secchi, . .	16	1865·28	76 33	74 34	+ 1 59	2 ·88	2 ·55	+ 0 ·33
Dembowski, .	17	1857·22	78 19	78 57	− 0 38	2 ·57	2 ·47	+ 0 ·10
,,	18	63·23	76 42	75 43	+ 0 59	2 ·51	2 ·53	− 0 ·02
,,	19	66·08	74 56	74 8	+ 0 48	2 ·56	2 ·56	0 ·00
,,	20	70·08	72 16	72 0	+ 0 16	2 ·54	2 ·60	− 0 ·06
Smyth, . .	21	1836·40	90 30	90 16	+ 0 14	2 ·4	2 ·26	+ 0 ·14
,,	22	39·32	87 42	88 42	− 1 0	2 ·4	2 ·29	+ 0 ·11
,,	23	43·38	86 0	86 27	− 0 27	2 ·5	2 ·33	+ 0 ·17
,,	24	53·29	81 18	81 4	+ 0 14	2 ·5	2 ·43	+ 0 ·07
Dawes, . .	25	1840·29	87 36	88 9	− 0 33	2 ·44	2 ·30	+ 0 ·14
,,	26	41·28	86 46	87 38	− 0 52	2 ·52	2 ·31	+ 0 ·21
,,	27	42·27	85 20	87 3	− 1 43	2 ·45	2 ·32	+ 0 ·13
,,	28	43·26	85 17	86 34	− 1 17	2 ·63	2 ·33	+ 0 ·30
Fletcher, .	29	1853·29	82 4	81 6	+ 0 58	2 ·36	2 ·43	− 0 ·07
Talmage, .	30	1866·28	75 48	74 1	+ 1 47	2 ·91	2 ·56	+ 0 ·35
,,	31	67·24	76 49	73 31	+ 3 18	2 ·91	2 ·57	+ 0 ·34
,,	32	68·21	76 52	73 1	+ 3 51	3 ·06	2 ·58	+ 0 ·48
,,	33	69·19	78 59	72 27	+ 6 32	2 ·85	2 ·59	+ 0 ·26
,,	34	71·37	78 13	71 17	+ 6 56	—	—	—
,,	35	72·41	77 2	70 42	+ 6 20	2 ·70	2 ·62	+ 0 ·08
Brünnow, . .	36	1869·25	74 4	72 25	+ 1 39	2 ·73	2 ·59	+ 0 ·14
Dunér, . .	37	1875·31	70 36	69 8	+ 1 28	2 ·58	2 ·65	− 0 ·07

XX.—*Report on the Superinduced Divisional Structure of Rocks, called Jointing; and its Relation to Slaty Cleavage.* By William King, Sc. D., Professor of Mineralogy and Geology, Queen's University in Ireland, and Queen's College, Galway. With Plates XXXIV. to XXXVIII.

Read June 14, 1875.

Contents.

Part I.—Jointing.

I. Introduction.
II. Character of Jointing.
III. Extensions of Jointing.
IV. Jointing in its Original Position.
V. Dislocating Jointing.
VI. Jointing developed in different Geological Periods.
VII. Origin of Jointing.

I.—Introduction.

Jointing, though more or less treated of by Daubuisson, Berger, Bakewell, Sedgwick, Phillips, Murchison, De la Beche, Hopkins, Harkness, Haughton, Jukes, Du Noyer, Sharpe, Kinahan, Foote, Emmons, Enys, David Forbes, and others, has received much less attention than its importance in Physical Geology ought to command.

On former occasions, as at the Meetings of the British Association held in Dublin in 1857,* and in Leeds in 1858,† I brought under the notice of geologists certain points relating to Jointing, which had escaped

* Athenæum, 1857, p. 1153.

† Report, British Association, 1858, p. 83.

their notice, or, as it appeared, had not engaged much of their attention—I allude to its remarkable development in the counties of Clare and Galway, its identity with the divisional structure called cleat, in coal, its various modifications produced by stratial disturbances, and its relation to slaty cleavage.

Since the occasions referred to, and aided to a considerable extent by a grant from the Royal Irish Academy, I have gained much additional information on the above points. Though much remains to be done, I feel that no further delay ought to occur in my placing before the Academy the result of the investigations I have been more or less engaged upon by favour of their grant, and which have been made in numerous places in the three kingdoms.

II.—Character of Jointing.

Most stratified rocks are more or less characterised with the structure under consideration, which consists of parallel divisional planes, independent of, and intersecting, those which form their bedding partings. Ordinary cracks occasionally assume a degree of regularity, so as to give rise to the idea that they are related to Jointing; but the planes of the latter, from their running in definite directions, and possessing other peculiarities, are obviously differentiated from a phenomenon of mere local origin. Jointing also characterises the so-called igneous rocks, not, however, except in a few cases,* to the same extent as it does the stratified deposits. In the latter, whether calcareous, argillaceous,† siliceous, or carbonaceous—in the finest

* Jointing appears to be highly developed in the granite of the Scilly Islands. It is mentioned in Murray's "Guide-book for Devon and Cornwall," p. 473, that in one place in St. Mary's, the "joints are so closely arranged as to resemble the cleavage of slate rock."

† Hitchcock states that unconsolidated clay beds "in West Springfield and Deerfield, in Massachussetts, are intersected by numerous and distinct joints, while those above and below are unaffected. This clay has certainly never been subjected to any great degree of heat, being of very recent origin."—Elementary Geology, p. 22. It is to be apprehended that there is some oversight in this statement.

limestones, shales, or sandstones—or in the coarsest conglomerates, it is marked by distinctive features. Occurring in rocks of such dissimilar character, jointing presents itself under different aspects. A joint may be defined as formed of two juxta-posed or cognate planes in immediate contact, or more or less separated by an open space. Closed joints are often manifested by mere lines on the surface of a bed; in some cases the contact of the planes being so complete that it is impossible to insert the finest blade between them: I assume this to be the *normal or original condition* of jointing.

In the case of open joints, it frequently happens that cognate planes are separated by spaces, which may be a few, or several, inches in width; caused obviously by the chemical and mechanical action of atmospheric water, or stratial disturbances.

In most rocks, joints are usually two or more feet apart; but cases are not uncommon in which the separation is a foot more or less; even considerably less distances are not unfrequently seen. Argillaceous rocks often display the finest kinds of jointing: it is also occasionally observed in limestones and sandstones. Coarse kinds are usually characteristic of igneous rocks and conglomerates.

Between any two adjacent joints, the rock or intermediate space breaks irregularly across from one plane to another. Indications of a divisional structure occasionally occur, but so faint as to escape detection. Allowing for such exceptional cases, it may be assumed that interjointal spaces cannot be sub-divided in directions parallel to the joint planes.

Coarse-grained rocks, especially sandstones, conglomerates, and granites, have the roughest joint planes; limestones have finer: in argillites the planes are usually smooth (that is, in their normal condition), as if cut with a knife; in cherts and carbonaceous shales, they are still smoother and bright; while in coal they rival in lustre the cleavage surfaces of calcite, fluor, and other minerals.

Jointing in homogeneous rocks has usually the most regular and even planes: the same thing also occasionally occurs in rocks of the coarsest structure, even when inclosing foreign bodies, as fossils, concretions and pebbles.

A strong conglomerate (Upper Silurian), abundant near Killery Harbour, incloses large water-worn blocks of Syenite and the like, which are divided by jointing continuous with that intersecting their matrix. Haughton, noticing the old red sandstone conglomerate of Waterford, mentions that "the large quartz pebbles, often five inches in diameter, which it contains, are cut clean across, as with a knife, forming a plane rigorously parallel to the general plane of the joint."*

Associated or interstratified beds, of a totally different character, are occasionally similarly affected. Near Dresden, where porphyry occurs interbedded with a Permian freestone (*rothe todte liegende*), both the igneous and the sedimentary rocks are jointed alike.†

As already stated, the divisional structure under consideration runs in definite directions. There may be only one direction, but it frequently happens that there are two or more. Where two occur together, their intersections meet obliquely, or nearly at right angles; causing the rock to be divided into solids often regular in form, with quite an artificial appearance. A remarkable instance of the kind, resembling a mosaic pavement, occurs in the bed of the River Swale, beneath the walls of Richmond Castle, Yorkshire: the blocks are so uniform and square, that it was seriously contemplated to remove them for the purpose of paving the streets; but, fortunately, on becoming known to Mr. Edward Wood, F. G. S., the well-known geologist of the place, the design was frustrated. At Callish Burnbridge, near Alnwick, Northumberland, there is a similar example, to which my attention was drawn by the late Mr. G. Tate, F. G. S.

When jointing of this kind occurs in thick beds, it gives rise to columns often remarkably regular.

Speaking of columnar jointing reminds me of the remarkable divisional structure described by Jukes as occurring in the gypsum of Chaumont, near Montmartre, Paris, where there are two beds of this rock, six and eight feet thick, divided by "three sets of vertical equidistant planes, crossing each other at angles of 60°." There are consequently produced prisms

* Transactions of the Royal Society, Vol. cxlviii., p. 337.

† Murchison and Morris, Quarterly Journal Geological Society, Vol. xi., p. 423.

"pretty regularly triangular and hexagonal."* The equidistant planes, and the constant angle of intersection, cause this case of "prismatic jointing" to differ from anything that has ever occurred to me. Strictly speaking, it cannot be considered to be mineral cleavage, inasmuch as the interdivisional spaces do not appear to be cleavable: still, as will be seen hereafter, this peculiarity, which has its analogy in certain minerals, may help to throw light on the origin of jointing.

"Columnar basalt is a peculiar example of the prismatic jointing," so said Jukes; but I see no relationship between these two structures. Whatever the relation may be between jointing and the prismatic structure of gypsum, I certainly feel that the columnar division of basalt ought not to be regarded as an example of either: in both of the former there is always a parallelism of the planes; but there is nothing of the kind in columnar basalt.†

When crowded jointing occurs in thin layers, the two depositional surfaces, added to the four joint planes (where two intersections are associated), often give rise to six-sided solids, remarkably simulating cubical, rhombohedral, and prismatic crystals. I exhibited several singular examples of the kind at the Dublin Meeting of the British Association (1857). A layer of chert in the middle Arran Island is so affected that it readily falls to pieces in slender square prisms: the layer *in situ* resembles a tesselated pavement. But the most remarkable example of this fine divisional structure is to be seen in coal, and has long been known under the name of cleat. So closely does cleat resemble mineral cleavage, that some have been led to ascribe its origin to crystalline forces. At the Dublin Meeting, having several specimens before me, I maintained that cleat is nothing more than jointing in a highly developed condition. My conclusion, I showed, was proved by the coincidence in the direction between both divisional structures in any district where they occur together. No stronger proof of their identity can be offered than the fact that in the north of England, while the quarryman is working his limestone or freestone, at the surface, according to the directions

* Jukes' "Manual of Geology," 1st and 2nd editions; also the 3rd edition (edited by Geikie), p. 181, 1872. Phillips had noticed this case earlier than Jukes.

† *Vide* Note [1] at end of Report.

of the jointing, the pitman, it may be a thousand feet below, is guided, in driving his "headings" and "boards" in the coal seam by the corresponding directions of the cleat.*

Cleat characterizes a great many different kinds of rock. The chert layer, already noticed, is an example in a silicious deposit allied to flint. In the carbonaceous shales at Lough Shinney it is often well exhibited. A bed of light-coloured limestone in a cutting of the Midland Great Western Railway, between Athenry and Oranmore, is in many places entirely divided by rectangular intersections approximately resembling the cleavage of galena. It often makes its appearance very remarkably in beds of shale at the cliffs of Moher: these beds, often affected by ordinary jointing, usually about two feet apart, are cleated for an inch or so on each side of the leading divisional planes. Another case occurred to me at Kilbride, on the shore of Lough Mask, where a bed of Upper Silurian grit displays pretty regular jointing, an inch or less apart: in one place the bed contained a large segregated mass of shale, also affected by a divisional structure, but of a character identical with that of cleat (see Plate XXXV., fig. 1): both were strictly parallel, and running about N. and S. No one could observe the example without concluding that the cleat was caused by the fineness of the material composing the segregated shale.

Often the fossilized cuticle of Lepidodendron, Stigmaria, and other Carboniferous plants, exhibits cleat very beautifully. In such cases the cuticle, converted into coal, adheres to the associated shale, from which, however, it may be readily detached: but, while the coally cuticle is wonderfully divided by cleat, the shale is not affected by it, except that its divisional lines are distinctly impressed on the adhering surface of the shale. A specimen of decorticated Stigmaria is so highly instructive, appearing to throw important light on a point to be discussed hereafter, that I cannot avoid giving a representation of a small part of it—natural size (Plate XXXV., fig. 2). It will be seen that the cleat runs in two directions, one of which

* Cleat in coal is mentioned by Professor Johnston, in his "Economy of a Coal Field," pp. 31, 32, 1838; also by Phillips, "Treatise on Geology," p. 395. The directions it follows are given by both authorities.

is represented by perfectly continuous lines, from $\frac{1}{4}$ to $\frac{1}{2}$ an inch apart; the other by much more numerous lines: both strike each other at right angles. It is necessary to observe that the latter are usually interrupted by the former; which fact seemingly proves that the one originated after the other.

Numerous other specimens have fallen into my hands with the cleat running only in one direction; causing them to split into thin laminæ, closely resembling those obtained by splitting selenite parallel to its planes of eminent cleavage; and, as in the latter, dividing the coal to the utmost possible extent.

Whether, then, cleat runs in one or more directions, the specimens under consideration may be regarded as representing miniature fac-similes of jointing, such as prevails in many stratified rocks. I have already described jointing in two intersecting sets, which divide layers into solids, in some cases so regular and symmetrical as to resemble pavement. I shall next notice a few cases of single jointing. One, occurring in the north Island of Arran, consists of a superficial bed of limestone, about eighteen inches thick, divided by jointing usually eighteen inches apart. On account of this bed being readily detached from the underlying one, the joint planes and the bedding surfaces together have caused it to break up into long four-sided columns, so abundant in some places, and lying about in all directions, as to remind one of the ruins of a city of ancient Egypt.*

De la Beche mentions, "that in one or two beds of Lias, on the west of Church Cliffs, Lyme Regis, divisional planes have caused a regular appearance resembling planks on a floor."† Emmons states, that masses of Potsdam sandstone with "a hundred square feet of surface may be raised, and afterwards split into rails six inches wide, and ten feet long."‡ But the most remarkable case of single jointing, which cannot be surpassed in its illimitability, occurs near Carragrian, about three miles from Galway, on the road to Oranmore, where there are several large fields with their surfaces consisting chiefly of exposed beds of Carboniferous limestone, and here and

* These are unlike columns produced by the columnar jointing already mentioned, each of whose faces is a joint plane.

† "Geology of Cornwall," p. 274. ‡ Logan, "Geology of Canada," 1863, p. 87.

there diversified with patches and stripes of herbage. The beds, which dip very slightly to the south, are all affected with north and south jointing to such an extent that they frequently afford flags (whose surfaces are at right angles to the bedding) a yard or more in length, and three or four inches in thickness. In numerous places the beds are marked by zones or bands, a few feet or yards in width, of jointing, often so crowded as to cause them to split vertically into plates, varying from a quarter of an inch to the thickness of a stout sheet of brown paper.* This case, with others already noticed, shows that jointing may occur developed to a degree of fineness bordering on that of mineral cleavage.

III.—Extensions of Jointing.

With reference to the subject now entered upon, it must be stated that every joint may be considered as having two extensions,—(1) in its course along the surface of the earth, which is called its *strike*,—and (2) in a line perpendicular to its course, which is called its *dip*.

Dip.—In districts like Clare and Galway, where vast exposures of the surface of limestone beds are common, especially in the Burren, jointing may be observed running uninterruptedly, with little deviation from verticality, up the steep slopes of hills, 800 feet or more in height. So obvious is the phenomenon from the opposite or north side of Galway Bay, on a clear day, that the hills appear to be scored with lines from top to bottom. Plate XXXV., fig. 3, represents a sketch of the north slope of Abbey Hill, (795 feet), near New Quay, distant nine miles due south of Galway, as it appeared after a fall of snow. As usually takes place in that district, the snow soon disappears; but on the occasion when the sketch was made

* Noticed by G. H. Kinahan (to whom I mentioned the case), in "Explanation" of Sheet 96 of the "Irish Geological Survey," p. 23, &c. He has observed similar jointing at other places in Co. Galway (Sheet 105, and 114, p. 23). Jukes, when I first showed him specimens (deposited in the Geological Museum of the Queen's College, Galway), was strongly inclined to the view that the case was one of slaty cleavage.

it remained for some time after in the joints, which thus came out in striking contrast to the sombre grey colour of the limestone.

Many of the perpendicular cliffs in Arran are joint planes: Nalhea, a narrow promontory projecting into the Atlantic, has a cliff 500 feet or more in length, and about 100 feet in height; the whole being one huge perpendicular joint plane, which intersects continuously beds of limestone and seams of shale.

A remarkable exception to these cases is occasionally seen at Liscannor, Co. Clare. Here, the "Malbay flags," usually from two to three inches thick, and marked by jointing that traverses their entire thickness, sometimes display a close superficial kind, which cuts into them to no more than half an inch in depth.*

Strike. Any one examining jointing in districts where it is well developed cannot but be struck with its remarkable persistency of direction along and intersecting the plane of the horizon. Very frequently the direction is obscured by irregular shrinkage cracks running deviously among the joint planes; but a little observation will soon reveal the distinction between them. Difficulties are also often caused by the sudden appearance of joint planes acutely intersecting the more frequent ones: nay, cases are not uncommon, as at Lough Shinney, showing different superimposed layers (carbonaceous shales), in each of which there is a difference in the direction of the jointing.

Still, observations made by geologists, who have paid attention to the subject, combine to show that the strike of jointing falls within a limited number of leading systems. De Saussure and Bakewell long ago observed facts tending to this conclusion. Sedgwick, in 1821, mentions that "whenever any natural section in Devon and Cornwall exposes an extended surface of the granite, we find portions of it divided by fissures, which often, for a considerable extent, preserve an exact parallelism among themselves;" also that these "masses are not unfrequently sub-divided by a second system of fissures, nearly perpendicular to the former, in consequence of which structure the whole aggregate becomes separated into blocks of

* *Vide* Note [2] at end of Report.

rhomboidal form."* Phillips, whose labours extended over Yorkshire and Durham, arrived at nearly the same result. By means of a diagram, representing the various directions which characterise the joints of these counties, he made it clear that there are two principal systems of divisions in the rocks, at right angles to each other, the primary one having the direction N.N.W. to S.S.E., the secondary, the direction E.N.E. to W.S.W.† De la Beche, in 1834, remarked that "Professor Phillips' diagram would, with only a few modifications, represent the course" of the joints in Devonshire and Cornwall; "the greatest number holding their courses within a few degrees of N.N.W. and S.S.E., these lines being cut by others which chiefly form angles from 70° to 90° with them."‡ Hopkins obtained the same results, particularly in the limestone districts, in Derbyshire and Nottinghamshire.§ In Ireland, at least in the west and south-west, the leading joints have their northern point bearing more easterly, while the less prominent ones run from south of east to north of west. In my papers read at the Dublin and Leeds Meetings of the British Association, especially in the last one, I pointed out the general bearings of the joints as they occur in the Carboniferous limestone of Clare and Galway; that there are in these counties four distinct sets, two of which lying more or less conformably to north and south (the most prominent set being about N.N.E.), and the other two approximately at right angles to them (the principal set being south of east).‖ Du Noyer, Foote, Kinahan, Jukes, and others of the Irish Geological Survey, have carefully noted the strikes in the course of their labours. According to the diagram constructed by Foote,¶ and the tables prepared by Kinahan,** jointing in Galway and the

* "Cambridge Philosophical Transactions." Vol. i.

† "Geology of Yorkshire," Part 2, p. 98, 1836; also "Treatise on Geology," in Lardner's Cyclopædia, Vol. i., p. 65, 1852. Other joints, but much less numerous, run N.W., also due N. and S.: these are rectangled by others running E.N.E., also due E. and W.

‡ "Geology of Cornwall," pp. 274, 275.

§ "British Association Report," 1836, 2nd Pt., p. 80.

‖ *Id.*, "Leeds Meeting," 1858, 2nd Pt., p. 83.

¶ Explanations to accompany Sheets 114, 122, and 123, pp. 21–26.

** *Id.*, 96, 97, 106, and 107, pp. 14, 22, 23.

north of Clare falls into two systems, one, consisting of "master" or "main joints," bearing for the most part between N. 5° W., and N. 25° E., the other of "cross joints," lying chiefly between E. 10° N., and E. 15° S.

In the beginning of 1858, a memoir by the Rev. Dr. Haughton was read at the Royal Society, in which he classified the jointing of the "old red sandstone conglomerate" of the Co. Waterford under four "conjugate systems;" * and in a subsequent memoir this classification, with some modifications, is adhered to.† In the latter essay the author has practically arranged these systems under two groups—"North and South joints," and "East and West joints;" from which it will be learnt that his observations so far are confirmatory of the classification proposed by Phillips and others.

Believing that the object of this paper will be more advantageously served by adopting the dual classification, I shall in future designate one of the groups, from its including divisional planes that run more or less conformably to the magnetic meridian (that is, allowing east and west variations), the *meridianal system;* and the other, from its including divisional planes that run more or less conformably to a line parallel with the equator (allowing corresponding north and south variations), the *equatorial system.* In some localities there are joints running in other directions, but which, nevertheless, must be included in either the one or the other of these systems: these may run east of the north joints, and be intersected by others running west of them; or they may run north of the east joints, intersecting others that run south of them. Distinguishing these sub-groups as *sets*, I would name the first two separately, *east meridianal* and *west meridianal*, and those respectively at right angles to them, *south equatorial* and *north equatorial.*‡

As already made known, meridianal jointing is highly developed in the west of Ireland. In the elevated district of Co. Clare, known as the Burren,

* Transactions of the Royal Society, Vol. cxlviii., pp. 333-348.

† *Id.*, Vol. cliv., pp. 154, 393, 411; read in the beginning of 1864.

‡ I proposed these terms in my paper read at the Leeds Meeting of the British Association, 1858: see Report, Transactions of the Sections, p. 83. Through an error, the north equatorial set of jointing is called "east equatorial," in the abstract referred to.

where occur some splendid examples of limestone terraces, of marine origin, rising one above another along the sides of the hills, and reaching the height of 1200 feet,* this system is unequalled in development. Scaling the slopes of these hills, we pass over a succession of broad terraces, often some hundred yards in width, and many miles in length, defined by high scarps; every terrace being formed of bared limestone, in massive beds, with a slight inclination to the south-west. An admirer of Nature may be excused becoming enraptured when he takes a view from any of these noble terraces. Looking north, or south, his eyes are riveted on vast surfaces of grey limestone rocks, split up to an extent, and with a regularity of direction, truly wonderful. Tops and sides of hills, valley-bottoms, and wide terraces, are alike marked with myriads of deep perpendicular fissures, running for hundreds of yards without interruption, and all falling into one direction. The observer becomes so absorbed with the scene that he unconsciously begins to feel as if the rocks under and around him were in process of being illimitably cleft from north to south—as if the earth's crust were in course of splitting up from one pole to the other; and he only rids himself of the feeling to become bewildered with the question, as to what mysterious agent produced the singular phenomenon he is contemplating.

The botanist may there revel in abundance; neither the plough, nor the spade has broken up the primæval patches of soil lodged here and there, and crowded with the rarest plants—remnants of an Arctic and a Sub-Tropical flora, having their opposite longitudinal range conterminous in the Burren.† To him and the geologist there are loveliness and grandeur

* First made known by myself, I imagine, as of Post-glacial age: see Geologist, Vol. vi., p. 172, 1863. They have evidently been formed as the land rose out of the sea (that in which the shells found on the Three-Rock Mountain, near Dublin, by the Rev. Maxwell Close were living, also those in the shell drift of Moel Tryfaen, Carnarvon), from a depth of not less than 1000 feet: every terrace indicates a stoppage in the uprising.

† *Dryas octopetala*, which, with a few doubtful or rare exceptions, has its southern limit in the Burren, is one of the last plants seen in the highest Arctic latitudes (nearly 80°). *Adiantum Capillus-veneris*, which, in plentifulness, has its northern limit in the Burren, extends south as far as Madeira. The former is a remnant of the Glacial flora, and the latter a representative of a climate now prevailing in much lower latitudes.

in all that lies around; but it is to be feared that to many, even with the highest attainments, if uneducated in the natural sciences, the Burren will be little better than a rocky desert.*

Hitherto I have spoken principally of jointing belonging to the meridianal system. One reason for which is, that the other system is rather uncommon in Clare and Galway. Still, examples of equatorial jointing are not wanting; a number having been noticed by Foote and Kinahan. According to these authorities, with whose observations my own are in accordance, the joints of this system are in general continuous for only short distances, and they seldom cut through more than one or two beds of rock.† The same differences prevail in England; but, as already noticed, the equatorial joints are occasionally well developed; and when associated with the meridianals, as often happens, the beds so affected are cut up into rectangular and rhomboidal blocks. At View Edge, north of Ludlow, the pentamerus beds, there horizontal, are intersected by both systems; the meridianal striking N. 12° West (which is in slight force), and the equatorial, E. 30 South. The planes of the latter are best developed; as I found was also the case in places near Ludlow.

A peculiarity in the difference of degree of development of the two systems requires to be made known. On the east of Kilbride, Lough Mask, there are several small promontories of Upper Silurian rocks, a hundred yards or more in length, displaying both systems. At the origin of one of the promontories meridianal jointing occurs in great force; but at the termination it becomes faint, and the equatorial takes its place. A similar case occurred to me in a bed of hard calcareous shale, on the strand near Howick, Northumberland, where both systems co-exist, well marked, and intersecting each other in about equal extent: but at a hundred yards distance equatorial joints are abundant; while the meridianals have nearly

* Plate XXXIV. is a representation of a scene north of a small hill behind Finvarra, near New Quay, on the west side of and overlooking Galway Bay. The beds, nearly horizontal, are remarkably intersected by perpendicular joints, running N. 8° East: a few others run N. 27° West. Equatorial joints are very rare and indistinct.

† Irish Geological Survey; Explanation to Sheets 105-114, p. 22.

disappeared. I shall have to refer to these cases again, when speaking of the relation between the two systems.

IV.—Jointing in its Original Position.

Reverting to the subject of *dip*, it will have been remarked that in Galway and Clare jointing of the highest type, and occurring in beds scarcely showing any disturbance, stands approximately perpendicular to the plane of the horizon or vertically. An important inference is suggested by this fact, viz., that *vertical* joint planes, intersecting horizontal beds, lie in their original position of dip.

I propose now to bring forward a few examples, selected from a large number, collaterally supporting this inference. Pl. XXXV., fig. 4, represents a section of Devonians, about a mile from Neyland in Milford Haven, Pembrokeshire. Five beds of purple shale are exposed, dipping east of north, at an angle of about 20°: three are separated by the intercalation of the other two, which are somewhat arenaceous. The shales are intersected by numerous fine joints; and the sandy beds by others more separated, and of a coarser character: both kinds, which belong to the equatorial system, are strictly parallel, and rectangled to the plane of bedding.

At the mouth of Ingleton Beck, Yorkshire, there are several beds of Carboniferous limestone extensively worked, and of great aggregate thickness, dipping S.W. at an angle of about 30°; as represented in Pl. XXXV., fig. 5. Most of the beds are affected by equatorial jointing of the ordinary kind; but some of them are so much divided by it as to produce flags, rather coarse, from one to six inches in thickness. The divisional planes of the flags stand, like the jointing in the other beds, at right angles to the planes of bedding.

It must be obvious that the jointing in these examples is not in its original dip. Instead of being highly inclined to the horizon, the joint planes must have been perpendicular to it, as will be understood by restoring the beds to their original position. No other conclusion can be come to than that in these cases the high inclination of the dip is due to the beds having been thrown out of their original horizontality.

In connexion with the above cases, this seems to be a fitting opportunity to refer to two or three others (though their consideration will have to occupy our attention further on), inferentially similar, and throwing light on what appears to be an anomaly,—I allude to cases of horizontal jointing.

In Figs. 6 and 7, Pl. XXXVI., are given two different representations of jointing in a flagstone quarry of Silurian tilestones, above Horeb Chapel, Brecknockshire. As in Fig. 6, the beds, occasionally thin and flaggy, dipping S.S.E., and lying at an angle of about 45°, are intersected by both systems: the leading one, which is meridianal, is on the line of the section or in the direction of the dip of the beds; the other intersects the beds rectangularly, and is highly inclined to the horizon. So far this is merely a repetition of the cases under Figs. 4 and 5. The next, Fig. 7, however, contains the point I am about referring to: it shows on the face of the quarry the respective surfaces of the two beds seen on the right side of the last figure. The long lines represent meridianal jointing,—a zone of which is so well developed that Sir Roderick Murchison mistook it for "imperfect slaty cleavage" (Sil. Syst., p. 183); but it is exactly the same as the zones of paper-like jointing already described. The cross lines represent equatorial jointing, which here, through the high inclination of the beds, is actually lying horizontally. Similar horizontal jointing occurred to me near Haverford West: and Du Noyer noticed an example "in the townland of Ballyknockane, between Clogheen and Ardfinnan" (which I examined a few years ago), as shown in Fig. 8, Pl. XXXV.: the beds (the surface of one of which is represented) stand at 67°, and are crossed by both systems; the nearly perpendicular lines representing the meridianal joints, and the nearly horizontal ones the equatorial. The latter, as Du Noyer remarked, "must have once been vertical, if they existed before the elevation of the beds."*

It having been made evident, from the Neyland and other sections, that joints have been thrown out of their original dip by stratial movements,—why may not their original strike have been altered by similar agencies? A few cases seemingly of this kind have occurred to me in Connemara, but

* Data, &c., to accompany second sheet, 45 S. E., of Irish Geological Survey, p. 21. Du Noyer has represented the section; but his sketch has got reversed.

they appear to be merely local phenomena. It is likely, however, that examples on a large scale may form a geological feature in areas or regions that have been ravaged by intrusive masses. It may be that in the south-east of Ireland, where the beds have been broken through, and apparently heaved out of the prevailing strike, by the great outbursts of porphyry that lie between Tramore and Ballyvoile Head, jointing with an altered strike, due to the same agencies, is present.

V.—Dislocated Jointing.

On the Limerick and Foynes railway, near Curraheen, and east of the Rathkeale station, beds of black argillaceous limestone (Lower Carboniferous), broadly undulated, are more or less affected by a vertical equatorial divisional structure, called "cleavage" and "coarse cleavage" by the officers of the Irish Geological Survey. It varies in different beds, being finest or platy, half an inch in thickness, in the most argillaceous, and coarsest, from one to two inches in thickness, in those that are more calcareous (see Fig. 9, Pl. XXXVII.): the former kind occasionally obliterates the bedding. The associated beds of ordinary limestone show no divisional structure.

Figure 10, Pl. XXXVIII., represents a number of Upper Silurian beds, *b*, *c*, *d*, *e*, unconformably overlying a series, great in thickness, of metamorphic schists, *a*, exposed on the side of a mountain nearly opposite the village of Munterowen, and in the valley leading from Maam to Killery Harbour. The beds (*b*, brown fossiliferous argillite—*c*, hard greywackite—*d*, red argillite—and *e*, coarse conglomerate), are all conformable *inter se*, and dip at an angle of about 20°. The fossiliferous brown argillite is remarkably intersected by nearly vertical lamina-like jointing, apparently belonging to the meridianal system (? N. 35° west, true). In general the laminæ are from an eighth to a quarter of an inch in thickness: their planes are somewhat irregular and undulating. The fossils occasionally display distortion to a small extent,—that is, circular corals and encrinal stems are slightly elliptical, with the longest axis of the ellipse parallel to the divisional planes.

I may here mention that encrinal and other fossils are present in the Rathkeale shales; but in no case do they evince that compression or distortion exhibited in rocks possessing true slaty cleavage. Therefore, notwithstanding the opinion of the Irish Geological Surveyors, as implied by their using the term "cleavage" for it, I have no hesitation in regarding the divisional structure occurring near Rathkeale as jointing; and I entertain the same opinion respecting that of the brown fossiliferous argillite in the Maam valley. I see no difference between these cases and the paper-like jointing in zones, near Galway, except that in the former its dip relatively to the bedding has been disturbed, while in the latter its original relativity is retained.

These two examples lead to the consideration of some others of a similar character. The ellipticity of the fossils in the Maam valley argillite may be held as evidence that this rock has undergone a slight compression perpendicular to the lamination planes. The fossils in the Rathkeale shales do not exhibit any such evidence; nevertheless, the verticality of their laminæ must be regarded as equally testifying to the same conclusion.

The point I have introduced will, I think, be understood by the following illustration, though it may be defectively expressed:—Suppose a number of books arranged in tiers, and strapped together as in a frame, to become crushed in on one side by a force exerted in a direction deviating somewhat from a line perpendicular to this side: the tiers of books (representing the shale beds) would be thrown into an inclined position; while the books (representing the laminæ) would slide against one another, and retain their verticality; as in Figures 11 and 12, Pl. XXXVIII. Such is the explanation I have to offer for cases of vertical jointing in inclined beds; and I consider it to be demonstrated by the following example. In a quarry near Studfield, on the east side of Ribblesdale, Yorkshire, there are Silurian beds dipping 50° South, and forming flags extensively worked: the beds are occasionally separated by thin layers of carbonaceous shale, from half an inch to one inch in thickness, often crowded with cleat-like jointing (locally called "bate") belonging to the meridianal system. This divisional structure cuts the layers into long thin strips.

Figure 13, Pl. XXXV., represents a section with a layer of the kind, an inch thick, displaying jointing; and Figure 14, Pl. XXXV., is an enlarged representation of it. What is specially noteworthy is, that the upper surface of the layer is highly puckered, presenting ripple-like ridges along the strike or direction of the divisional lines; and that the planes of division are curved: the ridges are highest on the lowest side of the strips. Now, it appears to me that we have the tops of the books represented by the peculiar ridging of the interjointed spaces; while the curving of the divisional planes affords a clear proof that they have been subjected to lateral pressure, which has held them in their original position, notwithstanding that the flags have been thrown out of their original horizontality. The Bray Head rocks, containing Oldhamia often show appearances similar to those I have described: their divisional planes are separated by spaces from a quarter of an inch to one inch in width; and they intersect the deposition layers at an angle of about 45°. The layers have been subjected to pressure both vertically and laterally; the former shown by the flattened appearance of the ridges or puckerings on the interdivisional spaces, and the latter by the fossils being drawn out in a direction parallel to the divisional planes.

The three sections represented in Figs. 15, 16, 17, Pl. XXXVI., are seen in the cliffs at the South Stack Lighthouse, near Holyhead, where the beds, consisting of semi-metamorphosed Cambrians (quartzose grits principally), are very much crumpled, and occasionally cut up by numerous joints, generally regarded as slaty cleavage, from half an inch and more apart, and belonging to the east meridianal set.

The section, Fig. 15, a few hundred yards in length, and about 100 feet high, as seen in the cliff looking from the Lighthouse, exhibits the jointing well developed. Fig. 16 represents another portion of the same cliff, not so high, nor so long, as the last, showing well-developed jointing at the south end; and Fig. 17, some beds on the south side of the east end of the Stack, much jointed: the planes, however, of the latter, instead of being uniform in dip, have a fan-like arrangement. The sharp curves in these sections, it will not be disputed, are due to the beds

having been violently disturbed by a lateral thrust; and it seems equally indisputable that the general uniform dip of the joint planes is due to the same operation: occasionally the thrust has not been powerful enough to act uniformly on the planes, as displayed in Fig. 17, which shows them preserving to some extent their original dip relatively to the planes of bedding.

Reverting to the Neyland and Ribblesdale sections, we have, in them, cases of simple vertical movements; but had pressure been exerted against the joint planes, there can be no doubt that they must have stood differently in relation to the bedding surfaces. Again, the Rathkeale jointing shows pressure, as in the Stack cliffs; but not sufficient to alter the internal structure of the rock by distorting the fossils. The Maam valley case, in its compressed fossils, shows that a greater amount of lateral pressure had been exerted. There is, therefore, no difficulty in admitting the existence of cases manifesting even a greater amount of pressure. This brings me to a point, at which we have arrived step by step—the consideration of certain cases of "slaty cleavage," generally so called by those who have described them.

It has long been known from the descriptions by Jukes, Harkness, Du Noyer and others, that in various places in County Cork, between Ringabella in the south, and Mallow in the north, a vast thickness of brown shales occurs, regarded by some as belonging to the upper division of the Devonian system, by others as being lower Carboniferous, strongly marked by a divisional structure, "looking very like beds, but probably joints or cleavage. In passing through calcareous or argillaceous beds it is cleavage, but in passing through hard compact silicious grits it is lost, or assumes the aspect of ordinary joints."*

I have made notes of a number of cases occurring at Monkstown, Shanbally, and other places: one will suffice for the present. Figure 18, Pl. XXXVII., represents a section on the new road in Ballyfouloo glen, leading from Shanbally: the beds have a strong dip, about 45° to the south. At the south end of the glen, a number of beds are vertically divided;

* Du Noyer, Irish Geological Survey, Explanation to Sheets 185, 186; p. 34.

the divisional planes more or less curving, and striking E. 27° North. This structure gives rise to laminæ, which, unlike those developed in true slate rocks, do not split up *ad infinitum*. There are cases of the kind; but usually the laminæ cannot be divided indefinitely: indeed the divisional structure closely resembles that of Rathkeale and the Maam valley. Call it by any name; but I contend that it is crowded or laminar jointing, whose planes have been kept in their original vertical dip by pressure exerted against them; and that the laminæ, when their thickness admits of mechanical appliances, are not illimitably divisible like those characteristic of slate rocks.

But it may be suggested that the Shanbally divisional structure is not typical slaty cleavage; also that it passes elsewhere, as in the west of Cork and Kerry, by intermediate gradations into a variety forming roofing slate. This I am ready to admit; but the admission does not in the least invalidate the truth of my proposition. The Stack jointing has also been called slaty cleavage; but it agrees with that at Rathkeale. The name in both cases has been given in consequence of the dip of the planes being constant or uniform, and usually vertical, in a series of beds; while the beds themselves may present any angle of inclination, or no end of undulation.

Instead of the name "slaty cleavage," I feel persuaded that it would be more correct to call the divisional structure in the preceding cases *dislocated jointing*.

I am not aware that hitherto any one has mentioned or suggested that jointing may occur modified in the way that has been spoken of. The oversight, I cannot help thinking, has completely vitiated the observations made by previous writers on the so-called slaty cleavage in the south of Ireland.

In regard to the Rathkeale and Maam valley examples, some may demur to my conclusion, because they are not in a locality where the divisional structure, just referred to, prevails; and because the rocks in these places belong to periods different from that of the Cork brown shales. I shall next bring forward another case to meet these objections.

In a quarry in the neighbourhood of Bandon, Co. Cork, on the road leading to Kilpatrick, hard brown shales are in great force. I have represented in Fig. 19, Pl. XXXVIII., a natural section, which exhibits numerous beds from one foot to three feet in thickness, dipping north at an angle of about 70°, and highly charged with fossils in the state of casts: the section lies in the direction of a set of vertical joints striking N.N.E. true, and separated in a number of places by interspaces from a quarter of an inch to six inches in thickness. Another set occurs lying like the horizontal joints at Horeb Chapel: its consideration, however, may be conveniently deferred for the present. There is also a third set standing vertically with an east and west strike: this corresponds with the so-called cleavage near Cork; but its laminæ, which exhibit only a slight appearance of compression, as indicated by the condition of the fossils, are somewhat thicker; each showing, in consequence, the two depositional surfaces of some width. These surfaces are marked with linear rugæ, impressed by the divisions between the laminæ that belong to the subjacent, or the suprajacent bed; as in the Bray Head and Studfield flags. The facts last stated reveal, in the clearest light, that the third set is dislocated jointing; and they leave no doubt on my mind that the Cork divisional structure is the same.

Many years ago Dr. Maculloch described a divisional peculiarity in the horizontal beds of (?) Jurassic sandstone, in the peninsula of Strathard, Isle of Skye, which he called "oblique lamination." A similar structure has been noticed in Kerry by Du Noyer, who describes its occurrence in a bed of purplish red sandstone, and states that it is "traceable for the distance of seven or eight miles," at Minard Head, near Cahirconree.* I failed in getting a sight of this example; but I was more fortunate with the one in Skye. In this last the "oblique lamination," which is finer in some beds than in others, although not so regular as it appears on the section given by Maculloch, is sufficiently striking to attract the notice of the least observant tourist sailing under the cliffs of that remarkable peninsula.

As sandstones often exhibit false lamination, this "oblique lamination" might be taken for the same; but it is too regular, and intersects different

* Irish Geological Survey, Explanation to Sheet 160, &c., pp. 38, 39, Fig. 8.

beds, disappearing and reappearing in the same bed, at a pretty uniform angle of dip, usually 45°: some portions of it are fine, and others much coarser. In certain respects it closely accords with jointing. But how are we to account for its being at such an angle in nearly horizontal beds, considering that, according to the view already advanced, it ought to be vertical? I would suggest the following explanation. The beds, when in their original position, were not affected by jointing: afterwards, when they became highly inclined, say at an angle of 45°, vertical jointing was developed in them. Next, the beds were thrown down again to about their original position, and as at present; which caused the jointing to stand as it now occurs. The tremendous stratial movements which the Strathard district must have undergone, as evidenced by the surrounding geological phenomena, strongly countenance the above explanation. I would offer a similar one for the case at Minard Head. Accordingly, I look upon "oblique lamination" as originally vertical laminar jointing that was developed in inclined beds, but remaining unaffected by pressure; while the beds themselves have been subsequently thrown into other positions. In short, I consider it to be a modification of dislocated laminar jointing, which has lost its original angle of intersection with the beds it traverses through being affected by pressure.*

VI.—Jointing developed in different Geological Periods.

It has long been thought that the divisional structure under notice is not the development of any one particular term in geological chronology. Judging, however, from anything known to the contrary in Europe, and especially in the British Islands, it would appear that the phenomenon was of rare occurrence in the Tertiary Periods. Antecedent periods, from the Jurassic back to the Cambrian, witnessed the development of jointing on the grandest scale. The fact, recorded by Kinahan, of the Lower Silurian rocks, near Altoir Ultach in Slieve Aughta, Clare, exhibiting wide joints, filled with Devonian Cornstone,† proves that they were produced before the

* *Vide* Note [3] at end of Report.

† Irish Geological Survey, Explanation to Sheets 124, 125, pp. 24, 25.

Upper Division of the Devonian Period: and the presence of numerous divisional planes in the large blocks of Silurian grit, imbedded in the conglomerate of the Skerries, apparently Devonian, testifies to a similar conclusion. Some years ago, Mr. Pengelly obligingly took me to Berry Head, Devonshire, where there are joints in the Devonian limestone filled with new red sandstone—a fact which makes it evident that they were developed between the periods represented by the two widely different systems of rocks occurring in this locality.* Jointing is tolerably well developed in the Permian limestones of Durham and Yorkshire: according to Harkness, the meridianal system occurs in the Triassic rocks at Howrig, Cumberland.† De la Beche has described the same jointing finely developed in one or two beds of Lias, on the west of the Church Cliffs, near Lyme Regis; and Phillips has noted the occurrence of joint planes in the Oolites of Yorkshire. In the latter four examples the evidences as to age are not so satisfactory as in the preceding: it is therefore difficult to determine this point; but there can be no doubt that, whatever it may be with the others, the Oolitic jointing is post-Jurassic. Doubtless, it will come to light that the development of jointing was a phenomenon that obtained during the Cambrian and Silurian Periods; since the rocks characterising these terms are eminently divided, and the physical conditions under which they must have been placed are favourable, theoretically, to its production.

I am not acquainted with *true* jointing in rocks of later periods than the Jurassic in the British Isles. But it would appear that evidences of the kind are not wanting on the Continent of Europe. The columnar structure of the gypsum at Montmartre may constitute a proof that a peculiar variety of it was formed during the Miocene Period: and, if it be not incorrect to add the divisional structure said to mark the nagelfluhe of the Rhigi, and equally its included pebbles, this may be regarded as an example of Pliocene jointing.

* Since I had the pleasure of accompanying Mr. Pengelly to Berry Head, he has described this example of jointing in the Geological Magazine, Vol. iii., pp. 19–22, 1865.

† Quarterly Journal of the Geological Society of London, Vol. xv., Pt. i., p. 96.

The evidence next to be adduced will show that the same rocks have been jointed in different or successive geological periods.

In previous pages it has been made clear that the original position of joint planes is vertical or perpendicular to the horizon,—that, where inclined, the planes have been made so by the beds containing them having undergone movements of disturbance subsequent to their development. A remarkable example of the kind, with some others, has been made known by Harkness, in the Carboniferous limestone at Blackrock, near Cork: it consists of three sets of joints crossing one another: one, which is vertical, strikes north and south; another lies approximately horizontally; and the third intersects the horizon at an angle of about 45 degrees. The beds are highly inclined—vertical in some places—with an east and west strike.* I have already touched upon a similar case, which occurs near Bandon.

Before making any further remarks, I must express my decided opinion, as I did in the paper read at the Leeds Meeting of the British Association, that the Blackrock and Bandon horizontal jointing belongs to the equatorial system, and was originally vertical. This opinion will be readily comprehended by restoring the beds in the quarry near Bandon to their original position. Both the other sets—vertical and oblique—I assign to the meridianal system.

Equatorial jointing, retaining its original relativity, is rare in the south of Ireland, owing, as will readily be understood, to the rocks throughout the entire region having been thrown into numerous folds, with an east and west axis. Granting that the equatorial system was developed before the beds had undergone these movements, it must now be either dislocated, or inclined, or horizontal. Still, as ought to be expected, cases of a normal character do occur. Du Noyer states that, near Knocklofty Bridge, the limestones, which are regularly bedded, are "smoothly jointed in lines striking north and south and east and west." †

Reverting to horizontal jointing: figure 20 represents a section of beds of limestone in a quarry near Shanbally, Co. Cork, stated to overlie the

* Quarterly Journal of the Geological Society of London, Vol. xv., pp. 86–104.

† Data, &c., to accompanying Quarter Sheet, 45 S. E., Geol. Survey of Ireland, p. 15.

brown shales, which come up in Ballyfouloo Glen. The beds, *b*, dip north at a high angle, and are rudely laminated; the laminæ, of a rude flaky character, striking east and west, and highly inclined. There are also other two divisional sets; one, *c*, a little out of the horizontal, and dipping north-west; the other, which lies in the direction of the section, is vertical, and runs north, 10° east. The latter represents the meridianal system, and the former (*c*) for the reasons given, the equatorial: but this is not so clear as in other cases, owing, very probably, to the beds having undergone complex movements, of which the district affords abundant evidence. With respect to this example, I hold that it warrants the following conclusion. The lamination, like that of the brown shales, represents an early equatorial jointing that has been dislocated by the crush which folded the beds into east and west ridges. Afterwards the beds were again affected by both systems of jointing; and next they underwent a repetition of the same movements, which threw the last equatorials into nearly a horizontal position, but which left the meridianals still retaining their original verticality.

Fig. 21 represents another section, at Black Rock, near the Railway, in the contrary direction, that is, east and west; yet with the same meaning. The bedding, not represented, is very obscure; but what I took for it dipped to the south at a high angle, about 70°. I take the vertical joints, which run north 20° east, as the meridianals, and those lying horizontally to be the equatorials. The oblique joints also run east of north, and therefore may be regarded as belonging to the meridianal system. The beds do not display much appearance of having been crushed or disturbed. The presence of the two sets of meridianals is in favour of this jointing having been produced at different periods; but it is difficult to determine which was the earliest. The divisional structures of this section have been differently interpreted by Professor Harkness, whose views will be subsequently noticed.

In order to avoid all complication when the Bandon case was described

I merely noticed three of its divisional planes—the vertical (running N. N. E.), the horizontal, and the dislocated. Besides these, there are other three sets, all coinciding more or less in their strike with the great N. N. E. set: they differ from the latter, however, in their dip (one set intersecting the horizon at an angle of about 45°); while their planes, especially those of one of the sets, are uneven and roughened. These three sets show that meridianal jointing has been developed at different epochs. The one with the roughest planes may be assumed as the oldest: and as all the sets are cut through by the N. N. E. one, which has clean smooth planes, they were evidently in existence before the latter was produced. The horizontal also the vertical dislocated jointing represent equatorial sets.

In accordance with my proposition respecting the original position of the joint planes, I assume that during the *first* stage the beds were normally circumstanced, and became affected with vertical equatorials, as in Fig. 22: in the *second* stage the beds became inclined—say to the north—without having the joints thrown out of their original dip; but they were consequently dislocated, as in Fig. 23: in the *third* stage the beds were thrown down again, but with a reverse dip, as in Fig. 24: and while in the latter position they became affected with another set of equatorials, standing vertically (*fourth* stage), as in Fig. 25: in the *fifth* stage the beds were forced up again into their present inclination, so that the last jointing became horizontal, as in Fig. 26.

The movements that have been assumed, judging from the rapid curvatures of the beds in County Cork, represent successive waves of disturbance, which caused the Bandon beds in the first instance to form at the south side of an anticlinal (second stage); next, the north side (third and fourth stages); and finally, the south side again (fifth stage). I have endeavoured to represent these movements in the diagram under Fig. 26. It may also be assumed that the beds have undergone disturbance in a north and south direction, from the fact that one of the meridianals intersects the horizon, with a dip to the west, at an angle of 45 degrees. Numerous cases occur in the south of Ireland showing that stratial dis-

turbances have taken place at different times; contorted beds of one rock system (Devonian) unconformably overlying sharply folded beds of an earlier one (Silurian).*

So far, jointing has been shown to have originated in different geological periods irrespective of any difference of kind between its systems; I shall, in the next place, pass on to the consideration of certain cases of meridianal and equatorial joints existing together, and affording evidence, in the opinion of many, of the former having been produced subsequently to the latter.

Mineral veins and their associated fissures resemble jointing in so many respects that most geologists are disposed to assign a common origin to both. The veins principally worked in Cornwall, and in many places in Durham, have the same bearing as the adjacent equatorial joints; while the cross courses, generally unprolific in Cornwall, take the direction of the meridianals. The Rev. Dr. Haughton, describing the joint planes of Cornwall, finds the mean of the equatorials (his System A) to coincide within a trifle with the main lodes, and the meridianals (his System C) to coincide as closely with the cross courses. The bearings which he gives of the counterlodes would allocate them to the equatorial system, but as a set distinct from the main lodes.† Thus, there can be little doubt that main lodes and cross courses, including counter-lodes, &c., were originally joints that have been widened by stratial disturbances. Taking this view, I shall include them in the two systems of jointing, according to their respective courses.

Dr. Pryce, one of the earliest observers of the phenomena, writes as follows: "Because the cross gossauns, or cross flukans" (meridianal jointing), "run through all veins of opposite directions, without the least interruption from them, but, on the contrary, do apparently disjoint and dislocate all of them, it seems reasonable to conclude that the east and west veins were antecedent to cross veins."‡ De la Beche§ and others have held to the same view.

* Irish Geological Survey; Explanation to Sheet 160," p. 38.

† Transactions of the Royal Society, Vol. 154, pp. 405–407.

‡ Taylor, Report British Association, for 1833, p. 7.

§ Geology of Cornwall, p. 296.

Disregarding their metalliferous, or non-metalliferous character, I shall continue to call veins and cross gossauns respectively meridianal and equatorial joints, though there can be no doubt of their having been considerably modified by Earth-crust movements: they might, indeed, be appropriately called disrupted joints.

The dislocations to which Pryce referred are usually termed "heaves" by the miners on the supposition that the ground, with its vein, on one side (say the west) of the joint has been carried further north, or south than the ground, with its vein, on the other (or east) side. The opinion of miners is, that in the Wheal Plover, near Redruth a vein is heaved by a cross lode (meridianal joint) fifty fathoms north of the line it would have otherwise pursued. Whether this be the fact or not, there are some grounds for suspecting that the so-called heaves have not been in most cases correctly considered.

Before, however, proceeding further, it is necessary to draw attention to a point in connexion with the Berry Head jointing. Mr. Pengelly believes he has proved that its meridianal is younger than the equatorial system. If this were the case it would be singularly corroborative of the prevailing view as to the relative ages of the right veins and cross courses in Cornwall. I do not attach much importance to the perceptible, though comparatively slight, difference in the colour of the sandstone of the two systems, observed by Mr. Pengelly; and I need not dwell upon certain difficulties which present themselves to my mind. I must confess, however, that the occurrence of calcareous veins in the sandstone of the equatorial joints only, and their disruption by the meridianals, are good evidences for the posterior origin of the latter. Without being able to adduce any argument against Mr. Pengelly's conclusion, I shall leave it to bring forward evidences which appear to support an opposite conclusion in regard to another case of divisional structure.

The specimen of cleat, represented in Figure 2, exhibits one set of well-marked lines, running continuously for a considerable distance, with an average separation of about half an inch, and another set in which the lines, shorter and more crowded, are from one-sixteenth to one-eighth of an inch

apart, and running in a direction nearly at right angles. The short lines, although in some cases crossing them apparently without a break, generally stop at one side of the long lines, and have no connexion with their parallels on the other side. It is also necessary to observe that, while the long lines are finely and sharply cut, the short ones are coarse and somewhat jagged. No one can look at the specimen without leaning to the conclusion that the short lines are mechanical consequences of the long ones.

This example may be looked upon as a tolerably fair representation of jointing in its original condition, as it occurs in districts that have not undergone anything approaching the amount of stratial disturbances which mark the rocks of Devonshire and Cornwall. In the comparatively undisturbed limestones of Clare and Galway, and the approximately paralleled beds in the north of England, the meridianal joints form the leading system. Wherever the equatorials are seen, there is no more appearance of "heaves," in the sense usually understood by miners, than in the cleat that has been delineated. I cannot, therefore, but entertain a doubt as to the "heaves" of the Cornish miners having been correctly interpreted. Viewing them by the light afforded by the cleated specimen, the "heaves," including the right veins, appear to be mechanical consequences (as apparently are the short lines in the latter specimen) of the cross courses (meridianal); so that, instead of being posterior, they may turn out to be synchronous. If there are any "heaves" that cannot be thus explained, it is quite possible that they are faulted joints, caused by the country on one side (say the west) of a cross course having subsided slantingly or obliquely and to the south along the line of a cross course. In this case, the country on the unsubsided or east side, after being planed down by denudation, would show the eastern continuation of the vein further to the north than the continuation on the other side. "Heaves" occur where no cross course is present, which is in favour of their being caused by a slanting subsidence; and it is rare for right veins to intersect a cross course without being interrupted—in which respect there is an agreement with what is seen in the short lines of cleat.

However, before any decided opinion can be formed on the question last raised it will be necessary that more investigations should be made in mining districts. It must be evident that if the views of mining engineers respecting "heaves" be erroneous, the sooner the correction is made the better for practical work; for, according to such view, when a vein disappears at its junction with the cross course, the engineer is warranted in adopting means to recover it on the other side: on the contrary, if the view be erroneous, a much more economical step would be the most prudent to take.

VII.—Origin of Jointing.

Most of the theories that have been advanced on the subject now entered upon involve some form of mechanical force acting by itself, or supervening upon previously existing phenomena.

Haughton, who without any reserve advocates the purely mechanical theory, considers that he has demonstrated the existence of four conjugate systems of jointing—AC, AC′, AC″, AC‴, which are described with much fulness of detail in the two memoirs to which reference has already been made.* Each system consists of two series meeting approximately at right angles. Since the publication of his first memoir (in which is given an account of the joints of the old red sandstone of the Co. Waterford), the system AC has occurred to him in Donegal, Mourne, Cornwall, and elsewhere: having reason to believe the same to be the most important one, he calls it the primary system, and believing that the systems AC′, AC″ can be shown to be simple mechanical consequences of the primary, AC, they have been named correlated secondaries.†

In connexion with this theory, I shall again recur to the specimen of cleat marked on shale, represented in Fig. 2. It is difficult, as I have already remarked, to examine the specimen without becoming impressed with the idea that the short lines are mechanical consequences of the long ones; and it is equally supported by an examination of cleated coal, in which the

* Transactions of the Royal Society, Vol. 148, p. 333, and Vol. 154, p. 393.

† System AC‴ seems to be an unimportant one.

cleat planes corresponding to the short lines are much less smooth and regular than those answering to the long lines. But whatever view may be taken as to the origin of the short cleat, the appearances and circumstances of the long kind strongly militate against the idea that *it* is of mechanical origin.

The same remark is applicable to jointing. The mechanical theory depends altogether on stratial disturbances. But where jointing is well developed, or occurs in its normal condition, as in the Burren, there is no evidence whatever of adequate stratial disturbances of the kind; the beds being scarcely or only slightly moved out of their original horizontality. Phillips, referring to this point, remarks that, "in symmetry, extent, and frequency, joints are not at all less, but rather more, remarkable at points far removed from axes and centres of disturbance of the rocks."* Again, the idea of mechanical causation is irreconcilable with the fact that coarse conglomerates exhibit divisional planes passing evenly and continuously through both the matrix and its more resisting pebbles, occasionally little short of a foot in diameter: the latter are usually quartzite; but in some cases, as the Silurian conglomerate at Killery Harbour, they consist of syenite and other tough rocks.

Geologists, in adopting the mechanical theory, pure and simple, have, it is to be feared, too often overlooked the leading features of jointing in its original condition. As generally seen, the joints have been considerably modified by stratial disturbances and atmospheric agencies; converting them into open chinks, or wide fissures. In their normal and original state they were, I feel confident, the finest cuts, having opposite planes fitting against each other so closely as to be almost impervious to water, or between which it would be difficult to insert the thinnest knife-blade. In a field between Galway and Oranmore, though the limestone there has been freely exposed for countless ages to the searching and wasting power of the elements, yet juxtaposed planes lie as close to each other as if they belonged to calcite or felspar. When a hammer is applied to the lines, indicating their presence,

* Treatise on Geology, 2nd Ed., Vol. 2, p. 190.

the rock falls asunder, exposing planes as fresh as if they had been formed by the blow which yielded them.

Influenced with the idea involving an elevatory force, Sedgwick has remarked:—"We find in many slate rocks two sets of nearly vertical joints (one set running parallel to the strike of the beds, I have called strike joints, and the other set, which are nearly at right angles to them, dip joints*), which do accord with the mechanical action stated by Mr. Hopkins, and probably represent them." † There are many exceptions, however, to this rule. According to Kinahan, and others of the Irish Geological Survey, much of the jointing (dislocated), miscalled cleavage, in the south of Ireland, is equally out of parallelism with the strike of the bedding, in one case as much as forty degrees. The same has been attested by Phillips and other observers; so that it has become the custom to speak of the parallelism as *approximate.*

This want of strict conformity, also other peculiarities characteristic of jointing, led Phillips to attribute its general and immediate causation to contraction of the rocks after their partial consolidation; and its remarkable constancy of direction to some very general and long continued agency, pervading at once the whole mass of the rocks impressed with it. ‡ Jukes, referring to the well-developed meridianal jointing between Glengariff and Beerhaven, remarks that its unlimited character was very probably produced, not from an internal shrinking, or the mere consolidation of the beds, but from a simultaneous yielding of the whole mass to a great expansive or stretching force." § No evidence, however, of this force is to be seen where jointing is normally developed. Haughton contends that it is due to "the shrinking of the rock-mass, consequent upon evaporation, and drying. The shrinking and contraction of the mass

* I find that Dr. Berger in his Mineralogical Account of the Isle of Man, published in the Transactions of the Geological Society, Vol. 2, 1814, p. 42, was the first to introduce the term "dip-joints" for "rents perpendicular to the direction of the beds, corresponding therefore with their dip."

† A Synopsis of the Classification of the British Palæozoic Rocks; Introduction, p. xxxviii, 1855.

‡ Manual of Geology (Encyclopedia Metropolitana, Cabinet Ed.), pp. 41, 42, 1855.

§ Manual of Geology, p. 194, 1st Ed., 1857.

would naturally show itself in planes perpendicular to the lines of minimum compression." * From what has already been stated, it seems impossible that the smooth and even planes presented by typical jointing, or those which, in the Waterford conglomerate, are "cut clean across a quartz pebble as if with a knife,"—(Haughton), or are "just as clean as if they had been cut by a lapidary's wheel,"—(De la Beche,)† could have resulted from any "force of shrinkage."

The coincidence, as first pointed out by Sedgwick, between the strike and the dip of any series of beds, and the like extensions of jointing, has often been held to establish the mechanical origin of joint planes—that they represent tension-fractures produced by elevatory movements. Hopkins' idea is, that movements of the kind acting simultaneously over an ellipsoidal area will produce two systems of vertical rectilinear fissures; the longitudinal one coinciding in direction with the major axis of the area, and the transverse one coinciding with the minor axis. Hence, in an anticlinated area the longitudinal fissures will run parallel with the strike of the beds, and those that are transverse will run in the direction of the dips. Jointing, it must be admitted, often occurs with a similar, but approximate relativity. The circumstance, I am also ready to admit, is seemingly in favour of the mechanical origin of this divisional structure: and it must be equally acknowledged that the researches of Hopkins in Derbyshire, Nottinghamshire, the Wealden district, and other places tend in the same direction. But we have seen that in many cases jointing is well developed where there are little or no signs of stratial disturbance to be seen.

There is more probability that, instead of having resulted from elevatory movements, and following their axes, jointing has given a direction to the movements. This is one of the earliest conclusions in Dynamical Geology formed by Phillips (1834). Observing in the north of England "a clear general analogy in direction between" the predominant lines of jointing and those of certain great dislocations, it appeared to him as "extremely probable that this character of joints is the primary phenome-

* Transactions of the Royal Society, Vol. 148, p. 337.

† Geological Observer, p. 628.

non which, by presenting *lines of least resistance* to the disturbing forces concerned, has permitted the dislocations to follow certain principal parallels."* Hopkins, however, has expressed his disbelief that joints have determined the directions of the dislocations; for he has not found in the Carboniferous rocks of Derbyshire and Nottinghamshire that "*accurate coincidence of the joints and dislocations*" which "there must manifestly be," if the latter followed the former.† This point, however, is not beyond an explanation on Phillips' side, inasmuch as the dislocations may be in strict conformity with jointing belonging to older and underlying rocks (Devonian, or even more ancient rock-systems); yet the observed joints may be much younger,—belonging to the Permian, or some secondary period.

Thus we are driven to seek for a cause, otherwise than mechanical, to explain the phenomena presented by this divisional structure in its primary or unmodified condition.

It would appear that, with the exception of some vague allusions to polar and crystalline forces previously made by others, the first proposition involving a physical theory emanated from myself. In my earliest paper on the subject, read at the Dublin Meeting of the British Association, 1857, it was contended that "jointing and mineral cleavage are due to the same laws operating under modified conditions."‡ Haughton, in expressing the suspicion that the jointing in granite "is related to mineral cleavage," seems to take a similar view, restricting it, however, to igneous rocks.§ Harkness, in 1858, adopting the crystalline theory in a more extended sense, by applying it to the jointing of certain limestones, has ventured on giving a fuller explanation of it. Referring especially to the vertical meridianal joints, common in the neighbourhood of Cork, he conceives that they resulted from "mechanical force, operating in the form of pressure;" admitting afterwards, however, that "mere pressure fails to account for the complex systems of joints—horizontal, inclined and perpendicular—co-existing together in the purer limestones. The latter rocks, it appears probable, when pressure was applied to them, not only

* Report, British Association, 1834, p. 657.

† *Id.*, 1836, p. 80.

‡ Athenæum, 1857, p. 1153.

§ Transactions Royal Society, Vol. 148, p. 336.

obeyed the influence of the force which gave them the north and south, or perpendicular vertical series of joints; but the pressure caused them to divide in accordance with the mineral cleavage of calc spar, and produced in them a disposition to assume a rhomboidal form. The uniform size of the angles of masses intersected by the complex system of joints, their angles agreeing with those of cleavage planes of calc spar, and the pure mineral nature of these limestones, support the opinion that mineral cleavage is in part an important agent in the production of complex series of jointing in limestone."*

Though admitting in a certain sense the idea of mineral cleavage, I cannot agree with my colleague that pressure was in any way concerned in developing either the "complex series," or the meridianal vertical jointing. It is difficult to understand how pressure, acting on a mass of limestone, could produce north and south joints. The limestone would be squeezed, and, as a consequence, present a more or less flaky divisional structure,—the divisions running E. and W., or at right angles to the pressure; while any fossils it contained would be flattened and elongated in the same direction: this being actually the case. But it happens that the east and west flakiness of the limestone, and the imbedded fossils, prove that the pressure has been exerted meridianally: such being the case, it is impossible to conceive that the pressure can have produced the north and south vertical joints. As regards the idea of mineral cleavage, the intersections of the planes must not be taken to resemble those of the cleavage planes that form the rhombohedron of calcite; as it is necessary for the planes to have not only three directions, but to intersect in two distinct zones, or, in other words, two different horizons. The joints in three courses, occurring in the Cork limestone, and represented by Harkness (*Op. cit.*, Figs. 4 and 5), intersect but one horizon; and therefore the solid produced by them must be a rhombic or rhomboidal prism. I possess solids in jointing remarkably like rhombohedrons of calcite; but, while two sets of their planes belong to jointing, the third set is formed by the depositional surfaces.

We have had examples before us of jointing, both unlimited (as in the

* Quarterly Journal of the Geological Society, Vol. 15, pp. 99, 100.

papery variety), and intermitted. Mineral cleavage I hold to be a superinduced divisional structure, its planes having no relation to incrementation or depositional growth: * so it is also in jointing. Cleat, especially as developed in coal, is occasionally as unlimited as mineral cleavage; a specimen before me being as infinitely divisible in one direction as selenite is in its basal cleavage: but it also occurs intermitted, as seen in Figure 2. Mineral cleavage is characterised by the same differences. Examined by the highest magnifying powers, chlorophyllite, gigantolite, spartalite, nepheline, and many other species, exhibit it so much intermitted as to give rise to plates an eighth of an inch or more in thickness: it is impossible to split the plates any further; a blow yielding a hackly or conchoidal fracture between the planes. Even calcite, in its brachydiagonal cleavage, affords another instance verging so closely on cleat, also jointing in its finer developments, as to show, like the preceding cases, if not a genetic identity—pure and simple—at least, a close relationship between the divisional structures of rocks and minerals in their origin.

Jointing occurs best developed in primary rocks. Hence, there is every reason for believing that those exhibiting it have at one time been overlaid by superincumbent masses of great thickness, since swept off by denudation; as all researches in connexion with the latter phenomenon, especially those made known by Professor Ramsay,† amply prove. Buried deep in the crust of the earth—it may have been to the depth of ten thousand feet or more—rocks would necessarily be more or less charged with moisture, softer than they are when exposed to the atmosphere, and subject to an elevated temperature.‡ Under these conditions, adding the effect of consequent enormous vertical pressure, it can only be expected that argillaceous, siliceous, and calcareous beds, would be powerfully affected by electrical, chemical, and mineralizing agencies. Moreover, in consideration of the fact that Were Fox has succeeded in producing a divisional structure, form-

* *Vide* Note [4] at end of Report.

† Memoirs of the Geological Survey of England, Vol. 1, pp. 297–335.

‡ It has been calculated by some physicists, based on the temperature of mines, artesian wells, &c., that at the depth of 10,000 feet, sufficient heat may prevail to boil water.

ing parallel laminæ, in clay subjected to galvanic action—may not these agencies, influenced by the polar or magnetic forces of the earth, have produced meridianal jointing in its original condition?

As regards the equatorial joints, although disposed to make full admission in favour of a mechanical causation, I cannot prevent myself believing that physical agencies have had considerable share in their development. Not unfrequently they are continuous for good distances; and their planes are occasionally as remarkably smooth and parallel as those belonging to the meridianals. The joints of both systems frequently intersect each other obliquely:—ought the intersections not to have been rectangled if mechanical action had been involved? The equatorials, when associated with the meridianals, appear in many cases to be the first that were formed: they are the leading joints at View Edge, near Ludlow. In the case already cited, as occurring at Lough Mask, the meridianals, generally in great force, die out at one spot, the equatorials taking their place: the same thing is to be observed near Howick, in Northumberland. Every one of these facts, as it appears to me, decidedly opposes the idea of there being any consequent relation of a mechanical nature between the two systems. With the above facts before us, and in consideration of Ampere's supposition that electrical currents traverse the globe from east to west (which supposition is supported by his experiments showing that simple rotation in some unknown way produces magnetism in bodies of every description), the idea suggests itself, that such currents may *occasionally* have had some share in giving a parallelism and an equatorial direction to what, though originally mechanical consequences of meridianal jointing, would otherwise have remained as irregular cracks.

Fully believing that jointing is in the main a physical phenomenon, the particular hypothesis which most strongly suggests itself to my mind, to account for it, is, that it is the product of divisional forces akin to those which give rise to mineral cleavage. The difference between the two phenomena arises, perhaps, from the forces, in the case of jointing, being subordinated to terrestrial magnetism—to laws pervading the crust of the

earth; while those productive of mineral cleavage are obedient to crystalline polarity—to laws limited to specific mineral substances or solids. Or, to disarm adverse criticism, usually evoked when anything is ascribed to the mysterious agents that have been introduced (though I wish it to be understood that the circumstance indicates what my leaning is), it may be safer, expressing myself as I did in 1857, to regard both divisional structures as the result of different manifestations of one and the same force.

In connexion with the above hypothesis, I shall conclude the present subject by drawing attention to a remarkable agreement between jointing and a phenomenon of terrestrial magnetism; which seems to favour the idea that the leading directions of the one are in some way consequent on the other.

It is well known that there has taken place a secular variation or declination of the magnetic meridian in the longitude of Paris, between 11° 20′ east and 22° 34′ west of true north (amounting to about 34 degrees), during a period of 234 years, that is, from 1580 to 1814. The fact shows that the magnetic polar currents change their direction. As the amount of variation approximately agrees with the range of the joints belonging to the meridianal system, let it be supposed, that at some epoch of a remote geological period, any given series of rocks in the region of the British Isles were under conditions (as great depth, &c., and high magnetic intensity) favourable to the development of jointing, and that the magnetic meridian were at the same time $22\frac{1}{2}°$ west of true north, I could imagine that the divisional planes would be necessarily forced into a corresponding or N.N.W. course: again, if, during a subsequent epoch, the magnetic meridian became changed to $22\frac{1}{2}°$ east, all conditions productive of jointing being equally favourable, the newer divisional planes, it is conceivable, would as a consequence take a N.N.E. course.

Part II.—Jointing in its Relation to Slaty Cleavage.

Slaty cleavage is a divisional structure that agrees with jointing in certain respects, and differs from it in others. It agrees in being superinduced, and in running over great stretches of country in persistent directions. The differences are important. Jointing, although occasionally affecting beds to an unlimited extent, as in places near Galway, does not cause them to be divided in any part except where divisional lines are apparent: on the contrary, slaty cleavage, when it is typically developed, causes the beds to split everywhere between and parallel to any two of its divisional planes. In the former, interdivisional spaces are incapable of further division: whereas in the latter, there are no uncleavable species; at least, this is the accepted formula of slaty cleavage.

The structure of slate rocks very early attracted the attention of geologists. Robert Bakewell, who appears to have been one of the first to distinguish slaty cleavage from stratification, expressed the belief that it is the effect of crystallization.* Dr. M'Culloch, making the same distinction, referred it to concretionary action.† Professor Sedgwick, who devoted much labour to the subject, conceived it to be "only resolvable on the supposition that crystalline or polar forces had acted on the whole mass of the rock simultaneously, in given directions, and with adequate power."‡ Phillips, in 1843, threw some discredit on the above physical hypotheses by making known, along with other facts, that fossils of various kinds, occasionally occurring in slate rocks, have undergone distortion, which "seemed to be the result of a creeping movement of the particles of the rock along the planes of cleavage." The facts referred to "he regarded as positive steps towards a mechanical theory of the subject."§

* Introduction to Geology, p. 86 (1813). Travels in the Tarentaise in 1820, &c., Vol. 1, pp. 67, 337, Vol. 2, Appendix, p. 423, (1823).

† Journal of the Royal Institution, 1825; System of Geology, Vol. 1, p. 139; Vol. 2, p. 186 (1831).

‡ Transactions of the Geological Society, 2nd Ser., Vol. 3, p. 477 (1835).

§ British Association Report, 1843, p. 61.

Next, Daniel Sharpe—after closely investigating facts similar to those notified by Phillips, observing also that slate rocks have their bedding surfaces more or less puckered in lines parallel to the cleavage planes, and that any concretions they contained are flattened in the same parallelism—expressed himself decidedly that "cleavage has been produced by causes solely mechanical," and that "pressure appears to have been concerned in the operation; for the cleavage is uniformly at right angles to the direction in which pressure is seen to have taken place."*

The attention of Sorby, Tyndall, and Haughton having been drawn to the subject, further light was thrown on it by these authorities.

By examining thin sections of cleaved rocks, Sorby obtained evidences enabling him to state that, "compression has so changed the position and form of their particles, that such a structural weakness has been produced in a plane perpendicular to the pressure, that they may be split along it in the masses characteristic of slaty cleavage." † Tyndall followed by showing that wax, puff-paste, graphite, and other substances, subjected to great artificial pressure, assume a laminated structure or "perfect cleavage,"—the laminæ being "perpendicular to the line in which the pressure was exerted." ‡ Haughton, in consideration of the results he obtained from several measurements of distorted fossils, confirming in the main those got by Sharpe and Sorby, enunciated in terms more definite than had been done by others the law, that "compression in a cleaved rock is greatest in a direction perpendicular to the plane of cleavage." And, with reference to the conclusion put forward with some degree of reservation by Sharpe, he makes no hesitation in declaring that pressure is quite sufficient to produce slaty cleavage.§

Sedgwick has expressed himself decidedly against the mechanical hypo-

* Quarterly Journal of the Geological Society, Vol. 3, p. 104, (1846): also see *Id.*, Vol. 5, pp. 111–129, (1848).

† Edinburgh New Philosophical Journal, Vol. 55, July, 1853. Philosophical Magazine, January, 1856.

‡ Philosophical Magazine, July, 1856, pp. 35–48.

§ Philosophical Magazine, December, 1856.

thesis, maintaining that cleavage structure is the compound effect of crystalline forces acting on the mass, and that it cannot be due to mechanical action.* Rogers, who is the only additional opponent I have to mention, after urging a number of objections, unreservedly "adopts the explanation originally proposed by Professor Sedgwick."†

Notwithstanding the existence of the clearest evidences that slate rocks have undergone considerable compression, varying as much as from a quarter to half of their original dimensions,‡ I have long felt myself compelled to dissent from any hypothesis limiting the causation of slaty cleavage to pressure alone. Though fully admitting that pressure is able to produce a laminated or fissile structure, I was unable to believe that it alone could originate the large even planes, occasionally several feet in length and width, presented by slates. Such planes, it struck me, could only belong to jointing. Furthermore, taking into consideration all the facts I had collected in connexion with the last divisional structure, but influenced by the evidences of compression which Sharpe, Sorby, and others had made known, I was induced, in 1857 and 1858, to bring forward the hypothesis that slaty cleavage was originally jointing,—the separated planes of which having been brought into proximity, and its cognate or opposite planes welded together, by pressure exerted against them.§

Sedgwick contends that the mechanical hypothesis "offers no explanation as to the disappearance and reappearance of cleavage-planes in mineral masses which have been under the same conditions of mechanical pressure."¶ In the absence of any cited cases, I may mention one which seemingly answers to the phenomenon alluded to. "Veins" of true slate are of common

* Transactions of the Geological Society, 2nd Ser., Vol. iii., p. 477. Introduction to M^c^Coy's British Palæozoic Fossils, p. xxxvi.

† Transactions of the Royal Society of Edinburgh, Vol. xxi., Part 3, 1856.

‡ Phillips, British Association Report, 1856, p. 390. Referring to Sorby's account of the evidences which are displayed in the cliffs at Ilfracombe, Phillips says, "The amount of compression inferred is so great, that points which appear to have been thirty-eight inches apart are now at the distance of only nine inches." From what I saw myself of the remarkable instance referred to, I do not think there is the least exaggeration in this statement.

§ Athenæum, 1857, p. 1153. British Association Report, 1858, pp. 83, 84.

¶ Introduction to M^c^Coy's British Palæozoic Fossils, p. xxxvi.

occurrence in masses exhibiting imperfect cleavage; but there is no difference in composition between the "veins" and the masses. I cannot understand how a case of this kind is to be explained by the action of crystalline or polar forces. On the hypothesis I have proposed, it may be explained by the fact that zones of fine jointing occur in the midst of a coarser kind. An argillaceous rock characterised by such variation, if subjected to pressure, would, it is obvious, display the "reappearance" and "disappearance" spoken of by Sedgwick.

In connexion with this point, I may remark that it is customary to define slaty cleavage as an illimitable divisional structure; nevertheless, it is undeniable that slates occasionally occur of considerable thickness, and cannot be made any thinner. I have often met with such slates; but, though thick enough for the purpose, and possessing the characteristic flaky grain, they would only break obliquely across between the two bounding planes, leaving an irregular conchoidal fracture. Sharpe has noticed certain "beds at Applethwaite Common, in which the cleavage is still marked; but its planes are more distant than in true slates."* Apparently these beds are similar to those possessed of a "transverse" fissile structure, producing "flagstone slate," and which is described by Sedgwick as "imperfect cleavage, sometimes only exhibited by parallel planes at definite distances, in which case it may be difficult to say whether the phenomena are to be classed with joints or cleavage."†

If slaty cleavage were the effect of pressure alone, necessarily every portion of a bed that had been subject to it would be equally marked with fissility; therefore, on the purely mechanical hypothesis, the cases mentioned by Sedgwick and Sharpe would be impossible. On the other hand, grant pre-existing jointing of the fine kind, common in primary argillaceous rocks, and that its planes had not been brought into close proximity from the want of sufficient force, we should then have precisely the cases above noticed.

Slate rocks, when exposed to the elements, have their divisional planes more or less visible to the naked eye; and the slates, not considered good for commerce, readily fall asunder. Deep beds do not afford such

* Quarterly Journal, Geological Society, Vol. 5, pp. 126.

† Transactions, Geological Society, Second Series, Vol. 3, pp. 480, 481.

strongly marked indications, the lines of cleavage being for the most part denoted by nothing more than the direction of the grain or flakiness of the rock. In beds of the latter kind considerable force is employed to split the rock into slates; which shows how firmly the planes are welded together or intergrained. Huge blocks of rock, several tons in weight, may be thrown down from great heights, yet they remain unbroken: they have literally to be riven asunder; and, as a consequence, the resulting slates have flaky or flatly hackled surfaces, with no appearance of pre-existing planes. A blow of the hammer will yield surfaces with markings radiating from the point of percussion, as would have been seen if the rock had been compact unlaminated limestone. Occasionally, however, the surface of a slate unmistakably exhibits here and there spaces distinguished by their smoothness and glossiness, and showing no appearance whatever of welding or intergraining:—what else can these be but traces of old joint planes?

All slate rocks cannot have undergone the same amount of pressure: it therefore follows that cases, like the last one, instead of being exceptional, will be common in any rock which has undergone pressure to an inadequate extent. I have seen many which may be taken as such in North Devon and Wales. The imperfect slaty cleavage in the cliffs, seen from the South Stack Light-house, Holyhead, admits of the same interpretation; and, as mentioned in the first section of the present memoir, I am decidedly of opinion that the irregular glossy imperfectly adhering divisional planes of the so-called slaty cleavage, characteristic of the brown shales of County Cork, originally belonged to jointing. The divisional planes of the Shanbally limestone, well distinguished by their red ochreous coating from the associated grey fractured surfaces obtained by the hammer, are obviously old intermitted joint planes.

The divisional structure of the brown shales agrees with slaty cleavage in being approximately vertical to the horizon, and in preserving generally a uniform dip, however much and variously the beds may have been disturbed; nevertheless, there is apparent on its face the stamp of dislocated jointing. Another example of the kind, besides those already noticed,

occurring at Ringabella Bay, and beautifully represented by Du Noyer,* may be noticed. The "cleavage" is manifested by the usual flaky grain, and smooth glossy planes: the separated planes are generally very close to one another, their separation being usually an eighth of an inch, or rarely exceeding a quarter of an inch. A large slab before me (for which I am indebted to Colonel W. A. Shortt, late of the 57th, who got it while stationed near Cork) exhibits the planes stained dark brown, while the *fractured surfaces between the planes* are of a much paler colour. The "cleavage" is therefore clearly intermitted, just as it is in the cases of dislocated jointing already adduced. Referring once more to the Bandon case, which agrees very closely with the one just noticed, both as regards the colour of the divisional planes, and that of the interdivisional or uncleavable spaces: what difference there is simply consists in the latter spaces being the widest (from half an inch to one inch or more) in the Bandon rock.† But this difference is in reality a powerful evidence in favour of the "cleavage" of Ringabella Bay being nothing more than dislocated jointing. If any doubt on this point attaches to the Ringabella, or Monkstown cases, it must be effectually removed on observing their divisional structures completely graduating into that occurring near Bandon. Even on the view that all these cases afford slaty cleavage imperfectly developed, the conclusion I adhere to is in no way invalidated:—for what else than the want of sufficient force in the pressure has made it imperfect?‡ Evidences of pressure are quite

* Jukes' Physical Geology, pl. xvii; p. 131, 1853. Geological Survey of Ireland: Explanation of Sheets, 187, 195, and 196, p. 57.

† Professor Harkness (associated with the late Dr. Blyth), in his account of the "Cleavage of the Devonians in the South-West of Ireland" (Edinb. Philosophical Journal, 1855), appears not to have noticed the internal structure of the laminæ yielded by the Cork divisional structure; but, from his calling the latter "the usual perpendicular flaggy-like cleavage which pervades the Devonians of the south of Ireland," I have no doubt he will be ready to admit that it is intermitted.

‡ It is possible that planes resulting from mere fracture or mechanical tension, if affected by pressure applied against them, may have in certain cases been converted into cleavage planes. I am not, however, prepared to point out any instances of the kind; though some, which might be interpreted as such, have occurred to me: I nevertheless cannot consider them to be otherwise than belonging to the category of dislocated jointing. But, even admitting their existence, instances of the kind must be regarded as still proving the truth of my hypothesis, inasmuch as pre-existing planes form its fundamental element.

obvious to the naked eye in the flaky character of the interdivisional spaces, and the crushed condition of the fossils; but the inference is equally plain that the pressure has been insufficient to bring separated planes into immediate contact, or to weld together those that are cognate.

The cases that have been noticed may be safely accepted as proofs that pressure alone, instead of producing the large even planes characteristic of true slates, can only give rise to a rude flaky structure. Several more proofs, equally and similarly conclusive, have fallen under my observation.

Specimens of a Silurian argillite before me, from Westmoreland, possess a strong flaky grain, which, with the highly distorted condition of their fossils (heads of encrinites), proves that the rock has undergone considerable pressure; yet no regular divisional planes can be obtained by the most careful manipulation. The Upper Cambrian flags in the Tremadoc district, N. Wales, often exhibit surfaces with linear puckerings, corresponding to which there is an internal fibrous structure. As a further proof of pressure, the specimens of *Lingula Davisii* which they contain, when their longitudinal axis is parallel to the linear puckerings, are considerably reduced in width and elongated in length. A specimen, favourably preserved, enables me to determine that originally it must have been $\frac{5}{8}$ths of an inch wide, and nearly $\frac{7}{8}$ths long; whereas its width and length are now respectively $\frac{3}{8}$ths of an inch, and over an inch. Yet, notwithstanding the great amount of pressure which they display, these Tremadoc flags are completely divested of slaty cleavage.

It is important to bear in mind that in true slate rock, although the linear puckerings are often parallel to the divisional planes, this parallelism does not always obtain, for I have seen the one forming an acute angle with the other. An example in point is shown in Figure 27, which illustrates a portion of a slab of Penrhyn slate in the undivided condition; the lines parallel to the sides of the figure representing the cleavage divisions, and the intersecting or oblique lines the linear puckerings or grain of the rock.*

* Phillips has observed what appears to be a similar want of conformity,—"A scaly structure lying obliquely to the plane of cleavage." He asks—"Is this a case of secondary cleavage?" British Association Report, 1856, p. 372.

The discordance is only in one direction, which I am led to believe is that of the strike: in the other direction or dip, both structures are strictly conformable. We have in cases of this kind clear evidence of two independent laminar internal structures in a true slate rock; one represented by its grain or linear puckerings, and the other by the obliquely intersecting cleavage planes. I leave the advocates of the purely mechanical hypothesis to explain away such an opposing difficulty: it is, however, not the only one of the same import.

For some time after the current hypothesis was proposed, the idea prevailed that the force which produced slaty cleavage had thrown the beds affected by it into their present strike. It was consequently assumed that the strike of a series of beds and the cleavage must be strictly coincident. This assumption, which is the legitimate outcome of the mechanical hypothesis, as based on Tyndall's experiments and Haughton's calculations, ought to be considered by those who accept it to involve a more correct expression of facts than the conclusion, laboriously worked out by Sedgwick, between the years 1822 and 1835, that "the strike of the cleavage planes, when they were developed, and passed through well-defined mountain ridges, was *nearly* coincident with the strike of the beds." Numerous observations, however, have shown that the assumed coincidence is far from general; in short, it appears to be exceptional. Phillips states that "in North Devon, where the general strike of the beds is nearly east and west, the strike of the cleavage is nearly E.N.E. — W.S.W.;" and that he found in Charnwood Forest "the average strike of the strata, exactly measured, to form an angle of 19° 12′ with the average strike of the cleavage." He also speaks of a portion of the Lake district, where the strike of the beds varies from N. 15° E., to N. 30° E., while that of the cleavage is generally N. 60° E., but varies from N. 45° E., to N. 75° E.* In Ireland, where the cleavage and the beds are often approximately (never strictly) parallel in strike, there are many places where the parallelism is *decidedly* wanting.† Sedgwick, adding to his conclusion, quoted above, remarks—"where the slate is coarse and thin bedded, the direction of the cleavage planes is

* British Association Report, 1856, p. 375, and p. 378.

† Geological Survey of Ireland; Explanation 121, p. 33; Expl. 175, p. 12, &c.

often considerably inclined to the strike of the beds."* It will therefore be understood that these cases have necessitated a modification in the mode of expressing the relationship between the two phenomena. All, apparently, that is now contended for by the latest authority (Phillips) is the general proposition, that "cleavage is symmetrically related to axes of movement of the strata."† Nevertheless, by abandoning the assumption that the direction of the cleavage and the strike of the strata are strictly coincident, the idea must be equally abandoned that the force concerned has produced the divisional structure of slate rocks in the simple way which enabled Tyndall to obtain by pressure a "cleavage" in wax, graphite, &c.; for, in the latter cases, such "cleavage" was obtained by pressure *exerted perpendicularly* to the resultant planes, as it necessarily must be.

The hypothesis proposed by myself does not limit the action of force to the direction just stated: it admits of the force being applied in various directions *against* pre-existing divisional planes: therefore, cases answering to those noticed in the last paragraph do not stand out as antagonistic to it, but as coming within the category of its resultant phenomena. Obviously, a force acting in a line deviating to some extent from perpendicularity to joint planes will bring them into proximity, and weld cognate planes together; though, perhaps, not so effectively as if it had been exerted perpendicularly to them. Moreover, as noticed in page 638, jointing of both sets forming the meridianal system, one striking east of north, and the other west of north, may occur separately in a great thickness of superimposed rocks, representing two different geological periods—say Devonian and Silurian; therefore, pressure exerted uniformly and simultaneously throughout the mass, while acting perpendicularly to the strike of the east of north joint planes in the Silurian rocks, would also be acting obliquely to the strike of the west of north jointing in the Devonians: yet it may be consistently held that in both instances slaty cleavage, though necessarily of two discordant strikes, could result from one and the same movement.

It appears to me that the above points not only elucidate cases of non-

* Transactions, Geological Society, Second Series, Vol. 3, p. 473, 1835.

† Report, *Op. cit.*, p. 374.

coincidence between the strikes of cleavage and bedding, but they afford a fair and full explanation as to why it is that "cleavage is symmetrically related to axes of movement of the strata." So much cannot be said of the purely mechanical hypothesis.

Again, the coincidence between correlated stratial, and cleavage strikes—whether it be close, or remote—is further resolvable on the hypothesis I am attempting to establish. Assuming the truth of Phillips' idea, that joint planes, by presenting lines of least resistance to disrupting forces, have permitted the latter to follow their direction (it may be strictly or approximately), it is conceivable that any set of beds possessing *equatorial* jointing, when affected by a force of the kind, would, by following the course of such jointing, be crushed laterally, that is, to the north, or south; so that, not only would the component particles of the beds be made to lie east and west, but the joints would be dislocated, brought into proximity, and cognate planes more or less welded together; thus producing different varieties of equatorial divisional structure analogous to that occurring in the south-west of Ireland, and regarded as slaty cleavage.

Harkness and Blyth have drawn attention to the fact that, while certain beds in the Cork Devonians possess "cleavage," others intimately associated with them show nothing of the kind: in their view the difference is caused by differences of chemical composition.* But they have overlooked an important point, namely, that whether the cleaved beds contain five or six per cent. more alumina than those not cleaved, the mechanical theory of compression, which they embraced, necessitates in the latter a superinduced divisional structure. Similar cases have been noticed by others, but I have not yet had the opportunity of observing any, with the exception of those under Figures 9 and 18, that I can adduce as more conclusive on my side than one brought forward by Phillips under the head of "Cleavage varies in strata of unlike quality." Occurring at Aberystwyth, it consists of a hard arenaceous bed lying between two "softer and more argillaceous" beds. "The cleavage, which crosses both the latter uniformly, is interrupted in the former. Across the arenaceous bed, however, there are gene-

* Edinburgh New Philosophical Journal, 1855, p. 245.

rally formed a considerable number of joints, which are always more nearly perpendicular to the plane of the beds than the cleavage planes are. These joints have in some cases the same strike as the cleavage."* I give the following as my explanation of this case. All the beds were originally affected with jointing. The divisional planes in the argillites, probably the most numerous, have been brought together by pressure, producing slates; but those in the arenaceous bed, owing to its greater resisting power, have remained separated, as in their original condition.

On reading Mr. Sharpe's brief notice of the divisional structure which has produced the flags, extensively quarried near Brathay at the north end of Windermere Lake,† my suspicion as to its nature led me to visit the locality. The rock is Upper Silurian, and generally thick bedded: the beds, which dip south at an angle of about 45°, are divided vertically into flags by planes with a north-east strike. Opposite or cognate planes adhere so strongly that mechanical force is required to separate them. But the beds, which are only cleavable on lines from an inch to six inches apart, have clearly been subjected to great pressure directed against the divisional planes, as shown by the flaky grain of the rock, and the linear puckerings on the bedding surfaces—both being parallel to the strike and dip of the planes. The surfaces of the flags, though quite even, are somewhat rough. The divisional lines, in general obvious to the eye, may be observed intersecting uninterruptedly two or more beds; while occasionally intercalated beds are completely divested of all divisional structure, or the divisions occur at a foot or more apart; though such beds still exhibit evidences of pressure on their surfaces, and in their internal structure.

I have nowhere met with any attempt to explain the Brathay intermitted cleavage. It must, however, be evident, from what has been stated in previous papers, that we have here an example of jointing affected by pressure, with the joints still kept asunder, through the force not having been sufficiently powerful to bring them into contact; or, which seems to be more probable, in consequence of their being too distant from one

* British Association Report, 1865, p. 383.

† Quarterly Journal of the Geological Society, Vol. 5.

another. Obviously, however, the force has been sufficient to weld together the two cognate planes belonging to individual joints. This case leaves nothing more to be desired by way of establishing the mechanico-physical origin of slaty cleavage.

Dr. Haughton has described a remarkable "flaggy or platy structure," called "cleavage," present in the coarse conglomerate (old red) near Portoonaka Beg, Co. Waterford,* which it strikes me is similar in its significancy to the intermitted cleavage of the Brathay flags. The thickness of each plate varies "from one-fourth of an inch to four inches,"—the flat sides of the plate being "perfect planes, cutting through the quartz pebbles" ('generally from four to six inches in diameter') "in the most beautiful manner." Unfortunately, I missed this case when traversing the locality; therefore, in the absence of any observations of my own, I would remark, subject to correction, that, considering the divisional structure is regarded as being the result of mechanical pressure exerted perpendicularly to the cleavage planes, it is to be expected that the pebbles, as well as their matrix, would be illimitably divided. But evidently this is not the case; for, besides possessing "perfect planes cutting through" them, as described, the pebbles have acquired, in consequence of having been subjected to pressure, "internally a *latent* cleaved structure;" and the matrix, which presents "an inner structure of a peculiar character," is divided "with great regularity into plates of four or five inches in thickness."† This case, like the Brathay flags, can only be interpreted as meaning that the "perfect planes" existed as jointing in the rock before it was subjected to pressure; and that the "peculiar inner structure," the matrix, and the "*latent* cleaved structure" of the pebbles, are due to pressure.‡

* Transactions of the Royal Society, Vol. 148, pp. 333 to 348 (1858).

† *Vide* Note [5] at the end of the Report.

‡ The singularly flexured cleavage made known by Harkness in some of the Valentia slates (Edin. Phil. Journal, 1855), I refer to jointing that has had its planes affected by pressure exerted at intervals, and unequally. Examples of crumpled cleavage are common in Anglesea, as mentioned by Ramsay in Memoirs of the Geological Survey of Great Britain, Vol. iii., pp. 179–188 (1866). These cases bring up the subject of foliation of rocks; but it is one which, if pursued in the present paper, would compel me to go beyond the limits I originally marked out for it.

The last case to be adduced consists of the slate rocks, in nearly horizontal beds, that are worked at Delabole, Tintagel, and other places in Cornwall. Break up a portion of the rock in the direction of its cleavage, the planes that are yielded present precisely the flaky appearance that characterises the Bangor slates. The force employed to produce the slates proves that they are strongly welded together; but they differ singularly from those got in North Wales in having occasionally complete fossils, principally *Spirifer disjunctus*, exposed on their surfaces; and, though convex in their specific form, so flattened out as to appear like thin papery plates.* The fossiliferous cleavage planes are clearly *depositional.* What conclusion is to be drawn from the Delabole slates?

I am not prepared to affirm or deny that in no case has jointing been concerned in the development of the Delabole cleavage. To assume the negative would be tantamount to denying the existence of joint planes in the rock previously to its having undergone pressure. Besides, there are vertical beds (Upper Devonian), a few miles south of Glengarriff, Kerry, in which it is difficult to distinguish the depositional surfaces, containing numerous encrinal remains, from the cleavage planes—both being to all appearance coincident, or intersecting each other at an acute angle.

The enormous amount of compression which cleaved rocks have occasionally undergone has reduced, according to Phillips, one of their horizontal extensions (width) from a half to a quarter of what it was originally, and a corresponding enlargement in the extension at right angles, either horizontally (length), or vertically, or both: it is, therefore, conceivable in respect to beds already marked with dislocated jointing, and standing at a high angle (like the Cork brown shales), that if they were further subjected to pressure exerted perpendicularly to such joint planes, the deposition surfaces and the cleavage planes would be forced into approximate coincidence: nay, I see no reason why absolute parallelism between both should not occasionally obtain.

* Where Tintagel or Delabole slates are sold, there may occasionally be found specimens with surfaces showing fossils. I have been favoured with a specimen, exhibiting a flattened Spirifer, found some years ago by Mr. C. Spence Bate, F. R. S. of Plymouth, in a slate yard at Swansea.

Going back to the Delabole slates, Sharpe, who appears to have had more time than I had for their examination, besides mentioning that the cleavage is parallel with the surfaces yielding fossils, speaks of the one as also intersecting the other at an angle of ten degrees.* Neither of these facts, as just considered, would involve any great difficulty, except that, unlike those near Glengarriff, the beds are lying *nearly horizontally*. This is also their position generally at Tintagel, and frequently at Petherwyn. But the same difficulty has to be encountered by those who contend that slaty cleavage is due solely to pressure; for, it may be asked, from what quarter came the pressure—from above, or below? On the assumption that the beds were at an early period overlaid by heavy masses of rock that have been removed by denudation, an affirmative might be given to the first of these questions. But there are considerations which induce me to offer another solution of the problem. I assume that the Delabole slate rocks have been affected by great movements, which, throwing them at first into a vertical position, have tossed them back again into their present and original horizontality. That similar movements have taken place elsewhere there can be no doubt. The Cambrian sandstones of the north-west of Scotland, which are lying horizontally, might be said to be in their original position; but they are overlaid by Lower Silurians, standing at a high angle.† The latter fact demonstrates that the Cambrians at one time were highly inclined, and that when in this position the denuded surfaces of their outcropping beds became covered up with horizontally deposited Lower Silurians: subsequently, opposite vertical movements supervened, throwing down the Cambrians into their original position, but necessarily tilting the Lower Silurians into their present inclination. The Delabole slate rocks may not have been affected in this particular way. I would assume the Dartmoor granite to have attained a much higher elevation when it was protruded than it has at this time, so that the stratified deposits in the country around stood at

* Quarterly Journal of the Geological Society, Vol. 3, Pt. 1, p. 77.

† I am assuming Sir Roderick Murchison's section of these rocks to be correct; but it must be mentioned that Professor Nicol takes exception to this section.

steep angles; that, afterwards, Dartmoor subsided—dragging down with it the surrounding rocks, and leaving them in their present horizontal position.

At any rate, the Delabole slates, whether or not their fissile structure is in all cases fundamentally a depositional phenomenon,* equally contribute, with the other evidences I have adduced, to prove that SLATY CLEAVAGE IS ESSENTIALLY THE RESULT OF PRESSURE EXERTED AGAINST DIVISIONAL PLANES, CHIEFLY BELONGING TO JOINTING, THAT EXISTED IN ANY GIVEN ROCK PRIOR TO ITS BECOMING AFFECTED BY SUCH PRESSURE.

* If Sedgwick be correct, that the roofing slates occurring in the "frontier chain from St. Abb's Head, to the Mull of Galloway," are "not derived from oblique cleavage planes, but in every instance" where he has seen them, "are exfoliations parallel to bedding" (Introduction to M'Coy's Palæozoic Fossils, p. xxxvi), they will afford another example of the peculiar cleavage characteristic of the Delabole slates. It would be interesting to ascertain if the "Glarus slates," whose divisional structure is depositional, have anywhere their planes welded together or intergrained, like those worked at Delabole: this does not seem to be the case in specimens, usually seen in Museums, displaying fossil fishes.

NOTES ADDED IN PRESS.

[1] Note to Page 609.

Valuable observations have been published, within the last few years, on the columnar structure of basalt, by Dr. James Thomson, Professor of Engineering in the Glasgow University, and Mr. Robert Mallet, F. R. S. (Proceedings of the Royal Society, No. 158). Thomson's attention has been much directed to the transverse concavo-convex divisions which characterise the columns of the Giants' Causeway. From my own observations, I am strongly of opinion that he has succeeded in bringing forward an explanation of the origin of the divisions, which is right in the main,—that is, that they are the result of some explosive force which has flashed off from a point in or near the centre of a column towards its sides. Something similar appears to have taken place in flints that have conchoidal fractures, produced by natural agencies: several of such have fallen under my notice. When concave, some part, usually *sub-central*, of the area of the fractured surface, shows a peculiar roughened or jagged mark, which corresponds to the bulb of percussion, artificially produced, and well known, on the edges of flint implements. I have observed a similar mark, much exaggerated, in the centre, or near it, of the transverse fractures in the jointed columns at the Causeway.

[2] Note to Page 613.

In one of the "Worm flag" quarries worked near Liscannor, Clare, I found the main joints well developed, and running east of north: they were crossed at right angles by others, less developed. In general, the main joints are from three to four feet apart, remarkably regular and even-faced: the cross joints are from four to six feet apart. By means of this jointing, very regular-shaped rectangular flags are raised.

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[3] Note to Page 626.

The "cleavage planes," noticed and represented by Du Noyer in the Explanatory Memoir to Sheets 102–112 of the Geological Survey of Ireland, pp. 56–57, fig. 18, occurring on the shore near Malahide, I would suggest belong to modified dislocated laminar jointing. The beds (Carboniferous) lie at an angle of about 20° with the horizon; and the "cleavage planes" traversing them "dip N.W. at 60°." The latter, assuming them to have stood originally in a vertical position, have been affected by a crushing force, apparently acting in the direction of the movement which inclined the beds, that is, from below, and inclined to the plane of the horizon; or they may have been depressed by the weight of superincumbent rock-masses, since carried off by denudation.

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[4] Note to Page 639.

I have never been able to finish for the press a paper that has been long lying in my cabinet, treating of mineral cleavage. Nothing of a satisfactory character has yet been published on the *origin* of this divisional structure: and the general opinion seems to be, that it is a depositional phenomenon. On the idea that a crystal of mica (perfect ones are very rarely seen) has been formed by layers of increase deposited transversely to its longitudinal axis, the cleavage lam æ in this instance, conformable as they are to the presumed formation, certainly lend some countenance to the ordinary idea. Certain considerations, however, may be held as sustaining the opposite one; but neither these, nor a number of evidences belonging to other minerals, which could be adduced, can be more than merely alluded to just now. Nevertheless, one of my evidences is so obvious and conclusive in its significancy, that I cannot refrain from bringing it forward on the present occasion. It consists of a cleavage solid of transparent calcite, displaying in its interior two separated surfaces, over which are scattered a profusion of minute modified rhombohedrons of the same mineral. Obviously these surfaces belong to layers of deposition; and as they have not the most distant relationship to the associated three sets of cleavage planes, characteristic of calcite, the latter planes must be regarded as representing an independent and superinduced divisional structure, developed after the mineral was formed.

[5] Note to Page 654.

In the case of the Devonian conglomerate of Portoonaka Beg, I take it that the pebbles are lying with their largest sides parallel to the superinduced divisional planes. A similar example, but in a Silurian rock, has been noticed by Du Noyer at Ballyvoyle Head, further west (Irish Geol. Survey, Expl. of Sheets 167, 168, 179, pp. 60–61). The remarkable conglomerate that underlies the slate rocks near Llanberris, possessing a platy structure—"affected by slaty cleavage" (Ramsay, Mem. Geol. Surv. of Great Britain, Vol. iii., p. 145, fig. 53)—is another instance of the kind, but older still, being Lower Cambrian. The Llanberris conglomerate, it may be admitted, has undergone "intense pressure;" but some doubts may be entertained as to the view taken by Ramsay in connexion with its pebbles. They appear rather to have been *forced out* of their original parallelism with the bedding planes, than "elongated in the direction of the cleavage lines, and obliquely to the planes of bedding;" while their flattened form seems to be due more to abrasive action than to elongation by pressure. Ramsay speaks of this conglomerate as having undergone more or less alteration, and passing gradually into the condition of the adjoining porphyry. A large specimen before me shows the matrix here and there methylosed—changed into a green semi-translucent talc-schist, or imperfect serpentine; yet, the fact is, it contains, besides those of jaspery quartzite, &c., pebbles of *ordinary argillite unaffected by cleavage.* Have the latter pebbles been formed out of older beds, of which some, suspiciously like them in every respect, occur further southward, on the west side of Lleyn, Caernarvonshire, intercallated with metamorphic rocks? The beds of argillite I refer to, on account of their freedom from slaty cleavage, and their resemblance to some at Bray Head, may possibly yield Oldhamia if carefully examined.

EXPLANATIONS OF FIGURES IN THE PLATES.

Plate XXXIV.

This Plate represents meridianal jointing as displayed over a large space (consisting of bared Carboniferous limestone in horizontal beds), north of Finvarra, near New Quay, Co. Clare, and bordering on a portion of the south side of Galway Bay. The town of Galway, indicated by some light shading on the coast line, is situated on the opposite side of the bay. The diagonal lines represent an imperfectly developed set—west meridianal—of jointing. See page 616.

*** In the following figures, the red lines represent bedding, and the blue lines divisional planes—whether jointing or cleavage. The black lines denote outlines.

Plate XXXV.

Fig. 1. A segregated mass of fine shale in greywackite, the former showing laminar jointing parallel with a coarser kind in the latter. Kilbride, Lough Mask. See page 610.

Fig. 2. Specimen of shale—the remains of a decorticated Stigmaria, showing impressions of cleat, which characterised an investing layer of coal—originally the cuticle of this fossil plant. The long continuous lines represent the main divisional planes: these are met at right angles by others, which however in many cases do not cross them. See further account of this specimen in pages 610, 611, 632, 633, 634, and 635.

Fig. 3. Diagram sketch of the north side of Abbey Hill (795 feet high), near Corcomroo Abbey, Burren, Co. Clare, representing meridianal joints in their normal dip, as seen filled with snow from near Galway, right across Galway Bay, on March 19, 1867: distance nine miles. See page 612.

Fig. 4. Section of Devonian beds, as seen some years ago in a cutting on the Railway near Neyland, Milford Haven, Pembrokeshire, showing that stratial disturbance has only affected the jointing to the extent of throwing it out of its original dip. See page 618.

Fig. 5. Section of Carboniferous beds affected with jointing, as in the last figure. Ingleton Beck, Yorkshire. See page 618. The bedding is more inclined than represented in the figure.

Plate XXXVI.

Fig. 6. Meridianal (of which the bedded portion is a plane), and equatorial jointing, as seen in a quarry above Horeb Chapel, Brecknockshire.

Fig. 7. The same jointing. A portion of the meridianal of a laminar kind, which has been mistaken for "slaty cleavage," lies in a zone. The equatorial is horizontal. See page 619.

Plate XXXV.

Fig. 8. Surface of a bed of limestone (Carboniferous), showing meridiana and equatorial jointing. Near Ardfinnan. On account of the bed being inclined in the way it is, the equatorial joints have been thrown into a nearly horizontal position. See page 619.

Plate XXXVII.

Fig. 9. Section on the Limerick and Foynes Railway, showing black shales, *a*, and others more calcareous, *b*, (Carboniferous), affected with laminar jointing, *a*, which has become dislocated (not retaining its original dip relatively to the bedding), through the beds having been thrown into an inclined position: *c*, limestone not jointed. See page 620.

Plate XXXVIII.

Fig. 10. Diagram representing the flank of a mountain on the south side of Maam Valley (Connemara), showing a fossiliferous argillite (Upper Silurian) affected with dislocated laminar jointing. See page 620.

Fig. 11. Diagram representing a number of books, arranged in tiers, to illustrate a series of beds affected with jointing in its normal or original position.

Fig. 12. Diagram similar to the last, illustrating a series of beds which have undergone stratial disturbance, or have been affected by a lateral force or pressure exerted against the joint planes (sides of the books), denoted by the arrow; so that they still retain their original verticality, although the beds have been thrown into an inclined position. This diagram illustrates dislocated jointing. See page 621.

Plate XXXV.

Fig. 13. Representation of a thin layer of shale (between beds of flagstone), affected with dislocated jointing. Studfield quarry, near Ribblesdale, Yorkshire. See page 621.

Fig. 14. The same enlarged, showing the effect of lateral pressure on the upper and under surfaces of the layer of shale, causing them to be puckered into ripple-like ridges. See page 622.

Plate XXXVI.

Fig. 15. Dislocated jointing, called slaty cleavage, as seen in the cliffs from the South Stack Lighthouse, Holyhead.

Fig. 16. Similar.

Fig. 17. Dislocated jointing with a fan-like arrangement, as seen in the cliff of the South Stack Lighthouse. See page 622.

Plate XXXVII.

Fig. 18. Section seen in Ballyfouloo Glen, Co. Cork, showing brown shales (Devonian) affected with dislocated jointing, or what is usually called "slaty cleavage." See page 623. The beds are marked with dislocated jointing at the north end of the glen—not the south end, as stated by mistake in page 623.

Plate XXXVIII.

Fig. 19. Section seen in a quarry near Bandon, showing dislocated laminar jointing ("slaty cleavage") associated with typical jointing, the former under instructive conditions. See pages 625 and 630.

Plate XXXVII.

Fig. 20. Section seen in a quarry of Carboniferous limestone at Shanbally, Co. Cork, showing meridianal and equatorial jointing (the latter nearly horizontal), also a platy divisional structure produced by pressure, and belonging to another set (dislocated) of equatorial jointing. See page 628.

Fig. 21. Section in a quarry of similar limestone at Black Rock, near Cork, showing jointing—meridianal (vertical), another meridianal set (oblique), and equatorial (horizontal). See page 629.

Plate XXXVIII.

Figs. 22, 23, 24, 25, and 26. These figures are diagram-representations of beds, supposed to be affected by successive waves of disturbance, to show how the jointing in the Bandon quarry has been made to assume its different characters and positions. See page 630.

Plate XXXV.

Fig. 27. Section of a block of slate-rock from Penrhyn, Caernarvonshire, showing that its cleavage structure is not coincident with the lie of its grain (distinguished in the rock by a flaky structure), as required by pressure exerted perpendicularly to what are now the cleavage planes. The lines parallel with the long sides of the figure represent cleavage; the oblique lines the grain of the rock.—See page 649.

END OF VOL. XXV.—SCIENCE.

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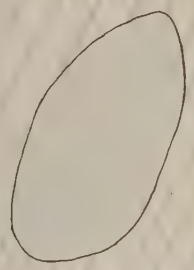

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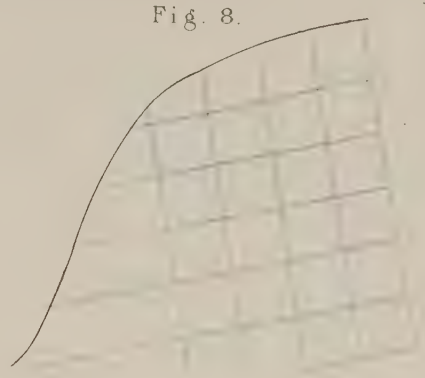

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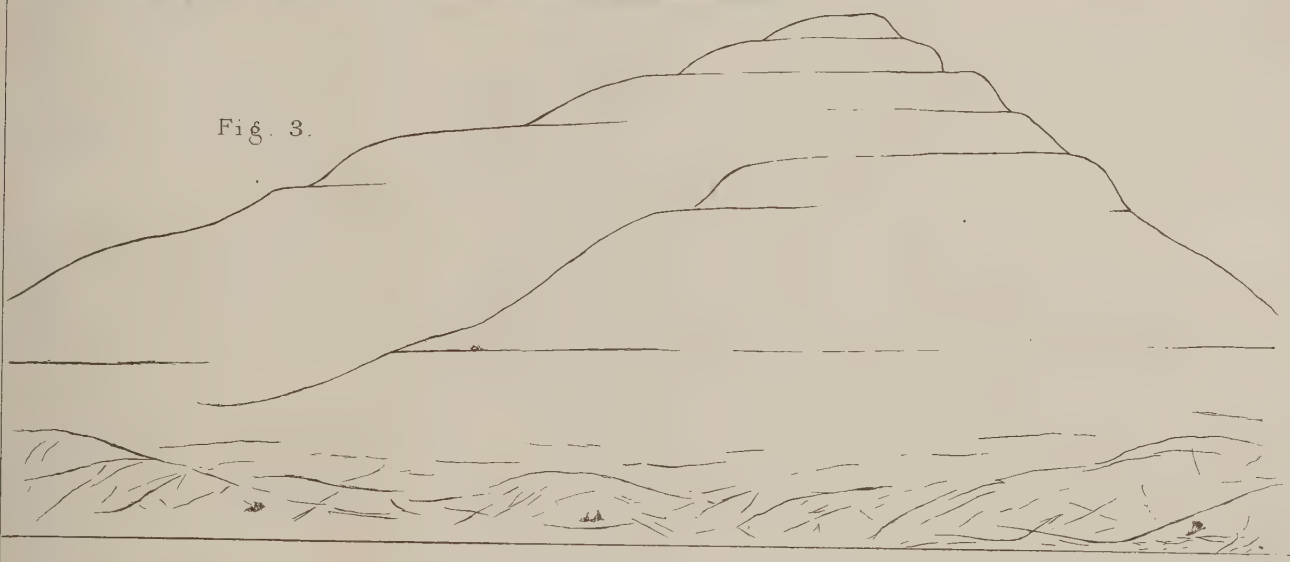

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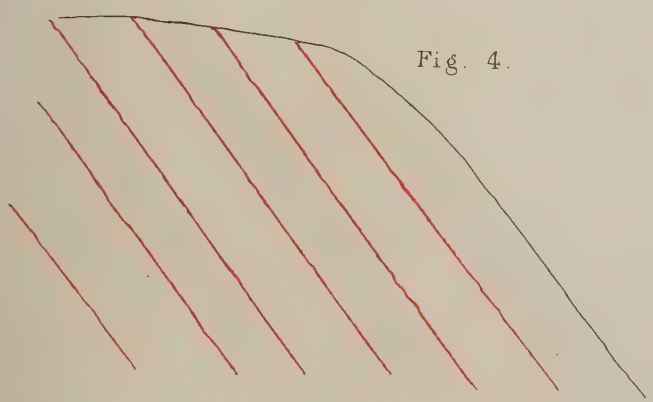

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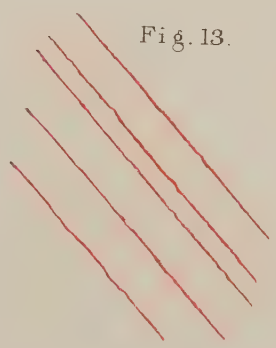

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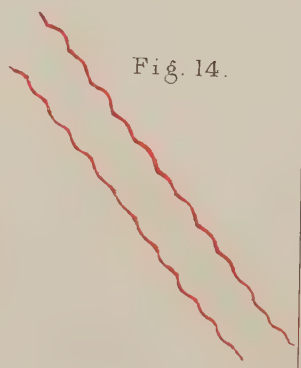

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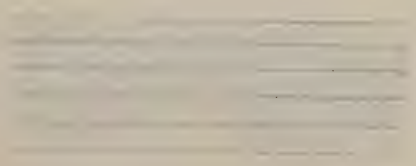

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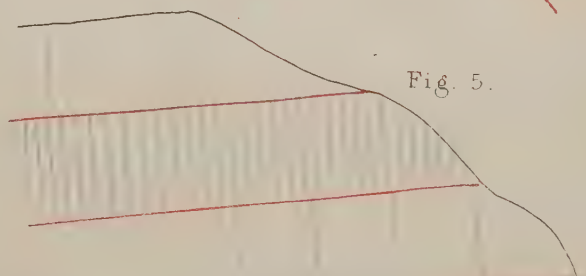

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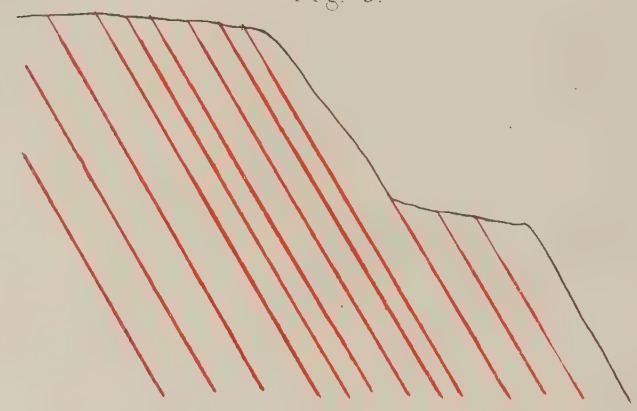

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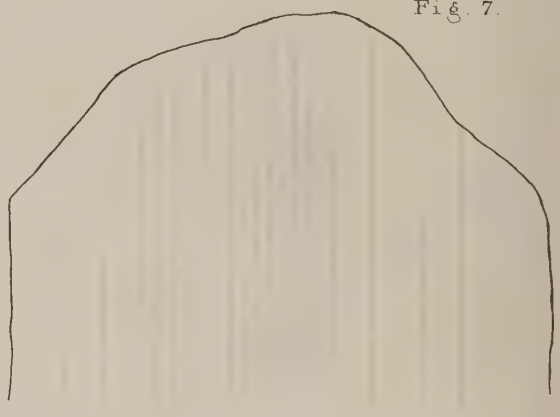

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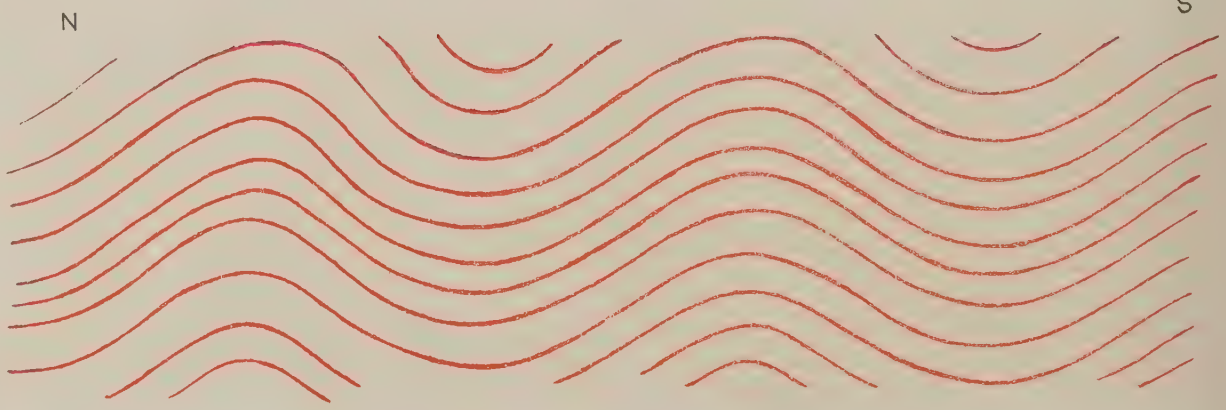

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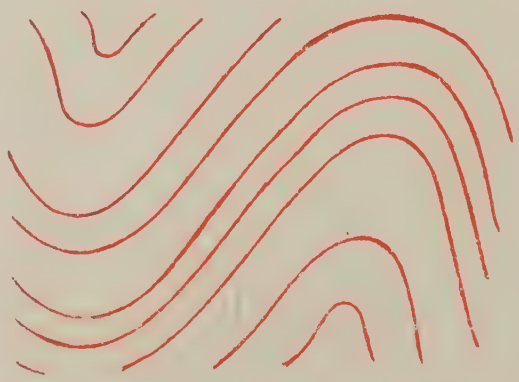

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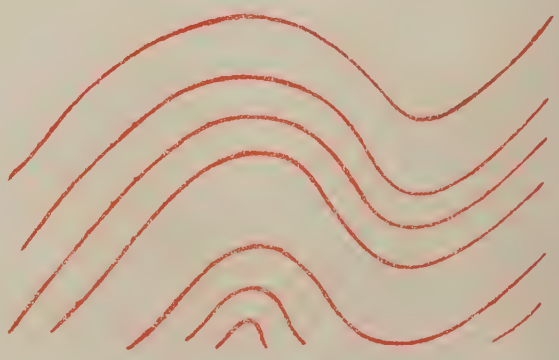

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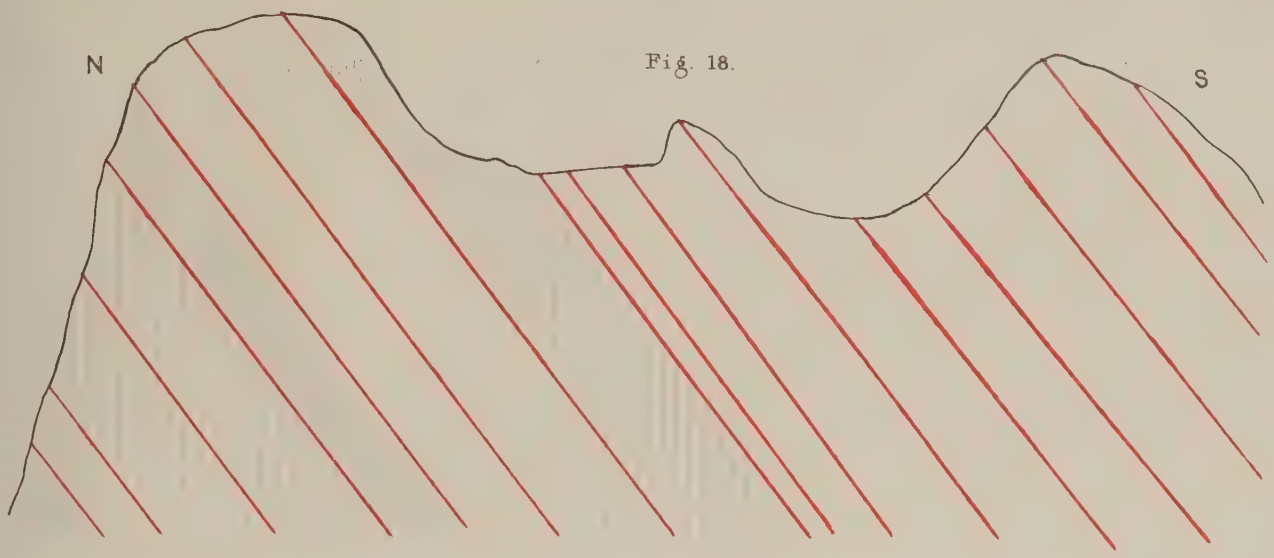

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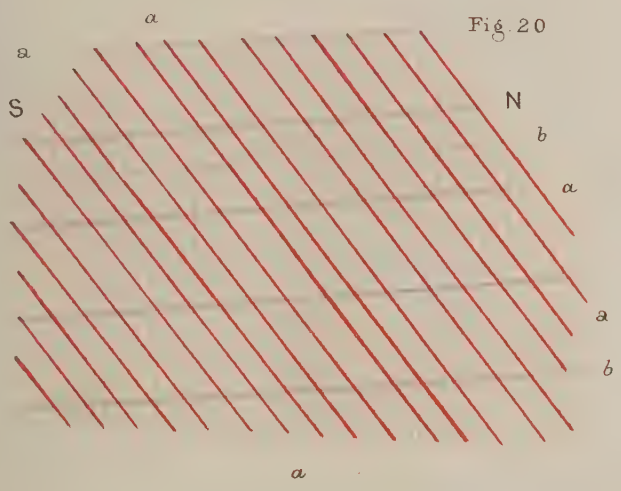

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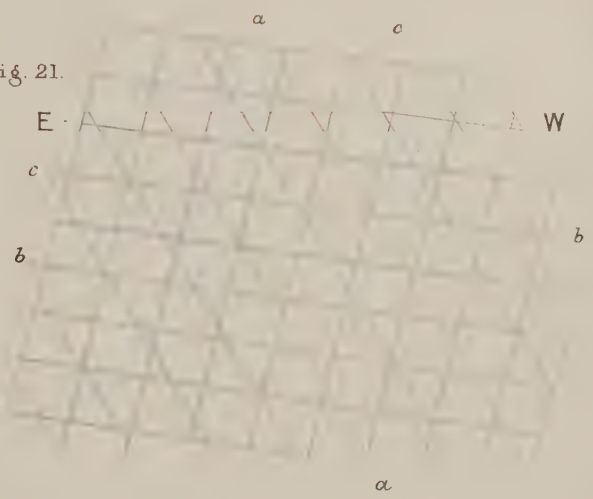

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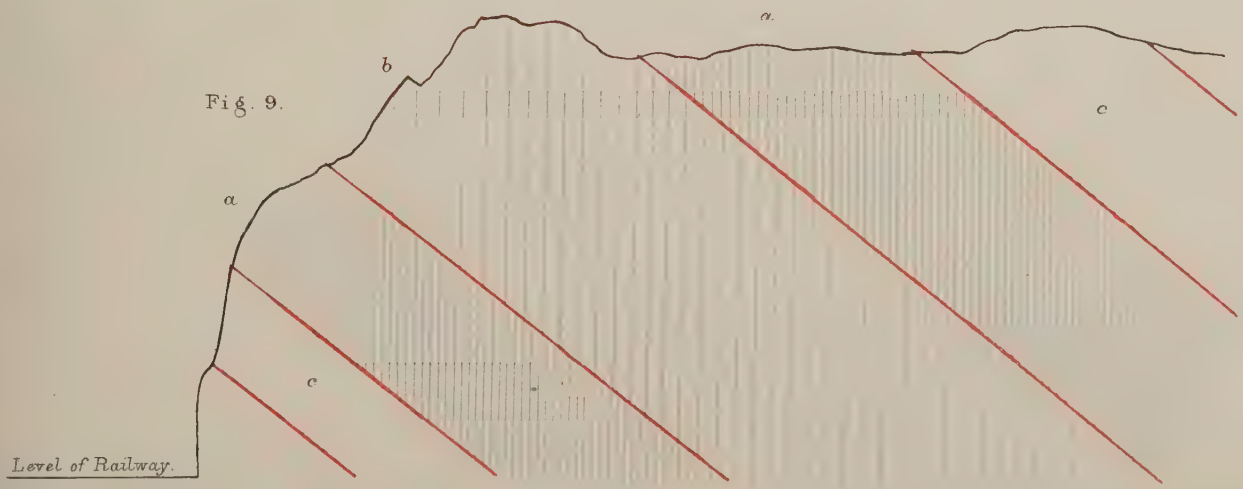

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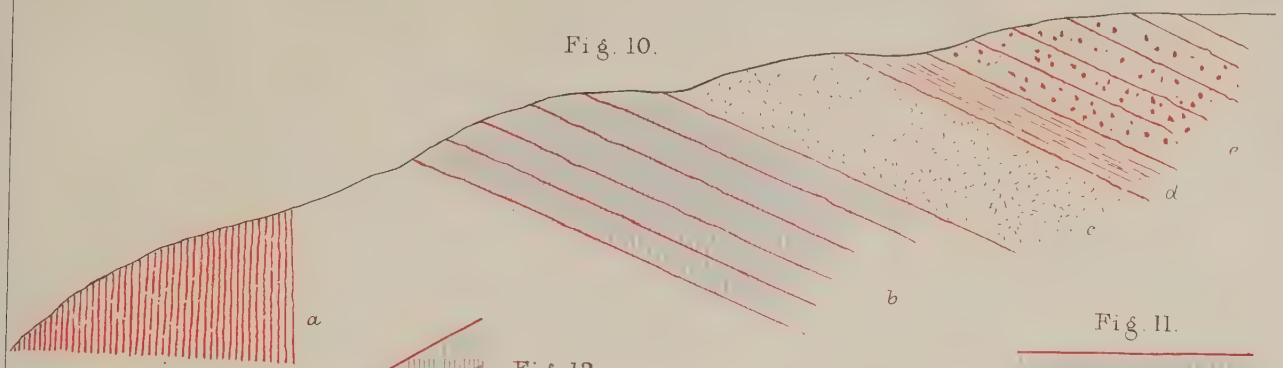

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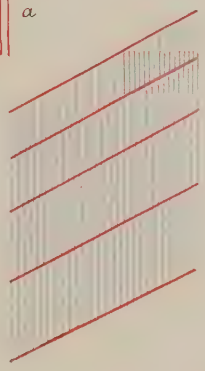

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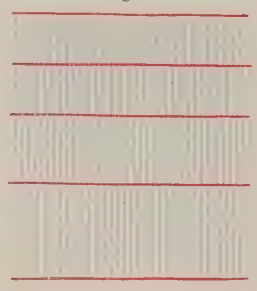

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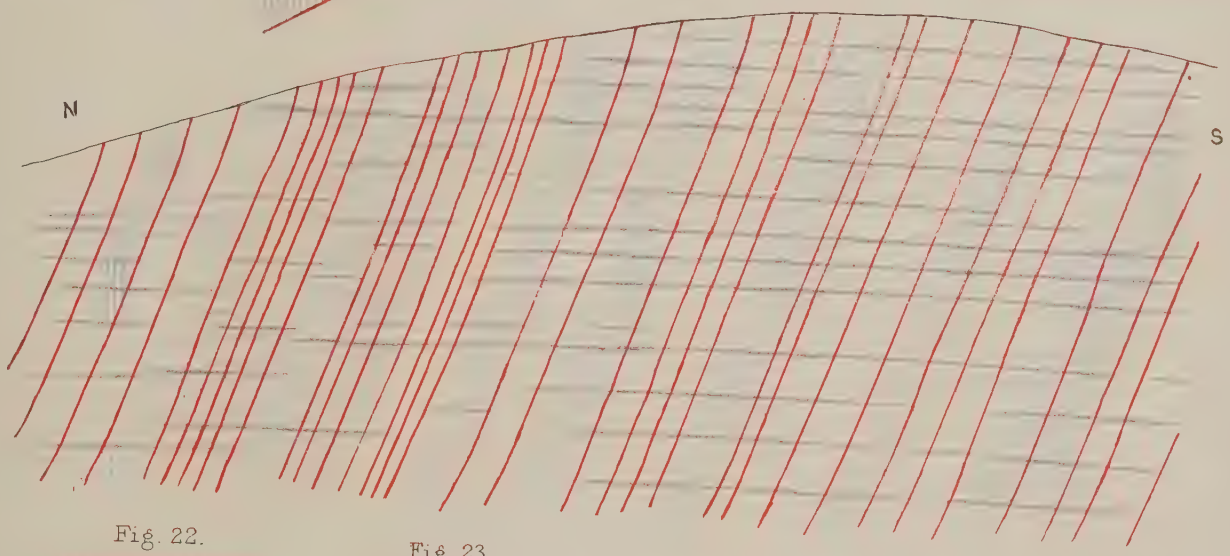

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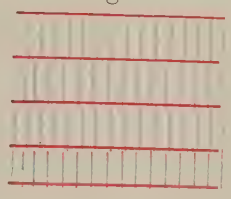

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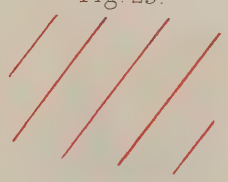

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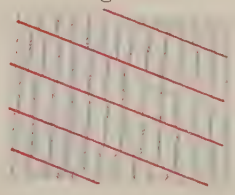

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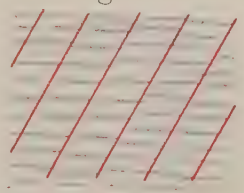

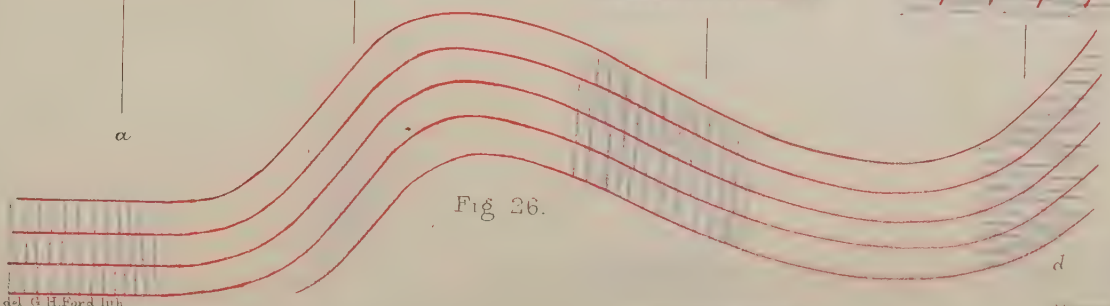

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CONTENTS.

PAGE.

The Theory of Screws: a Geometrical Study of the Kinematics, Equilibrium, and small Oscillations of a Rigid Body. By ROBERT STAWELL BALL, LL.D., M.R.I.A., Professor of Applied Mathematics and Mechanism, Royal College of Science for Ireland, ... 157

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